Optimal Design of Corporate Market Debt Programmes in the Presence of Interest-Rate and Inflation Risks

March 2011
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Foreword


The purpose of this research chair, led by Lionel Martellini, Scientific Director of EDHEC-Risk Institute, is to support research undertaken at EDHEC-Risk on the benefits of inflation-linked bonds from the issuers’ perspective as well as from the investors’ perspective. The chair also focuses on comparing and contrasting issuers’ and investors’ perceptions of inflation-linked bonds.

The current paper introduces a general framework with which a corporation subject to default risk may make optimal debt-management decisions. It attempts to answer the following question: given an exogenous revenue process for a corporation, what is the optimal liability structure when the issuer faces such instruments as fixed-rate debt, floating-rate debt, and inflation-linked debt?

The main contribution of this paper is to provide a joint quantitative analysis of capital-structure decisions and debt-structure decisions within a standard continuous-time model in the presence of interest-rate and inflation risks. The main findings are that debt management has an impact on capital structure and that an optimal debt structure can facilitate substantial increases in firm value. We also find that a number of corporations would benefit from issuing inflation-linked bonds, bonds usually associated with sovereign states.

In the current context of increasing inflation uncertainty, I would like to thank the co-authors, Lionel Martellini and Vincent Milhau, for their comprehensive analysis of optimal corporate debt-management policies. We trust that you will find the paper useful and will continue to monitor and contribute to our research in this area.

We would also like to extend our warm thanks to our partners at Rothschild & Cie for their collaboration on the project and their support of the research chair.

We wish you a pleasant and informative read.

Noël Amenc
Professor of Finance
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About the Authors

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Abstract
Abstract

This paper provides a joint quantitative analysis of capital structure decisions and debt structure decisions within a standard continuous-time capital-structure model. In the presence of interest rate and inflation risks, we are able to obtain quasi-closed form expressions for the price of various forms of indexed- and non-indexed bonds issued by the firm, which allows us to generate computationally efficient estimates for the optimal debt structure. Our analysis shows that debt-structure decisions have a strong impact on capital structure decisions. It also suggests that substantial increases in firm value can be generated by optimal debt structures.
Executive Summary

Inflation-Linked Bonds in the Corporate World

A recent surge in inflation uncertainty has whetted investor appetite for appropriate hedges. Inflation hedging is now of critical importance to pension funds—pension payments are often indexed to consumer price or wage indices—as well as to private investors, who consider inflation a direct threat to their purchasing power. Inflation-linked securities, first introduced by sovereign states, have been introduced in response to rising demand for inflation hedging. Although inflation-linked debt is still most closely associated with sovereign states, state-owned agencies, municipalities, and corporations—utilities and financial-services companies, in particular—are also expressing interest in it. In fact, intuition suggests that if a firm’s revenues grow with inflation, issuing some inflation-linked debt can be a natural hedge.

It is perhaps surprising, then, that some large corporations still do not issue inflation-linked bonds. This situation may be explained in part by the common belief that debt management should be governed by the desire to reduce the cost of debt financing. In particular, the standard argument suggests that a corporation should seek to issue fixed-rate debt if it expects an increase in interest rates and floating-rate debt otherwise. A similar intuition suggests that the firm should issue nominal bonds if it expects an increase in inflation and inflation-linked bonds if it expects a decrease. In this context, inflation-linked bonds would not seem attractive from the issuer’s perspective, since the cost of debt servicing would be expected to increase with inflation. This seemingly straightforward line of reasoning suffers, however, from one fatal flaw: the difference between fixed and floating (versus real) rates merely reflects market expectations and a risk premium, so the only non-trivial impact may come from the chief financial officer’s active views if they deviate from market expectations. In the end, the purpose of a corporation is arguably not to make profits by trading in financial markets.

On the Relevance of Debt Management

In this paper, we introduce a general framework with which a corporation subject to default risk may make optimal debt-management decisions. We attempt to answer the following question: given an exogenous revenue process for a corporation, what is the optimal liability structure when the issuer faces such instruments as fixed-rate debt, floating-rate debt, and inflation-linked debt? In fact, this problem is the exact counterpart of the standard asset/liability management problem for a pension fund, in which liabilities are exogenously given while it is the allocation decision that is optimised.

Although the theory of asset allocation decisions is relatively well understood, our understanding of liability management is comparatively limited. Two separate strands of the corporate-finance literature—the dynamic capital-structure literature, which has abstracted away from debt allocation to focus more closely on optimal capital-structure decisions in tractable quantitative settings, and the risk-management literature, which has provided some (mostly qualitative) analysis of the debt-management problem in isolation from...
the capital-structure problem–have, as it happens, dealt primarily with optimal corporate liability allocation decisions. The main contribution of our paper is to do an initial joint quantitative analysis of capital-structure and debt-management choices in a unified framework to tie together these two somewhat disparate strands of the literature.

A Formal Capital- and Debt-Structure Model
We show that debt-management decisions can be formally analysed in the context of a dynamic capital-structure model, with a tradeoff between the (bankruptcy) costs and (tax shield) benefits associated not only with leverage but also with debt-structure decisions. To do so, we abstract away from problems of agency and asymmetric information, and consider competing forms of liability classes (fixed-rate bonds, floating-rate bonds, and inflation-indexed bonds, in addition to equity) in a relatively rich stochastic environment involving interest-rate and inflation risks. Although the non-independence of default risk and interest-rate risk turns out to be a great complication, we have been able to obtain quasi-closed-form expressions for the price of both indexed and non-indexed defaultable bonds by focusing on a setting in which the distance-to-default is a log-normal process. The presence of these quasi-analytical expressions allows us to generate computationally efficient estimates for the optimal debt structure.

In an attempt to increase shareholder wealth, we find that the managers of the firm should seek to immunise debt servicing from exposure to interest-rate and inflation risk. In fact, what matters is not so much the variability of debt servicing as the volatility of firm cash flows net of debt payments. On the one hand, decreasing the share of fixed-rate bonds increases uncertainty about debt servicing since interest payments on floating-rate and inflation-linked bonds are uncertain. On the other hand, the increase in the volatility of the promised repayment may lead to an increase in the correlation between changes in liability and asset values if the correlation of asset values and interest rates or inflation is positive. In other words, issuing floating-rate bonds or inflation-linked bonds may increase risk from the perspective of pure debt management, but it may decrease risk from the perspective of integrated asset/liability management. From this tradeoff emerges an optimal debt structure, and one can show that under (mild) simplifying assumptions, minimising the volatility of assets net of liabilities is equivalent to minimising the (risk-adjusted) probability of default, which is in turn equivalent to maximising firm value.

Numerical Estimates of Debt-Management Benefits
We thus find that the optimal share of floating-rate bonds increases with the correlation between changes in interest rates and changes in the revenues of the firm. When the correlation of a firm’s operating cash flows (before interest expenses) and interest rates is positive, its floating-rate debt should be made to account for a greater share of its total debt to avoid the high (bankruptcy) costs associated with low cash flows and high debt servicing. When, on the other hand, this correlation is negative, floating-rate

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debt should account for a smaller share of total debt. Similarly, the optimal share of inflation-linked bonds increases when the correlation of changes in inflation rates and changes in the revenues of the firm rises. On the whole, we find that optimising the structure of debt leads to a smaller probability of default and thus to higher firm value. One key conclusion that we obtain is that debt-management decisions have a strong positive impact on firm value. Another is that, for reasonable parameter values, corporations should issue a non-zero share of inflation-linked (IL) bonds. We also find that the opportunity costs associated with failing to issue IL bonds are substantial. From an implementation perspective, one could also use derivatives to adjust interest rate and inflation risks, exposure to, but derivatives would not be the natural approach for long horizons in the presence of counterparty risk.

Risk and Asset/Liability Management for Corporations

From the normative standpoint—in other words, from the perspective of a firm seeking to maximise its value—the hedging motive is the key determinant of debt-management decisions: this is the conclusion of our statistical analysis of the relationship between the debt structure that explicitly maximises firm value and that which minimises the volatility of assets net of liabilities. Hence, the main benefit of optimising the debt structure is to allow firms to reduce the variability of their net cash flows and therefore to lower the probability of default. In other words, the main motive for debt management is not to lower the cost of debt financing but to hedge exposure to interest-rate and inflation risks. In fact, by matching the interest-rate and inflation exposure of the liabilities to that of the assets, the managers of a firm can lower the variability of cash flows. The likelihood of default—as well as the the cost of debt—is thus lowered; equity value increases.

For an intuitive understanding of the reasons specific risk factors in asset returns matter, consider the problem of optimal issuance of inflation-indexed bonds by corporations. Issuing inflation-indexed bonds leads to a reduction in the cost of debt since the issuing party is selling insurance against inflation and receives the associated premium. On the other hand, issuing inflation-indexed bonds rather than nominal bonds increases uncertainty in financing costs because of heightened uncertainty in coupon payments. This cost/risk tradeoff is the liability-management counterpart of the risk/return tradeoff in asset allocation. Taking into account the assets of the firm, however, leads to a different appreciation of the impact of increasing the share of inflation-indexed bonds. Indeed, because firms’ operating cash flows are often positively related to changes in inflation, increases in inflation do not necessarily lead to decreases in net revenues for the issuer. In other words, although issuing inflation-indexed bonds leads to an increase in risk from a pure liability perspective, it is not necessarily more risky from a perspective of asset/liability management. Inflation-indexed debt, then, would appear to have risk-and-return properties superior to those of nominal debt, and the optimal composition of a debt portfolio will be affected accordingly. In other words, intuition suggests that optimal debt structure should stem not solely from a preoccupation with minimising
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the cost of debt but also from efforts to hedge the risks to the revenues of the firm.

Conclusions and Extensions

The literature on liability management provides quantitative guidance for capital-structure decisions (allocation to equity and debt), but when it comes to debt-structure decisions (choice of fixed- and floating-rate debt), most insight is purely qualitative. The main contribution of this paper is to provide a joint quantitative analysis of capital-structure decisions and debt-structure decisions within a standard continuous-time model in the presence of interest-rate and inflation risks. Our main findings are that debt management has an impact on capital structure and that an optimal debt structure can facilitate substantial increases in firm value. We also find that a number of corporations would benefit from issuing inflation-linked bonds, bonds usually associated with sovereign states.

Our analysis could be further extended in a number of useful directions. It would be useful to include other instruments, such as convertible bonds, preferred shares, and other equity-linked structures in the liability mix, in addition to fixed-rate and floating-rate bonds. On a different note, our model, like much of the literature, considers the liability-allocation problem from the standpoint of the original owners of the firm, who are assumed to be risk-neutral with respect to the (diversifiable) source of uncertainty impacting firm value. In practice, however, the managers of the firm, unlike the owners, are in charge of making corporate risk-management and liability-allocation decisions. Academic research has documented the role of conflicts of interest and managerial incentives in corporate debt structure; incorporating the impact of these conflicts and incentives would thus be of interest.
Les obligations indexées dans l’univers corporate

Le récent accroissement des incertitudes liées à l’inflation a renforcé chez les investisseurs la volonté de se prémunir contre les variations aléatoires des prix. La couverture de l’inflation est même devenue une préoccupation essentielle des fonds de pension, dans la mesure où les retraites que ceux-ci doivent verser sont souvent indexées par contrat sur les prix ou sur les salaires. Les investisseurs particuliers se sentent également concernés par une inflation qu’ils perçoivent comme une menace pour leur pouvoir d’achat. Les obligations indexées ont été introduites en réponse à ce besoin croissant de couverture contre l’inflation, d’abord par les états souverains. Même si la majeure partie de la dette indexée est toujours d’origine souveraine, d’autres émetteurs se sont récemment montrés intéressés, à l’image de certains organismes publics et municipalités, mais aussi d’entreprises, en particulier dans le secteur des infrastructures et des services financiers. L’intuition suggère en fait que si les revenus d’une entreprise tendent à augmenter avec le niveau général des prix, émettre une fraction non nulle de dette indexée est un moyen naturel de se couvrir contre le risque d’inflation.

Il apparaît dès lors surprenant que certaines grandes entreprises restent en marge de ce mouvement, et continuent à définir leur structure de dette sans recourir aux obligations indexées. Cet état de fait tient probablement au moins en partie à l’idée généralement admise selon laquelle un émetteur devrait gérer sa dette de manière à en réduire le coût. Dans cet ordre d’idées, un argument souvent avancé est qu’une entreprise doit émettre de la dette à taux fixe si elle s’attend à une hausse des taux d’intérêt, et de la dette à taux variable dans l’hypothèse du contraire. De la même manière, si elle s’attend à une hausse significative du niveau des prix, elle doit privilégier la dette nominale, tandis qu’une prévision d’inflation négative doit logiquement la conduire à émettre davantage d’obligations indexées. De ce point de vue, les obligations indexées ne présentent guère d’intérêt pour l’émetteur, puisque celui-ci s’attend à voir le coût de sa dette s’alourdir quand les prix augmentent. Ce raisonnement en apparence parfaitement fondé souffre pourtant d’un présupposé faux : en effet, les différences entre le taux fixe, le taux nominal variable et le taux réel demandés par les investisseurs sont simplement dues à des primes de risque qui reflètent les anticipations du marché. De ce fait, le gérant de la dette ne peut, par ses décisions, avoir de contrôle sur le coût de celle-ci que si ses propres anticipations diffèrent de celles du marché. Pour résumer, on peut à bon droit soutenir que la raison d’être d’une entreprise n’est pas de gagner de l’argent par des opérations sur les marchés financiers.

De l’importance de la gestion des risques

L’article propose un cadre de travail général permettant d’analyser les décisions prises en matière de structure de dette par une entreprise susceptible de faire défaut. De manière plus précise, les auteurs cherchent à répondre à la question suivante : étant donné un flux de revenus, quelle est la structure de passif optimale pour un émetteur qui peut émettre plusieurs types d’obligations, notamment des obligations à taux fixe ou variable ou des obligations indexées ? Ce problème se présente en
Résumé

quelque sorte comme le « symétrique » du problème de gestion actif-passif posé à un fonds de pension : dans le cas de celui-ci, c'est le passif qui est donné, et c'est la structure de l'actif qui doit être optimisée. La gestion d'actif est un sujet désormais relativement bien maîtrisé du point de vue théorique. La littérature fournit des recommandations en la matière dans toute une gamme de situations parfois complexes, comme celles qui mettent en jeu des variations aléatoires des paramètres de risque et de rendement au cours du temps, voire de l'incertitude sur les valeurs de ces paramètres. En comparaison, la compréhension des effets induits par la gestion de passif apparaît bien plus limitée. En ce qui concerne les entreprises, il est possible de distinguer deux grands types de travaux. Une première branche porte sur la structure du capital, mais laisse de côté l'étude de la structure de la dette afin de pouvoir travailler avec des modèles mathématiques maniables. Une deuxième branche, au contraire, s'est intéressée à la gestion de la dette, mais surtout d'un point de vue qualitatif, et sans tenir compte de la structure du capital. Le principal apport de l'article à la littérature est de réconcilier ces points de vue, jusqu'ici assez nettement séparés, en proposant une analyse conjointe des structures du capital de la dette, menée dans un cadre de travail unifié.

Une modélisation des structures du capital et de la dette

Les auteurs montrent qu'il est possible d'analyser de manière formalisée la question de la gestion de la dette en travaillant avec des modèles de structure du capital semblables à ceux qui ont été proposés dans la littérature. Dans ces modèles, la structure du capital optimale provient de la confrontation entre deux effets contradictoires : un accroissement de l'endettement entraîne une hausse des coûts de faillite, mais génère aussi des économies d'impôt. Le modèle présenté dans l'article associe chacun de ces effets non seulement au niveau de la dette, mais aussi à sa structure. Tout en écartant les problèmes de conflits d'agence et de conflits d'intérêt, il tient compte de la présence de diverses formes sources de financement : obligations à taux fixe ou variable et obligations indexées, en plus des actions. Il suppose aussi les taux d'intérêt et l'inflation aléatoires. Ces caractéristiques, ajoutées à l'existence du risque de défaut, rendent a priori particulièrement ardue la résolution mathématique du modèle. Les auteurs parviennent cependant à obtenir des expressions quasi-explicites pour les prix des obligations indexées ou non soumises au risque de défaut. Mathématiquement parlant, la relative maniabilité du modèle est assurée par le fait que le processus de distance au défaut a une distribution log-normale. De telles formules permettent d'obtenir rapidement des estimations de la structure optimale de la dette par des méthodes numériques.

La principale conclusion est que s'ils cherchent à créer de la valeur pour les actionnaires, les dirigeants de l'entreprise doivent s'efforcer de couvrir les risques de taux d'intérêt et d'inflation auxquels sont soumis les paiements d'intérêts. Il s'avère en fait que le paramètre essentiel n'est pas tant la volatilité du coût de la dette que la volatilité des revenus de la firme nets du paiement des intérêts. Par exemple, il est vrai qu'une diminution de la part allouée aux obligations à taux fixe entraîne une
augmentation de la volatilité de la charge de la dette, dans la mesure où les intérêts payés sur la dette à taux variable et sur la dette indexée ne sont pas connus d’avance. Mais il est également vrai que si la corrélation entre les revenus et les taux d’intérêt ou l’inflation est positive, l’augmentation de la part allouée aux obligations à taux variable ou indexées qui se produit simultanément conduit à une augmentation de la corrélation entre les revenus et les intérêts à payer. En d’autres termes, émettre davantage d’obligations à taux variable ou indexées est une stratégie plus risquée en gestion de passif pure, mais peut se révéler au contraire une stratégie moins risquée si l’on élargit le point de vue à la gestion actif-passif. La tension entre ces deux effets opposés produit une structure de dette optimale. Il est même possible de montrer, sous des hypothèses légèrement modifiées, que minimiser la volatilité de l’actif net du passif, minimiser la probabilité de défaut (évaluée sous la mesure risque-neutre), et maximiser la valeur de l’entreprise, sont trois objectifs équivalents.

Estimations des bénéfices procurés par la gestion de la dette
Les résultats mettent en évidence que la proportion optimale d’obligations à taux variable augmente avec la corrélation entre les taux d’intérêts et les revenus de l’entreprise. Ainsi, un émetteur dont les revenus (mesurés avant prise en compte du service de la dette) sont positivement corrélés avec les taux d’intérêt devrait émettre davantage d’obligations à taux variable qu’une entreprise qui présenterait les mêmes caractéristiques mais avec une corrélation moindre, et ce afin d’éviter les situations dans lesquelles les revenus sont faibles et la charge d’intérêts trop importante. De telles conditions de marché pourraient en effet conduire au défaut, avec les coûts de faillite qui y sont inéluctablement associés. De la même manière, la part optimale d’obligations indexées croît avec la corrélation entre le taux d’inflation et le taux de croissance des revenus. Pour résumer, les résultats montrent donc que par l’optimisation de la structure de la dette, il est possible de réduire la probabilité de faillite et donc d’augmenter la valeur de l’entreprise. C’est une des conclusions essentielles de l’article : les choix relatifs à la gestion de la dette ont un effet significatif sur la valeur. Une autre conclusion importante se dégage : pour des valeurs réalistes des paramètres décrivant sa structure de revenus, l’entreprise devrait émettre des obligations indexées, et le fait de s’en priver induit des coûts non négligeables. D’un point de vue pratique, il est toujours possible d’utiliser des dérivés pour atteindre la bonne exposition aux risques de taux et d’inflation, mais lorsque l’horizon de l’émetteur est éloigné, cette technique pose le problème du risque de contrepartie.

Gestion des risques et gestion actif-passif (ALM) des entreprises
Les résultats suggèrent que le souci de gestion des risques devrait orienter les choix faits en matière de structure de la dette. Les auteurs développent cette intuition en effectuant une analyse statistique du lien entre d’une part la structure qui résulte de considérations de gestion des risques, et d’autre part celle qui maximise la valeur de l’entreprise. Il en ressort que le bénéfice à attendre
d'une optimisation de la structure de la dette est une réduction de la volatilité des cash-flows nets, et donc de la probabilité de faillite. En d’autres termes, la gestion de la dette ne devrait pas tant viser la réduction des coûts de financement que la couverture des risques de taux et d’inflation : en ajustant la composition de la dette de manière à ce que les sensibilités de l’actif et du passif à ces risques soient équivalentes, les dirigeants peuvent réduire la volatilité des cash-flows. Cette réduction se traduit d’une part par une diminution de la probabilité de défaut, donc du coût de financement par la dette, et d’autre part par une augmentation de la valeur des actions.

L’exemple de l’émission d’obligations indexées par une entreprise aidera à comprendre intuitivement en quoi l’existence de facteurs de risques spécifiques d’actif est importante. Proposer de telles obligations permet de réduire le coût attendu de la dette, dans la mesure où l’émetteur vend une assurance contre l’inflation, et reçoit en échange la prime de risque correspondante. Cependant, la charge d’intérêts devient plus incertaine qu’elle ne le serait avec des obligations nominales. L’émetteur est alors confronté à un dilemme entre le coût de financement attendu et l’incertitude sur le coût ex-post : c’est là l’équivalent, en gestion de passif, du dilemme entre rendement et risque qui apparaît en gestion d’actif. La prise en compte des actifs modifie sensiblement le contexte. En effet, puisque les revenus de l’entreprise sont souvent positivement corrélés avec l’inflation, il n’est pas nécessairement vrai qu’un accroissement des prix se traduise par une baisse des revenus nets de la charge d’intérêts. On peut exprimer cette idée en disant que si l’émission d’obligations indexées est une stratégie de gestion de passif plus risquée, elle n’est pas toujours une stratégie de gestion actif-passif plus risquée. De ce point de vue, les obligations indexées sont supérieures aux obligations nominales, ce qui devrait être pris en compte dans la composition de la dette. Plus généralement, l’intuition suggère qu’une structure de dette optimisée devrait être définie de manière à compenser l’exposition aux facteurs de risques qui affectent les revenus de l’entreprise, plus que dans le but de minimiser le coût de financement.

**Conclusions et perspectives**

L’étude de la gestion de passif se limite en général à celle de la structure du capital, c’est-à-dire à celle du choix entre financement obligataire et financement par actions. Quant à la question du choix d’une structure de dette, c’est-à-dire de l’allocation entre obligations à taux fixe ou variable ou indexées, elle n’est dans la plupart des cas abordée que sous l’angle qualitatif. La principale contribution de l’article est de mener une analyse quantitative conjointe des structures du capital et de la dette en se plaçant dans un modèle en temps continu semblable à ceux proposés dans la littérature, et en incorporant les risques de taux et d’inflation. Les conclusions essentielles sont que les choix de structure de la dette ont un effet sur les décisions à prendre en matière de structure du capital, et que des augmentations substantielles de la valeur de l’entreprise peuvent être obtenues par une gestion optimisée de la dette. De plus, il apparaît que bon nombre d’émetteurs corporate gagneraient à émettre de la dette indexée, rejoignant ainsi les états souverains qui fournissent...
aujourd'hui le plus gros contingent de ce type d'obligations.

Plusieurs prolongements à cette étude peuvent être envisagés. Tout d'abord, il serait utile d'introduire dans l'analyse d'autres types d'instruments financiers en sus des obligations à taux fixe ou variable considérées ici : il pourrait s'agir d'obligations convertibles, d'actions de priorité, ou de titres dont le paiement à échéance serait indexé sur les actions de l'entreprise. Par ailleurs, il convient de souligner que le modèle tel qu'il est présenté ici privilégie le point de vue des propriétaires de la firme, suivant en cela les travaux antérieurs. Ces agents sont supposés avoir la possibilité d'annuler leur exposition au risque lié à la valeur de l'entreprise, et donc être finalement indifférents à ce risque. En réalité, cependant, ce n'est pas à eux qu'est confiée la responsabilité des choix en matière de gestion des risques et de structure de la dette : ceux-ci incombent en effet aux dirigeants. Comme l'a montré la recherche sur ce sujet, les conflits d'intérêt et les incitations offertes à ces dirigeants prennent dès lors une importance non négligeable. La prise en compte de leurs effets dans le modèle fournirait la matière d'une autre extension intéressante.
1. Introduction
1. Introduction

Although the theory of asset allocation decisions is relatively well understood—prescriptions may involve a stochastic opportunity set or the presence of parameter uncertainty—our understanding of liability-management decisions is comparatively limited. Two separate strands of the corporate finance literature—the dynamic capital-structure literature, which has abstracted away from debt-allocation decisions to focus more closely on optimal capital-structure decisions in tractable quantitative settings, and the risk-management literature, which has provided some (mostly qualitative) analysis of the debt-management problem in isolation from the capital-structure problem—have, as it happens, dealt primarily with optimal corporate liability-allocation decisions.

For the early literature on capital structure, corporate-financing policy and the choice of liability structure are irrelevant in the absence of contracting costs and taxes; this is the fundamental insight of the Modigliani-Miller theorem (Miller and Modigliani 1958). Hence, the introduction of frictions provides one natural possible justification for a non-trivial capital-structure choice that is based on the tradeoff between the tax benefit of debt and the bankruptcy costs of debt. An initial elegant quantitative analysis of this tradeoff theory of capital structure was done by Leland (1994) in a model in which perpetuity was the single class of debt. In an attempt to account for more realistic debt characteristics, Leland and Toft (1996) extend Leland (1994) to examine the effect of (an exogenously assumed) finite debt maturity on bond prices, credit spreads, and optimal leverage. In all these (as well as other related) papers, however, corporate debt is represented exclusively by a fixed-coupon bond. In other words, these papers discuss the choice of the optimal mixture of debt and equity, but are silent on the optimal allocation to various types of debt instruments. Because most of these papers assume away interest-rate and inflation risk, there is in fact no role and no need for interest-rate or inflation hedging in such models, and as a consequence no need for floating-rate or inflation-linked bonds in the menu of liability classes. There is at least one example (Ju and Ou-Yang 2006) of a dynamic capital-structure model with stochastic interest rates, but the assumption of a single class of fixed-coupon debt is maintained for tractability and to focus on the derivation of the optimal maturity of the bond. This assumption is clearly at odds with corporate practice, in which several classes of debt, including floating-rate bonds, are routinely issued. Focusing on floating-rate debt only, as can be seen from table 1, it appears that around 40% of the firms in the Compustat database issue floating-rate debt, and that the median proportion of long-term floating-rate debt in total long-term debt exceeds 60%. It can even be seen that, for at least 25% of the companies, floating-rate debt is the only form of long-term debt! One of the objectives of this paper, then, is to extend the literature on dynamic capital structure to realistic settings involving both interest-rate and inflation risks (or either risk); it focuses specifically on a quantitative analysis of the associated choices for the structure of market debt.
1. Introduction

risk-management literature. **Hedging theories** posit that, by matching the interest-rate exposure of the liabilities to that of their assets, firms can reduce the variability of their cash flows and as a result lower their expected costs of financial distress or benefit from a greater shield (Smith and Stulz [1985] and Leland [1998]).

Hedging also allows firms to minimise the frequency with which they must raise costly external capital (Froot et al. 1993). These hedging theories stand in sharp contrast to **market-timing theories**, which posit that debt-management decisions are governed by **speculative** motives, with a focus on lowering the expected cost of debt servicing.\(^7\) In principle, of course, hedging and speculative motives are not mutually exclusive, and an optimal blend of debt instruments can be designed in an attempt to optimise the risk/cost tradeoff.

Although the non-independence of default risk and interest-rate risk turns out to be a great complication, we have been able to obtain quasi-closed-form expressions for the price of both indexed- and non-indexed defaultable bonds by focusing on a setting in which the distance-to-default is a log-normal process. The presence of these quasi-analytical expressions allows us to generate computationally efficient estimates for the optimal debt structure. We find that debt-management decisions have an impact on capital-structure decisions; an optimal design of the corporate debt programme leads to higher leverage ratios than a sub-optimal situation in which fixed-rate bonds alone are issued. We also confirm that the optimal allocation to fixed- and to floating-rate bonds depends on the correlation of the interest rate and the firm’s asset value, whereas interest-rate volatility and interest-rate risk premia have comparatively little quantitative impact on debt-management decisions.

The main contribution of our paper is to tie together these two somewhat separate strands of the corporate finance literature (the capital-structure literature and the debt-management literature) by providing the first joint quantitative analysis of capital-structure and debt-management choices in a unified framework. We show that debt-management decisions can be formally analysed in the context of a dynamic capital-structure model, with a tradeoff between the (bankruptcy) costs and (tax shield) benefits associated not only with leverage but also with debt-structure decisions. To do so, we abstract away from agency and asymmetric-information problems, and consider competing forms of liability classes (fixed-rate bonds, floating-rate bonds and inflation-indexed bonds, in addition to equity) in a relatively rich stochastic environment involving interest-rate and inflation risks.\(^9\)

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7 - From the empirical standpoint, the evidence is rather mixed, with Faulkender (2005), who finds support for the market-timing hypothesis for a sample of chemical firms and (see also Chava and Purnanandam (2007) for related evidence), whereas Vickery (2008), studying a sample of small private firms, finds evidence for the hedging theories.

8 - Also related are papers by Diamond (1991a), Diamond (1991b), or Bolton and Freixas (2000), who provide a theoretical analysis of the debt structure, a strand of the literature that is complemented by a number of empirical studies, including a recent paper by Rauh and Sufi (2008), who present an empirical analysis of the debt structure for large US corporations.

9 - This is contrast to the recent, somewhat related literature on optimal security design, e.g., DeMarzo and Fishman (2007), Tchistyi (2005), or Manso et al. (2010), which has focused on the presence of asymmetric information in stylised settings.
model comes in fact as no surprise: because we have assumed that the objective of debt-structure decisions is to maximise the total value of the firm, increasing the cost of debt transfers welfare from shareholders to bondholders and decreasing it transfers it from bondholders to shareholders. Either way, the final impact on the total value of the firm is minor.\(^\text{10}\) Hence, the main benefit of optimising the debt structure is to allow firms to reduce the variability of their net cash flows and therefore to decrease the probability of default. We provide a quantitative assessment of the impact of debt-structure choices on the tax shield and bankruptcy costs, and we show that very substantial increases in firm value can be generated by optimal debt-management decisions; in fact, debt-structure decisions are found to matter even more than capital-structure decisions for some reasonable parameter values. When inflation-linked bonds are made part of the liability mix, we find that they should account for a sizeable fraction of optimal bond issuance for reasonable parameter values.\(^\text{11}\)

Perhaps most closely related to ours is a study by Morellec and Smith (2007), who analyse the issuance—by a firm with cash flows positively related to the price of the commodity—of commodity-linked debt as a hedging mechanism. Their paper addresses a richer set of agency problems, including manager-shareholder conflicts, than ours, which focuses solely on the shareholder-bondholder conflict. In this framework they show that hedging can help not only to solve the underinvestment problem but also to keep incentives to overinvest under control. On the other hand, Morellec and Smith (2007) assume a constant interest rate, which prevents them from analysing the interest-rate-hedging properties of floating-rate debt.\(^\text{12}\)

More importantly, perhaps, they discuss the respective merits of fixed-rate debt and commodity-linked debt separately, whereas we solve for the optimal combination of indexed and non-indexed debt. As such, our paper complements their work on debt and risk management by formally showing that a mixture of forms of debt is, in general, optimal; we also analyse the main factors of this optimal mixture.

The remainder of the paper is organised as follows. In section 2 we introduce a general dynamic model with default triggered at a random date, and we summarise our main theoretical results regarding the joint analysis of the capital- and debt-structure decisions. Section 3 presents a numerical analysis of the optimal blend of floating- and fixed-rate debt in the debt of the firm. In section 4, we introduce inflation risk in the economy and consider inflation-linked bonds an additional class of market debt. Finally, in section 5, we do a quantitative assessment of the importance of the hedging motive in debt management. In section 6, we present our conclusions and suggestions for further research. Technical details on pricing formulas are relegated to an appendix.
2. Debt-Management Decisions in a Dynamic Capital-Structure Model
We make standard assumptions regarding the nature of uncertainty. Let [0, T), with T > 0, denote the (finite) time span of the economy. Uncertainty in the economy is described through a probability space (Ω, A, P). The set of available information at time t is referred to as A_t, and the filtration (A_t)_{t∈[0,T]} is denoted by F. We assume that there is a risk-neutral probability measure Q, under which discounted asset prices are martingales.

2.1 Asset Value

\( V_t \) is the total before-tax value at date t of the firm’s unlevered assets. We assume that this variable evolves as:

\[
dV_t = V_t \left[ (R_t - \delta) \, dt + \sigma_V \, dz_V^t \right] \quad (2.1)
\]

where \( z_V^t \) is a \( Q \)-standard Brownian motion, \( R_t \) is the (nominal) short-term interest rate and \( \delta \) a constant payout rate. An important and standard assumption is that the process \( V \) is unaffected by capital-structure choices. We extend this assumption by postulating that the firm’s value process is also independent of debt-structure decisions. It can easily be checked that for \( \delta > 0 \) we have:

\[
V_t = E^Q \left[ \int_t^\infty \delta V_s e^{-\int_t^s R_u \, du} \, ds \right]
\]

In other words, \( V_t \) is the present value of receiving the cash flow \( \delta V \) per unit of time from date t to infinity and \( V \) can thus be interpreted as the present value of all future cash flows generated by the firm.

The short-term interest rate \( R \) is stochastic and assumed to follow an Ornstein-Uhlenbeck process:

\[
dR_t = a(b - R_t) \, dt + \sigma_R \, dz_R^t \quad (2.2)
\]

where \( z_R^t \) is a \( Q \)-standard Brownian motion correlated to \( z_V^t \), with correlation coefficient \( \rho_{RV} \).

We can rewrite (2.1) and (2.2) using a single two-dimensional Brownian motion \( z \), as follows:

\[
dV_t = V_t \left[ (R_t - \delta) \, dt + \sigma_V \, dz_t \right] \quad (2.3)
\]

\[
dR_t = a(b - R_t) \, dt + \sigma_R \, dz_t \quad (2.4)
\]

Under our assumption regarding the short-term interest rate process, the price \( B(t, T) \) at time t of a risk-free zero-coupon bond paying 1 at time T is given by:

\[
dB(t, T) = B(t, T) \left[ R_t \, dt + \sigma_B(t, T) \, dz_t \right] \quad (2.5)
\]

where \( \sigma_B \) is the deterministic function:

\[
\sigma_B(t, T) = \frac{\sigma_R}{\sigma_R} = \frac{1 - e^{-a(T-t)}}{a} \sigma_R \quad (2.6)
\]

2.2 Promised Liabilities

Throughout this section we assume that the firm has access to two types of debt instruments, both of which we model as zero-coupon bonds with identical maturity \( T \) (the analysis will be extended in section 4 to include inflation-linked bonds in the liability mix):\(^{14}\)

- **fixed-rate bonds**, with promised payout at time \( T \) given by \( B(t, T) \), where \( R_{0,T} \) is the zero-coupon rate at time 0 with maturity date \( T \), defined as \( R_{0,T} = -\frac{1}{T} \ln B(0, T) \). The present value of the promised payout at time \( t \) is \( B(t, T) \) \( \exp \left( T R_{0,T} \right) \).
- **floating-rate bonds**, with promised payout at time \( T \) given by \( \int_t^T R_u \, du \).

13 - Goldstein et al. (2001) suggest an alternative approach relying on EBIT and show that, under the assumption of constant parameters (constant drifts, volatilities, and interest rates), the unlevered asset process and the EBIT process have the same dynamics under the risk-neutral pricing measure. It is unclear, however, how to extend that approach to the presence of stochastic interest rates, and the dynamic that we assume for \( V \) is similar in that respect to what is assumed in the literature on dynamic capital structure with stochastic interest rates (Collin-Dufresne and Goldstein [2001]; Ju and Ou-Yang [2006]).

14 - We make the simplifying assumption that corporate debt pays no coupons because a coupon-paying debt would involve the pricing of the stream of a constant coupon \( c \) and of a stochastic coupon \( R_t \). Although Leland (1994) obtains a closed-form expression for the price of the fixed-coupon bond in the absence of interest-rate risk, the non-independence of default risk and interest-rate risk greatly complicates the pricing exercise. We have not been able to find a closed-form expression, or even an efficient numerical approximation, for defaultable coupon-paying bond prices (Ju and Ou-Yang (2006) work around this problem by assuming that corporate debt is issued at some given fraction of par, but their approach does not translate to a setting with multiple types of debt instruments). As a result, introducing coupon payments in our model would come at the cost of increased numerical complexity (we would lose the quasi-closed-form expressions for the tax shield, and Monte-Carlo simulations would be needed in pricing exercises) without necessarily having a substantial impact on our main results.
2. Debt-Management Decisions in a Dynamic Capital-Structure Model

the cumulative short-term interest rate over the period \([0, T]\). The present value of the promised payout at time \(t\) is \(\exp \left(\int_0^t R_s \, ds\right)\).

The promised payment on fixed-rate bonds is certain because \(R_{0,T}\) is a constant, whereas the promised payment on floating-rate bonds is uncertain, as seen from the initial date, because of the stochastic nature of \(R\). If the fixed-rate and floating-rate bonds were not subject to default risk, their value at time 0 would be 1. Because of default risk, there is a positive probability that the payoff of each bond will be lower than the promised payoff, so the initial price of the defaultable bond will be strictly less than 1.

We define \(\theta\) (respectively, \(1 - \theta\)) as the weight allocated to fixed-rate debt (respectively, floating-rate debt) within the corporate-debt programme, so we obtain the following dynamics for the market value of promised liability payments, denoted by \(L_t\):

\[
\frac{dL_t}{L_t} = \theta \frac{dB(t,T)}{B(t,T)} + (1 - \theta) R_t \, dt \quad (2.7)
\]

which can be rewritten using equation (2.5) as:

\[
\frac{dL_t}{L_t} = L_t R_t \, dt + \theta \sigma_B(t,T)\gamma \, dz_t \quad (2.8)
\]

The parameter \(\theta\) will be used as a control variable governing the debt structure when it comes to maximising firm value. Finally, we assume that the firm cannot renegotiate its debt between 0 and \(T\) and that it can credibly commit to this strategy, so neither equity- nor bondholders expect any deviation from these rules.\(^{15}\)

2.3 Default Time

Because of the presence of default risk, the total promised liability value process \(L\) is different from (and in fact, takes greater values than) the market value of debt process, denoted by \(D\), which we turn to now. Following Black and Cox (1976), we assume that bankruptcy is triggered when the process \(V\) first hits a barrier. Several barriers have been considered in the literature, with the final choice most often imposed by tractability constraints. Black and Cox (1976) and Longstaff and Schwartz (1995) consider fixed exogenous barriers, whereas Leland (1994) or Leland and Toft (1996), as well as many subsequent papers by others, consider a constant but endogenous barrier. In both cases, explicit solutions can be derived for the optimal capital structure because the prices of equity and debt are obtained in analytic forms. On the other hand, stochastic barriers are introduced in Briys and de Varenne (1997) and Ju and Ou-Yang (2006), which simplifies the pricing of the fixed-rate debt in a context of stochastic interest rates.\(^{16}\) We follow these papers by considering a stock-based notion of default: bankruptcy is triggered as soon as the unlevered asset value of the firm is less than a fraction of the promised liability value. Formally, default is triggered as soon as the distance-to-default process \(e^{-\delta(T-t)}V_t/L_t\) hits some exogenously specified level \(q\) from above:\(^{17}\)

\[
\tau = \inf \left\{ t \geq 0 \mid V_t \leq qL_t e^{\delta(T-t)} \right\} \quad (2.9)
\]

where it is implicitly understood that \(V_0 > qL_0 e^{\delta T}\).\(^{18}\) The debt-structure decision will impact the default threshold because it impacts the promised liability value process \(L_t\) through (2.8).

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15 - We make this assumption because there is no reason for optimal decisions at time 0 to be optimal at later dates. In Leland (1994) time-homogeneity (stemming from infinite maturity of debt and constant interest rate) induces a form of time-consistency, which ensures that optimal decisions at time 0 are still optimal at later dates.
16 - These barriers are expressed as fractions of the present value of promised repayment. A possible justification for this choice can be found in Leland (1994): indeed, this paper shows that when debt is protected by a positive net-worth requirement there are no agency conflicts between shareholders and debtholders after debt is in place.
17 - For \(q = 1\) and \(\delta = 0\), we recover a positive net worth condition.
18 - Considering a flow-based notion of default would be an alternative (Kim et al. [1993]). In fact, both notions would be equivalent in an EBIT-based model, in which the unlevered value of the firm, \(V\), is a multiple of the EBIT process (Goldstein et al. [2001]).
The distance-to-default process is thus a key state variable in our problem. Using Ito's lemma and equations (2.3) and (2.8), we obtain its dynamic:

\[
\frac{d}{L_t} \left( e^{-\delta(T-t)}V_t \right) = \frac{e^{-\delta(T-t)}V_t}{L_t} \left\{ [\theta \sigma(t, T)]' \sigma(t, T) \right\} + \delta V_t \frac{dL_t}{L_t} + \sigma V_t \frac{d\xi}{\xi}.
\]

In particular, in our setting with constant asset and interest rate volatilities, the process is a geometric Brownian motion with deterministically time-dependent drift and volatility under $\mathbb{Q}$. This property will be crucial for the tractability of our model.

### 2.4 Bankruptcy Costs and Tax Shields

We make two main assumptions about what happens on default. First, an exogenous and constant fraction $\alpha$ of the asset value is lost to bankruptcy costs, so the total amount distributed to debtholders after they take over the firm is $(1 - \alpha)V_T$. We also assume that the holders of fixed-rate debt and the holders of floating-rate debt enjoy equal seniority, so they both receive a fraction of the remaining asset value proportional to the respective promised payment. In other words, we assume that holders of fixed-rate debt collectively receive $\theta(1 - \alpha)V_T$ at date $T$, whereas holders of floating-rate debt collectively receive $(1 - \theta)(1 - \alpha)V_T$. These two assumptions are relatively standard. At time $T$, if default has not yet occurred, all bondholders receive what they were promised, that is, $\Theta L_T$ for fixed-rate debtholders and $(1 - \Theta)L_T$ for floating-rate debtholders.

We let $D^1_t$ be the market value at time $t$ of the fixed-rate bonds held by bondholders as a whole and $D^2_t$ be the value of the floating-rate bonds. $D_t = D^1_t + D^2_t$ is the value of the portfolio held by bondholders. As we assume equal seniority, we have, for $t < \tau$:

\[
D^1_t = \mathbb{E}^\mathbb{Q} \left[ e^{-\delta(T-t)} \alpha V_{\tau} \right] + (1 - \alpha) \mathbb{E}^\mathbb{Q} \left[ e^{-\delta(T-t)} \theta V_{\tau} \right]
\]

\[
D^2_t = (1 - \theta) \mathbb{E}^\mathbb{Q} \left[ e^{-\delta(T-t)} \alpha V_{\tau} \right] + (1 - \alpha) \mathbb{E}^\mathbb{Q} \left[ e^{-\delta(T-t)} \theta V_{\tau} \right]
\]

As a consequence, the allocation to the two types of bonds after default is accounted for is given by $D^1_t = \theta D_t$ and $D^2_t = (1 - \theta)D_t$, as is the case for present values of promised repayments. Hence, the debt-structure control variable $\theta$ can be regarded as the split between fixed-rate and floating-rate debt in terms of present value $L_t$ of promised payments or in terms of present value $D_t$ of actual payments. Bankruptcy costs are paid at time $\tau$ and they are equal to a fraction $\alpha$ of the remaining asset value at that time, with a present value given by:

\[
BC_0 = \alpha \mathbb{E}^\mathbb{Q} \left[ e^{-\delta(0)} \alpha V_0 \right] \left( 1_{\{T \leq T\}} \right)
\]

The present value of the tax shield at time 0 is defined as the fair price of the payoff $k(L_T - L_0)$, which is paid at time $T$ only if default has not occurred before:

\[
TS_0 = k \mathbb{E}^\mathbb{Q} \left[ e^{-\delta(0)} \alpha (L_T - L_0) \right] 1_{\{T > T\}}
\]

We have assumed a tax shield over the entire interest payment at terminal date, mostly for tractability reasons. The total value of the firm equals unlevered asset value plus the present value of the
2. Debt-Management Decisions in a Dynamic Capital-Structure Model

tax shield minus the present value of bankruptcy costs:

\[ v_0 = V_0 + TS_0 - BC_0 \]

and equity value is obtained as the difference between total value and market price of debt:\(^{20}\)

\[ E_0 = v_0 - D_0 \]

Capital and debt-structure decisions are made at time 0 by the initial owners of the firm.

We take the literature farther by assuming that their objective is to maximise the total value of the firm with respect not only to the variable \( L_0 \) (the capital-structure decision) but also to the variable \( \theta \) (the debt-structure decision). This objective can be mathematically written as:

\[ \max_{L_0, \theta} v_0 \]  
(2.14)

The optimal values of \( L_0 \) and \( \theta \), if they exist, are denoted by \( L_0^* \) and \( \theta^* \).

2.5 Pricing Exercise

In this subsection, we present detailed expressions for the prices of claims written on the firm’s assets. These expressions allow numerical computation of \( v_0 \) for a given policy \((L_0, \theta)\). It will turn out to be useful to use the processes \( L \) and \( B(\cdot, T) \) as numeraires and to work with the probability measures \( Q^L \) and \( Q^T \) defined as:\(^{21}\)

\[ \frac{dQ^L}{dQ} = \frac{L_T}{L_0} e^{-\int_0^T R_s \, ds}, \]

\[ \frac{dQ^T}{dQ} = \frac{1}{B(0,T)} e^{-\int_0^T R_s \, ds} \]  
(2.15)

The following proposition explains how to obtain the market value of corporate debt (2.11), the bankruptcy costs (2.12), and the tax shield (2.13) using the probability measures introduced in (2.15):

**Proposition 1** The present value at time 0 of bankruptcy costs is given by:

\[ BC_0 = \alpha q L_0 \left[ Q^L(\tau \leq T) + \delta e^{\delta T} \int_0^T e^{-\rho(T-t)} \, dt \right] \]  
(2.16)

and the present value at time 0 of the tax shield is:

\[ TS_0 = k L_0 Q^L(\tau > T) - k L_0 B(0,T) Q^T(\tau > T) \]  
(2.17)

The market value of defaultable debt at time 0 is:

\[ D_0 = L_0 Q^L(\tau > T) + \frac{1 - \alpha}{\alpha} BC_0 \]  
(2.18)

**Proof.** See appendix A.

We must now compute \( Q^L(\tau \leq t) \) for \( t \leq T \) and \( Q^T(\tau > T) \). The default probabilities \( Q^L(\tau \leq t) \) can be obtained in closed form because the distance-to-default process follows a martingale under \( Q^L \). Obtaining an expression for the term \( Q^T(\tau > T) \) is less straightforward because the distance-to-default process is not a martingale under \( Q^T \). But it is still a log-normal and Markovian process, which allows us to use the numerical approximation introduced in Longstaff and Schwartz (1995) and generalised by Collin-Dufresne and Goldstein (2001). The following proposition gives the detailed expressions.
Proposition 2 • We have, for \( t \leq T \):

\[
Q^t(\tau \leq t) = \mathcal{N} \left( \frac{-\ln \frac{1-e^{-\alpha T}}{\alpha T}}{\sqrt{\phi_1(t)}} \right) + \frac{Ye^{-\Delta T}}{qL_0} \mathcal{N} \left( \frac{-\ln \frac{1-e^{-\alpha T}}{\alpha T}}{\sqrt{\phi_1(t)}} \right)
\]

(2.19)

where \( \mathcal{N} \) is the standard normal cumulative distribution function and:

\[
\phi_1(t) = \mathbb{E}^{\mathbb{Q}} \left[ \ln \frac{V_t}{L_T} \right] = \int_0^t \| \sigma_V - \theta \sigma_B(s, T) \|^2 \, ds
\]

(2.20)

• The survival probability \( Q^T(\tau > T) \) can be approximated by:

\[
\hat{Q}^T(\tau > T) = 1 - \sum_{u=0}^{n_T-1} \hat{g} \left( (2u + 1) \frac{\Delta t}{2} \right) \Delta t
\]

(2.21)

where \( n_T \) is a large integer, \( \Delta t = T/n_T \), and \( \hat{g} \left( (2u + 1) \frac{\Delta t}{2} \right) \) is recursively computed through:

\[
G_1(t_i) = \sum_{u=0}^{i-1} G_2 \left( t_i, (2u + 1) \frac{\Delta t}{2} \right) \hat{g} \left( (2u + 1) \frac{\Delta t}{2} \right) \Delta t, \quad i = 1, \ldots, n_T
\]

\[
G_1(t) = \mathcal{N} \left( \frac{-\ln \frac{1-e^{-\alpha T}}{\alpha T}}{\sqrt{\phi_1(t)}} \right), \quad 0 \leq t \leq T
\]

\[
G_2(t, s) = \mathcal{N} \left( \frac{-\frac{1}{2} \phi_2(t) - \phi_2(s) - \phi_3(t) + \phi_3(s)}{\sqrt{\phi_1(t) - \phi_1(s)}} \right), \quad 0 \leq s < t \leq T
\]

\[
\phi_2(t) = \mathbb{E}^{\mathbb{Q}} \left[ \ln L_T \right] = \int_0^t \| \theta \sigma_B(s, T) - \sigma_B(s, T) \|^2 \, ds, \quad 0 \leq t \leq T
\]

\[
\phi_3(t) = \mathbb{E}^{\mathbb{Q}} \left[ \ln V_T \right] = \int_0^t \| \sigma_V - \sigma_B(s, T) \|^2 \, ds, \quad 0 \leq t \leq T
\]

Proof. See appendix B.

Given that no analytical expression is available for the optimal values \( L_0^* \) and \( \theta^* \) of the control variables in (2.14), we must resort to a numerical approximation of the solution. In section 3, we present comparative static analysis that will help us understand the factors impacting these decisions.

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3. Numerical and Comparative Statics Analysis
3. Numerical and Comparative Statics Analysis

3.1 Base-Case Parameter Values

We first define a set of base-case parameters inspired by the literature on the pricing of corporate debt with stochastic interest rates. Table 2 summarises these parameters and lists the base-case parameters used in the related papers by Longstaff and Schwartz (1995), Briys and de Varenne (1997), and Ju and Ou-Yang (2006). The asset return volatility, $\sigma_v$, is set at 20%. This value is similar to those used in the three papers mentioned above, and is also near the 23% that Leland (2006) reports for firms issuing AAA-rated debt. The minimal distance-to-default acceptable to equity holders, $q$, is set to 70%, which is a value similar to that used by Leland (2006) and Chen et al. (2009). The speed of mean reversion in the nominal interest rate, $a$, is set to 0.261 as in Ju and Ou-Yang (2006).

As we want to analyse the impact of the market price for interest-rate risk $\lambda_r$ on debt-management decisions, we set the long-term mean under the real probability measure $\mathbb{P}$, denoted $\hat{\bar{b}}$, and thus let $b$ vary with $\lambda_r$ in keeping with the well known expression $b = \hat{\bar{b}} - \sigma_b t$. In the base case, $\hat{\bar{b}}$ is set at 5% and $\lambda_r$ at −0.00075, as in Sørensen (1999). The volatility of the nominal short-term interest rate, $\sigma_R$, is set at 3%, between the 2% used by Briys and de Varenne (1997) and Munk and Sørensen (2004), and the 3.16% of Longstaff and Schwartz (1999). The choice of a base-case value for the correlation parameter $\rho$ is more subtle, because the sensitivity of firm’s cash flows to interest rates depends on the activities undertaken by the firm. For instance, Vickery (2008) estimates the beta of a firm’s cash flows with respect to short-term rates in different sectors of the economy and finds positive as well as negative values. Non-depository financial institutions and personal services firms exhibit negative beta, while mining and petroleum refining firms have positive beta. Faulkender (2005) also finds empirical evidence for both signs of the correlation. In the base case, we take $\rho = 0$ as is done, for example, by Ju and Ou-Yang (2006), and we subsequently perform comparative static analysis involving both positive and negative values in an attempt to capture the two main types of firms.

3.2 Comparative Static Analysis

Tables 3, 4, and 5 summarise our comparative analysis. In each table, panel A reports the firm value-maximising $L_0^*$ and $\theta^*$ levels, which define the optimal capital and debt structure respectively. In panel B we take $\theta = 1$, which corresponds to the standard assumption in the capital-structure literature of a single class of fixed-rate debt, and then search for the optimal capital structure. In other words, we solve for the optimal $\hat{L}_0$ in the programme:

$$\max_{L_0} v_0 \quad \text{s.t.} \quad \theta = \hat{\theta} \equiv 1 \quad (3.1)$$

In both cases, the optimal capital structure is described by the leverage ratio $D_0/L_0$, which is the ratio of the market value of debt to the total value of the firm. If we let $v_0^*$ be the total value of the firm when both the face value of debt and the debt-structure are optimally chosen, and $\hat{v}_0$ be the total value when only the face value is optimally chosen, we can compute the ratio $(v_0^* - \hat{v}_0)/(\hat{v}_0 - V_0)$: it measures the marginal gain of optimisation of debt composition relative to optimisation of capital structure. Finally, we report the volatilities of $\ln V_T$ and $\ln(V_T/L_T)$, and the correlation of $\ln V_T$ and $\ln L_T$ in an attempt to analyse the impact of debt management on the uncertainty in the ratio $V/L$, which
appears in the definition of the distance-to-default process (see (2.10)).

We first provide a quantitative estimate of the impact of correlation on the allocation to fixed- and to floating-rate debt, a subject that has been discussed on mostly qualitative grounds in the literature. In panel A of table 3, we present the results of a comparative statics analysis when the correlation coefficient $\rho$ ranges from $-0.99$ to $+0.99$. These results show that the optimal allocation to floating-rate debt, $1 - \theta^*$, is an increasing function of the correlation of changes in asset value and changes in interest rates. We also observe that the standard deviation of $\ln(V_\ell/L_\ell)$ is lower than that of $\ln V_\ell$ or equivalently that $\ln V_\ell$ and $\ln L_\ell$ are positively correlated. So the joint optimisation of $L_0$ and $\theta$ leads to a decrease in the uncertainty on the cash flows net of promised debt payment. This finding confirms the tenets of hedging theories (Froot et al. 1993 or Smith and Stulz 1985) that state that if the operating cash flows before interest expenses and tax (EBIT) of a firm are positively correlated with interest rates, the firm should maintain higher floating-rate debt to avoid the default costs associated with low cash flows. In other words, if these cash flows are positively correlated with interest rates, floating-rate debt can lower volatility, even though it involves an uncertain interest-rate payment: by matching the interest-rate exposure of their liabilities to that of their assets, firms can reduce the variability of their cash flows net of debt repayment. As a result, firms may lower their expected costs of financial distress and increase the tax-shield benefits. When fixed-rate debt alone is issued, the promised repayment is a constant, so there can be no diversification effect: the variance of $\ln(V_\ell/L_\ell)$ is the same as that of $\ln V_\ell$.

For some parameter values, the allocation to either floating-rate or fixed-rate debt instruments is negative, which is to be expected since we have not ruled out such a possibility. The interpretation of a negative $\theta$ value is that the firm should receive a net fixed-rate cash flow rather than pay it out. Of course, such a position cannot be taken using cash instruments alone, since the net supply of corporate bonds must be positive. But it is possible with a swap contract. For example, $\theta = -50\%$ corresponds to a net long position in fixed-rate bonds, i.e., the firm should receive 50% fixed and pay 150% floating. Such a debt mix can be implemented by issuing 100% floating-rate debt and entering into a swap contract with a notional equal to 50% of the total debt value, by which the firm will pay the floating rate and receive the fixed rate. As a result, one would obtain the target $-50\%$ fixed and 150% floating net cash flows. Although optimal debt-management decisions can be acted on with or without derivatives when $\theta$ lies between 0 and 1, using a derivatives contract can prove an effective and cost-efficient means of adjusting an interest-rate exposure that deviates from the target exposure. As a general comment, it should be noted that our paper provides guidance to the optimal debt structure regardless of how the firm implements its target exposure to interest-rate risk. This approach is the same as that taken in the literature (Smith and Stulz 1985 and Froot et al. 1993) that has focused not on how firms achieve a particular exposure but on what final exposure they should have.
A comparison of panels A and B of table 3 shows that debt-management decisions have a great impact on capital-structure decisions. We find that optimal leverage is always higher, and often significantly higher, when the debt structure is optimally chosen (with a proportion \( \theta = \theta^* \) of fixed-rate debt in panel A) than it is when the firm issues fixed-rate debt alone (\( \theta = 1 \) in panel B). Hence, by optimising the debt structure, corporations can maximise the advantages of debt and thus issue more of it. We also find that in the event of high (positive and negative) correlation of changes in the firm value and changes in interest rates, optimal debt management leads to an increase in firm value much greater than that to which optimal capital-structure choice alone leads. When the correlation of interest rates and changes in firm value is 50\% (respectively, 70\%), we find that the marginal increase in firm value arising from optimal debt management accounts for more than half (respectively, more than 100\% of) the increase in firm value arising from capital structure decisions only. In the event of near-perfect 0.99 correlation, for example, the joint optimisation of capital and debt structures even leads to an additional increase in firm value three times greater than that arising from the optimisation of capital structure alone.

In tables 4 and 5, we do a comparative statics analysis with respect to the market price of interest-rate risk, denoted \( \lambda_R \), and interest-rate volatility, denoted by \( \sigma_R \). We find that \( \theta^* \) is a decreasing function of the absolute value of \( \lambda_R \), which can be explained by the fact that an increase in the cost of fixed-rate debt should lead to a decrease in the optimal share of fixed-rate debt issued by firms. On the other hand, in table 5, we find that an increase in interest-rate volatility, which makes the payout of floating-rate bonds more uncertain, makes it optimal to issue more fixed-rate debt. Again, this property of the optimal issuance policy is not unexpected. The most striking result that we obtain here is perhaps that changes in interest-rate volatility and risk premia have relatively little impact on the optimal structure of the corporate debt programme, at least in comparison to changes in the correlation of interest rates and firm value. This finding suggests that the optimal choice of debt structure is driven more by hedging considerations than by an attempt to time the market. We revisit this question in section 5.

As a conclusion, we find that debt-management decisions have an impact on capital-structure decisions, and those decisions should be analyzed simultaneously in the context of a unified framework. We also find that issuing a single class of fixed-rate debt, as typically assumed in the capital-structure literature, is sub-optimal in a stochastic interest-rate environment and can induce a significant loss in firm value. As can be seen from table 3, the difference between \( v_0^* \) and \( \bar{v}_0 \) increases with the absolute value of the correlation of changes in interest rates and changes in firm value; optimal management of debt structure matters more for firms whose cash flows have either a strong negative or a strong positive correlation with interest rates. This property suggests that any attempt to increase the correlation of the asset value process and the liability value process would be valuable from the perspective of maximising the total levered value of the firm. One possible approach to enhancing the
hedging properties of the debt portfolio with respect to the asset portfolio consists of further expanding the liability mix by introducing inflation-linked bonds, which is what we turn to in the next section.23
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4. Introducing Inflation-Linked Bonds

We now extend the analysis and make inflation-linked bonds part of the liability mix. Inflation-linked securities are a relatively recent innovation in the debt markets; the introduction of Treasury inflation-protected securities (TIPS) marked a starting point. Inflation-linked debt issuance has risen steadily over the last ten years, particularly for companies in the regulated electricity and water sectors in the United States and the United Kingdom, where rate increases are linked by law to inflation indices such as the retail price index (RPI). Although inflation-adjusted bonds account for only a tiny share of outstanding corporate debt, and although the bulk inflation-linked debt is issued by sovereign states, utilities and financial-services companies have recently shown interest in issuing inflation-linked bonds. It is perhaps surprising that some large corporations are still sitting on the sidelines, since intuition suggests that if a firm’s earnings tend to grow with inflation, having some inflation-linked issuance can be a natural hedge. In this section, we propose a formal model that will allow a quantitative assessment of the optimal allocation to inflation-linked bonds in corporate debt programmes.

4.1 A Formal Model of Inflation Risk

As in Brennan and Xia (2002), we model the stochastically time-varying price process under the pricing measure $\mathbb{Q}$ as:

$$\frac{d\Phi_t}{\Phi_t} = (\pi_t - \sigma \Phi_t) dt + \sigma \Phi_t dz_t^\pi$$

$\frac{d\Phi_t}{\Phi_t}$ can be interpreted as the actual inflation rate over $[t, t + dt]$, while the drift process $\pi_t$ represents the instantaneous expected inflation rate under the historical measure $\mathbb{P}$, and $\Lambda = \Lambda_t$ is the market price of inflation risk. For the sake of tractability, we assume, as in Brennan and Xia (2002), that both expected inflation $\pi_t$ and the real interest rate $r_t$ follow Ornstein-Uhlenbeck processes, namely:

$$d\pi_t = \kappa (\beta - \pi_t) dt + \sigma \pi_t dz_t^\pi$$
$$dr_t = a (b - r_t) dt + \sigma r_t dz_t^r$$

Regarding the asset side of the balance sheet, the model we use for the unlevered value of the firm is the same as in section 2:

$$\frac{dV_t}{V_t} = (R_t - \delta) dt + \sigma_V dz_t^V$$

where $R_t$ is still the nominal short-term interest rate at date $t$. As shown by Brennan and Xia (2002), the nominal rate in this setting is given by a modified version of Fisher’s relation:

$$R_t = r_t + \pi_t - \sigma \Phi_t$$

This relation is used for the pricing of default-free nominal and inflation-indexed bonds (see derivation in appendix C).

We let $l(t, T)$ be the price of a perfectly indexed zero-coupon bond paying $\Phi_t$ at time $T$. We also define the equivalent nominal and real zero-coupon rates at time $0$:

$$R_{0,T} = -\frac{1}{T} \ln B(0, T),$$
$$\rho_{0,T} = -\frac{1}{T} \ln I(0, T)$$

Regarding the liability side of the balance sheet, we now assume that the firm can issue a third class of debt–inflation-indexed bonds—in addition to nominal fixed-rate and floating-rate bonds. An inflation-indexed bond with maturity date $T$ is defined as a bond with real promised payoff $\exp(T\rho_{0,t})$ and nominal promised payoff $\Phi_T$ at time $T$. The value at time $t$ of the promised payments is given by $B(t, T) \exp(T\rho_{0,t})$, $\exp(T\rho_{0,t}) dz_t$.
4. Introducing Inflation-Linked Bonds

and \( l(t, \tau) \exp(T \rho_0, \tau) \) respectively. We finally obtain:

\[
\frac{dL_t}{L_t} = (1 - \theta_1 - \theta_2) R_t dt + \theta_1 \frac{dB(t, T)}{B(t, T)} + \theta_2 \frac{dI(t, T)}{I(t, T)}
\]

(4.5)

where \( \theta_1 \) is the fraction allocated to fixed-rate nominal debt, \( \theta_2 \) is the fraction allocated to inflation-linked debt, with the rest allocated to floating-rate debt. We set \( \theta = (\theta_1, \theta_2) \). We can rewrite (4.1), (4.2) and (4.3) using a single four-dimensional Brownian motion \( z \) under \( \mathbb{Q} \):

\[
\frac{d\Phi_t}{\Phi_t} = (\pi_t - \sigma_\pi \Lambda_t) dt + \sigma_\pi d\mathbf{z}_t
\]

\[
d\pi_t = \kappa (\beta - \pi_t) dt + \sigma_\pi d\mathbf{z}_t
\]

\[
dr_t = a (b - r_t) dt + \sigma_r d\mathbf{z}_t
\]

\[
\frac{dV_t}{V_t} = (R_t - \delta) dt + \sigma_V d\mathbf{z}_t
\]

In appendix C we show that the dynamics of the nominal and real fixed-rate non-defaultable bonds prices can be written as:

\[
dB(t, T) = R_t dt + \sigma_B(t, T)' d\mathbf{z}_t
\]

\[
dI(t, T) = R_t dt + \sigma_I(t, T)' d\mathbf{z}_t
\]

where \( \sigma_{\pi}(\cdot, \tau) \) and \( \sigma_r(\cdot, \tau) \) are the deterministic volatility vectors of \( B(\cdot, \tau) \) and \( I(\cdot, \tau) \). We define the volatility matrix of default-free bonds \( \sigma_L \) as:

\[
\sigma_L(t, T)' = \begin{pmatrix} \sigma_B(t, T)' \\ \sigma_I(t, T)' \end{pmatrix}
\]

so that:

\[
\frac{dL_t}{L_t} = R_t dt + \theta' \sigma_L(t, T)' d\mathbf{z}_t \quad (4.6)
\]

The dynamic evolution of \( L \) with inflation-linked bonds is thus similar to its dynamics with fixed-rate and floating-rate bonds only. In fact, equation (2.8) is a special case of equation (4.6), where \( \theta' = (\theta_1, 0) \).

A straightforward application of Ito’s lemma yields the dynamics of the distance-to-default process:

\[
\frac{dV_t}{V_t} = [\theta_1 \sigma_L(t, T)' \sigma_L(t, T) - \theta_1 \sigma_L(t, T)' \sigma_V] dt + [\sigma_V - \theta_1 \sigma_L(t, T)'] d\mathbf{z}_t \quad (4.7)
\]

4.2 Optimal Debt Structure with Inflation-Linked Bonds

As in subsection 2.4, we assume that in the event of default before time \( \tau \), a fraction of the remaining unlevered firm value is lost to third parties and that the recovery payment, \( (1 - \alpha) V^r \), is shared by bondholders of equal seniority. As a result, we can rewrite (2.11) as:

\[
D_1 = \theta_1 E_t^Q \left[ e^{-\int_t^\tau \sigma_1(\cdot, \tau)' dL_\tau} \mathbb{I}_{\{\tau > T\}} \right] + (1 - \alpha) \theta_1 e_t^Q \left[ e^{-\int_t^\tau \sigma_2(\cdot, \tau)' dV_\tau} \mathbb{I}_{\{\tau \leq T\}} \right]
\]

\[
D_2 = (1 - \theta_1 \theta_2) E_t^Q \left[ e^{-\int_t^\tau \sigma_1(\cdot, \tau)' dL_\tau} \mathbb{I}_{\{\tau > T\}} \right] + (1 - \alpha) (1 - \theta_1 \theta_2) E_t^Q \left[ e^{-\int_t^\tau \sigma_1(\cdot, \tau)' dV_\tau} \mathbb{I}_{\{\tau \leq T\}} \right]
\]

\[
D_3 = \theta_2 E_t^Q \left[ e^{-\int_t^\tau \sigma_2(\cdot, \tau)' dL_\tau} \mathbb{I}_{\{\tau > T\}} \right] + (1 - \alpha) \theta_2 e_t^Q \left[ e^{-\int_t^\tau \sigma_2(\cdot, \tau)' dV_\tau} \mathbb{I}_{\{\tau \leq T\}} \right]
\]

The market value at time \( t \) of the risky debt is \( D_t = D_1 + D_2 + D_3 \), and we have:

\[
D_1 = \theta_1 D_t, \quad D_2 = (1 - \theta_1 \theta_2) D_t, \quad D_3 = \theta_2 D_t
\]
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Present values of bankruptcy costs and the tax shield are still defined as in (2.12) and (2.13). Equations (2.16), (2.17), and (2.18) still hold when indexed bonds are introduced because they are independent of the liability mix. But the survival probabilities $Q^t(\tau > t)$ for $t \leq T$, and $Q^T(\tau > T)$, whose expressions are modified when inflation-linked bonds are introduced, must still be computed. The following proposition generalises proposition 2 to the case of three debt classes.

**Proposition 3**

- $Q^t(\tau \leq t)$ is given as in proposition 2, with
  \[ \phi_1(t) = \int_0^t \|\sigma_V - \sigma_L(s, T)\theta\|^2 \, ds \]

- The survival probability $Q^T(\tau > T)$ can be approximated as in proposition 2, with:
  \[ \phi_2(t) = \int_0^t \|\sigma_L(s, T)\theta - \sigma_B(s, T)\|^2 \, ds, \quad 0 \leq t \leq T \]
  \[ \phi_3(t) = \int_0^t \|\sigma_V - \sigma_B(s, T)\|^2 \, ds, \quad 0 \leq t \leq T \]

**Proof.** The proof is similar to that of proposition 2 once we have noticed that the quadratic variation of the process \((e^{-\delta(T-t)}V_t/L_t)\) is deterministic. After all, $\sigma_V$ is constant and $\sigma_t(\cdot, \tau)$ is deterministically time-dependent. We still assume that the objective is to maximise the total value of the firm at time 0 with respect to $L_0$ and $\theta$. $L_0$ is the control variable for the capital structure, whereas $\theta$ is the control variable for the debt structure. That is, we consider the following programme:

\[ \max_{L_0, \theta} v_0 \quad (4.8) \]

and we let the solution to it, which we obtain numerically, be $(L_0^*, \theta^*)$.
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5. A Formal Analysis of the Hedging Motive in Debt Management

We have found that the optimal blend of fixed- and floating-rate bonds depends on the correlation of the interest-rate process and firm asset-value process, whereas interest-rate volatility and interest-rate risk premia have comparatively little quantitative impact on debt-management decisions. We have reported similar effects for inflation. On the whole, these results suggest that, to maximise their value, firms should mostly choose their debt structure in such a way as to hedge the risk factors impacting their cash flows rather than to optimise the cost/risk tradeoff of their debt.

To confirm this idea, we now study the debt structure that minimises risk rather than the structure that maximises firm value. This perspective can be thought of as the one favoured not by the well-diversified risk-neutral shareholder but by an infinitely risk-averse manager. Our aim is to provide a formal comparison of the debt structures obtained by minimising risk and those obtained by maximising firm value. In the empirical applications that follow, we return to the model of section 2, in which there is no inflation risk, but our results can be extended in a straightforward manner to an economy with inflation risk.

5.1 A Generic Debt-Management Problem

Consider the following two programmes of risk minimisation:

\[
\min_{\theta} \mathbb{V}[\ln L_T] \quad (5.1)
\]

\[
\min_{\theta} \mathbb{V}[\ln \frac{V_T}{L_T}] \quad (5.2)
\]

The first programme (5.1) focuses on minimising the uncertainty of promised debt payment. This programme has a trivial solution: since the promised payment of fixed-rate debt is not random, a focus on risk minimisation unambiguously leads to issuing fixed-rate bonds only, which corresponds to $\theta = 1$. On the other hand, the manager in charge of debt-structure decisions might recognise that what eventually matters is not so much the variability of promised payment as the volatility of cash flows net of debt payments. This concern is captured by the programme (5.2), where risk is measured by the uncertainty in the "surplus" $\ln V_T - \ln L_T$ of log assets with respect to log liabilities, not by the uncertainty in $\ln L_T$ alone. To see that (5.2) differs from (5.1), we develop the expression for the variance of $\ln \frac{V_T}{L_T}$:

\[
\mathbb{V}[\ln V_T - \ln L_T] = \mathbb{V}[\ln V_T] + \mathbb{V}[\ln L_T] - 2\text{Cov}[\ln V_T, \ln L_T] 
\]

(5.3)

Hence, increasing the fraction in floating-rate debt (i.e., decreasing $\theta$) would lead to an increase in the $\mathbb{V}[\ln L_T]$ term, but it could also lead to a decrease or an increase in the $\text{Cov}[\ln V_T, \ln L_T]$ term (depending on the sign of the correlation of $V$ and $R$), with an ambiguous overall effect. This tradeoff between an increase in risk and an increase in covariance results in a non-trivial solution to the programme, as shown in the following proposition.

Proposition 4 • Consider programme (5.2) in the event the menu of liability classes includes fixed-rate and floating-rate bonds. Its solution is given by:

\[
\hat{\theta}_{\text{opt}} = \rho_{RV} \sigma_V \frac{\int_0^T \sigma_B(t,T) \, dt}{\int_0^T \sigma_B(t,T)^2 \, dt} 
\]

(5.4)
5. A Formal Analysis of the Hedging Motive in Debt Management

• If the firm can issue fixed-rate, floating-rate, and inflation-indexed bonds, the optimal vector of weights is given by:

$$\hat{\theta}_{hed} = \left( \int_0^T \sigma_L(t, T)' \sigma_L(t, T) \, dt \right)^{-1} \int_0^T \sigma_L(t, T)' \sigma_V \, dt$$

**Proof.** See appendix D.

The expression (5.4) is reminiscent of the liability-hedging portfolio arising in Merton (1993). Of course, a perfect hedge cannot be achieved since the liability market is incomplete: as long as there is imperfect correlation of asset risk and interest-rate risk, the equality $V_t = L_t$ cannot hold almost surely.

Programme (5.2) is ex ante fundamentally different from programme (2.14), the programme that is relevant from the perspective of the owner of the firm. But in our next numerical illustration, we show that, despite their difference, their respective solutions happen to be close to each other, which reinforces our previous conclusion that hedging considerations should be dominant in the choice of a particular debt structure. When the firm chooses the debt structure defined by $\hat{\theta}_{hed}$, we compute the optimal capital structure by maximising its total value with respect to the face value of debt, which yields optimal values $L_0$ and $v_0$. The results are shown in tables 8 and 9: panel A shows the capital and debt structures that maximise firm value, and panel C displays the capital and debt structures that minimise the volatility of assets relative to liabilities. By definition of $(L_0^*, \theta^*)$, the improvement brought by debt-structure decisions over capital-structure decisions is greater in panels A, where it is denoted as $(v_0^* - \bar{v}_0)/(\bar{v}_0 - V_0)$, than in panels C, where it is denoted as $(v_0 - \bar{v}_0)/(\bar{v}_0 - V_0)$.

Nevertheless, we observe that the ratio $(v_0^* - \bar{v}_0)/(\bar{v}_0 - V_0)$ is relatively close to the ratio $(v_0 - \bar{v}_0)/(\bar{v}_0 - V_0)$ for most parameter values. This finding is especially true for high absolute values of the correlation (see table 8), which suggests again that the hedging motive explains a large part of debt-allocation decisions. The fact that the speculative motive plays a limited role in a capital-structure model can be explained as follows: because we have assumed that the objective of debt management is to maximise the total value of the firm, increasing the cost of debt alone transfers welfare from shareholders to bondholders and, conversely, decreasing it leads to a transfer in the opposite direction; but the final impact on the total value is only small. On the other hand, by matching the interest-rate exposure of their liabilities and that of their assets, firms can reduce the variability of their cash flows and as a result lower the likelihood and expected costs of financial distress or capture a greater tax shield benefit (or both).

Our numerical experiments provide a quantitative assessment of the associated benefits in terms of increases in total value.

5.2 Regression of $\theta^*$ on $\hat{\theta}_{hed}$

In an attempt to quantify the importance of the hedging motive in optimal debt-structure decisions, we now run a regression of $\theta^*$ on $\hat{\theta}_{hed}$. In other words, we estimate the following regression equation:

$$\theta_i = c + a_1 \hat{\theta}_{hed,i} + \varepsilon_i$$

(5.5)

where homoscedasticity is assumed for the residuals. In (5.5), subscript $i$ refers to the $i$th observation. To estimate this model, we
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first create observations for the two weights \( \theta_i^* \) and \( \theta_{hed,i} \). We do this by letting one or more parameters of the model vary over a pre-defined grid, while maintaining the other parameters at their base-case values (see table 2). The output of the procedure is a total of \( N \) sets of parameters, and we compute \( \theta^* \) and \( \theta_{hed,i} \) for each of these \( N \) sets. We thus get \( N \) observations of each of these portfolios, which we use to run the regression (5.5). The resulting estimates of \( \hat{a}_1 \) and \( \hat{c} \), which we denote \( \hat{a}_1 \) and \( \hat{c} \), depend ex ante on the parameters set during the generation of observations.

Goodness-of-fit of the model is traditionally assessed by the coefficient of determination, \( R^2 \), for which we adopt the following definition:

\[
R^2 = 1 - \frac{\sum_{i=1}^{N} (\theta_i^* - \hat{a}_1 \theta_{hed,i})^2}{\sum_{i=1}^{N} (\theta_i^* - \bar{\theta})^2}
\]

where \( \bar{\theta} \) is the mean of \( \theta_i^* \) over all observations. This coefficient compares the model (5.5) and the intercept-only model through their respective sums of squared errors. It is a decreasing function of the sum of squared residuals, and is always less than 1, except in the event of perfect fit (when all residuals are zero).

We choose \( \rho_{RV} \) and \( \lambda_R \) as the free parameters, and leave the other parameters fixed. We then repeat this process for several figures for the volatility of firm value, \( \sigma_{V} \), and the volatility of interest rates, \( \sigma_{R} \), which leads to a total number of 126 joint observations of \( \theta_{hed,i} \) and \( \theta_i^* \). Tables 10 and 11 show the results of the regression. We obtain very high \( R^2 \) values, suggesting that \( \theta^* \) is close to an affine function of \( \theta_{hed} \). The hedging motive \( \theta_{hed} \) appears to be highly significant, both from an economic standpoint (magnitude of the coefficient) and from a statistical standpoint (t-value). These results can be taken as yet another confirmation that the hedging motive \( \theta_{hed} \) explains the quasi-totality of the cross-sectional differences in the optimal debt structure \( \theta^* \).

5.3 Introducing Time-Varying Debt-Structure Strategies

We have hitherto considered only fixed-mix strategies, in which the allocation to fixed-rate debt is assumed to be constant over time and states of the world. This is clearly a restrictive assumption, even though it makes possible a fair comparison with the results in section 3. Although it does not seem feasible to solve the problem of determining an “optimal policy” (\( \theta_t^* \) \( t \in [0,T] \)) in the sense of the programme (4.8) (an optimal time-dependent debt-structure strategy that would maximise \( v_0 \) over all \( \mathcal{F}_t \)-adapted processes), we introduce in this subsection deterministically time-dependent strategies. 29

First, we explain how to price the claims written on the firm value \( V \) when a time-dependent non-fixed-mix issuing strategy is adopted. In subsections 2.5 and 4.2 we have shown how we can price the tax shield and bankruptcy costs when the parameters \( \theta \) in (2.8) and \( \theta \) in (4.6) are constant through time and across states of the world. Examining the proof of proposition 2 in appendix B, we see that the key condition for carrying out the computation was for the volatility vector of \( L \) to be deterministic. This property is preserved when the constant \( \theta \) in (2.8) (resp. \( \theta \) in (4.6)) is replaced by a deterministic process \( (\theta_t)_{t \in [0,T]} \) (resp. a deterministic vector process \( (\theta_t)_{t \in [0,T]} \)). Hence the tax shield and the

29 - Using models where interest-rate and other related sources of uncertainty are represented by a discrete (Markov regime switching), as opposed to continuous, state space model, a recent body of literature has studied the impact of predictable changes in market conditions on capital-structure decisions (Hackethal et al. 2006; Chen 2006; and Bahmra et al. 2007, Chen et al. 2009). An attempt to complement this recent literature with a joint analysis of state-dependent capital and debt structure decisions would be a non-trivial worthwhile extension of our paper.
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Bankruptcy costs are still given by (2.16) and (2.17), with the survival probabilities being computed as in propositions 2 and 3. The only changes lie in $\phi_1$ and $\phi_2$, which are now defined as:

$$\phi_1(t) = \int_0^t \|\sigma_V - \sigma_L(s, T)\theta_s\|^2 ds,$$

$$\phi_2(t) = \int_0^t \|\sigma_B(s, T) - \sigma_L(s, T)\theta_s\|^2 ds$$

when the firm issues three classes of debt, and:

$$\phi_1(t) = \int_0^t \|\sigma_V - \theta_s\sigma_B(s, T)\|^2 ds,$$

$$\phi_2(t) = \int_0^t \|\sigma_B(s, T) - \theta_s\sigma_B(s, T)\|^2 ds$$

when it issues only fixed-rate and floating-rate bonds.

The following proposition shows that, although we cannot find an optimal time-dependent strategy maximising $v_0$, it is still possible to find such a strategy for a slightly different objective. It establishes in fact that minimising the variance of $\ln(V_T/L_T)$ is equivalent to maximising the total value of the firm plus an additional term, equal to the present value of a payment $kL_0$ at time $T$ conditioned on the absence of default before $T$, and that this objective is also equivalent to the minimisation of the present value of bankruptcy costs.

**Proposition 5** • **Consider the programme of minimising bankruptcy costs:**

$$\max_{(\theta_t)_{t\in[0,T]}} BC_0$$

and the programme of maximising firm value plus the present value of a payment $kL_0$ at time $T$ conditioned on the absence of default prior $T$:

$$\max_{(\theta_t)_{t\in[0,T]}} [v_0 + kL_0B(0, T)\mathbf{Q}^T (\tau > T)]$$

Both programmes are equivalent to:

$$\min_{(\theta_t)_{t\in[0,T]}} \mathcal{V}^P [\ln V_T - \ln L_T]$$

(all maxima are taken over deterministically time-dependent strategies).

- If the firm can issue only fixed-rate and floating-rate bonds, their common solution reads:

$$\tilde{\theta}_{\text{hedg}} = \frac{\sigma_B(t, T)' \sigma_V}{\|\sigma_B(t, T)\|^2}$$

- If the firm can issue fixed-rate, floating-rate, and inflation-indexed bonds, the solution is given by:

$$\tilde{\theta}_{\text{hedg}} = [\sigma_L(t, T)' \sigma_L(t, T)]^{-1} \sigma_L(t, T)' \sigma_V$$

**Proof.** See appendix E.

A consequence of this proposition is that it is not equivalent, in general, to perform maximisation of $v_0$ over time-dependent strategies $(\theta_t)_t$, or to perform minimisation of $\mathcal{V}^P [\ln (V_T/L_T)]$ over these strategies. The equivalence is recovered only in the very special case in which there is no tax advantage to debt (i.e., when $k = 0$). But the proposition also shows that a pure hedging issuing strategy, a solution to (5.8), minimises the present value of bankruptcy costs.

A similar property has been established in the context of hedging theories as that developed in Smith and Stulz (1985) and stipulating that firms can lower expected bankruptcy costs by reducing the variability of their net cash flows.

We now turn to a numerical implementation of the time-dependent strategy introduced in proposition 5. The results are presented in tables 12 and 13. Panels A still show the capital structure and the fixed-mix debt structure that maximise firm value, whereas panels D show the face value of debt that
maximises firm value when the firm takes the time-dependent strategy. In the vast majority of cases, the time-dependent hedging strategy that is a solution to problem (5.8) outperforms the optimal fixed-mix strategy of panel A. Intuitively, this strategy outperforms because introducing time-dependent strategies allows a further increase in the correlation \( \text{Corr}(\ln V_t, \ln L_t) \), which is always higher than what it is under the assumption of a fixed-mix strategy. In table 12, we find that it is for high absolute values of the correlation that the differences in performances of the two strategies are highest. Again, the ratio \( (\tilde{\nu}_0 - \tilde{\nu}_0)/ (\tilde{\nu}_0 - V_0) \), where \( \tilde{\nu}_0 \) is the maximum firm value obtained for the time-dependent strategy, is used to measure the improvement debt-structure choices make on capital structure choices. If \( \rho_{RV} = 70\% \), this marginal improvement is 119.09\%, and it rises to 262.80\% for \( \rho_{RV} = 90\% \). These ratios were only 106.08\% and 208.91\% respectively when the debt structure was a fixed-mix (see table 8). The effect is still present, although much weaker, for low correlations. These results show that using the time-dependent hedging strategy for problem (5.8) as a proxy for the solution to programme (2.14) can provide additional substantial increases in total firm value.

5. A Formal Analysis of the Hedging Motive in Debt Management
6. Conclusions and Suggestions for Further Research
6. Conclusions and Suggestions for Further Research

Our understanding of liability-management decisions barely extends beyond the capital-structure decision (equity versus debt allocation), and when it addresses the debt-structure decision (fixed-versus floating-rate debt allocation), it relies mostly on qualitative insight. The main contribution of this paper is to provide a joint quantitative analysis of capital-structure decisions and debt-structure decisions in a standard continuous-time model in the presence of interest-rate and inflation risks. Our main findings are that debt-management decisions have an impact on capital-structure decisions, and that substantial increases in firm value can be induced by optimising debt structure. We also find that a number of corporations would benefit from issuing inflation-linked bonds, a segment of the bond market now mostly occupied by sovereign states.

Our analysis could be further extended in a number of directions. In particular, it would be useful to include other types of instruments, such as convertible bonds, preferred shares, and other equity-linked structures in the liability mix, in addition to fixed-rate and floating-rate bonds. From the technical standpoint, however, an explicit analysis of the optimal liability structure that includes such ingredients might prove challenging. On a different note, our model, following previous literature, considers the liability-allocation problem from the standpoint of the original owners of the firm, who are assumed to be risk-neutral with respect to the (diversifiable) source of uncertainty impacting firm value. In practice, however, the managers of the firm, unlike the owners, are in charge of making corporate risk-management and liability-allocation decisions. Several papers have documented the role of conflicts of interest and managerial incentives in the design of corporate debt-structure programmes (Smith and Stulz 1985 and Stulz 1984, as well as Chava and Purnanandam 2007), and incorporating these aspects beyond the generic model discussed in section 5 would also be desirable. These and other related questions are left for further research.
Appendices

A. Proof of Proposition 1
The derivation of $TS_0$ from (2.13) and (2.15) is immediate. For $BC_0$, we first note that by definition of default time (see (2.9)), we have that:

$$BC_0 = o\mathbb{E}^\mathcal{Q}\left[e^{-\int_0^{T\wedge T} R_d u_t e^{\lambda(T-t)}qL_t}1_{\{T\leq T\}}\right]$$

Note that the random variable $e^{\lambda(T-t)}1_{\{T\leq T\}}$ is $\mathcal{F}_{T\wedge T}$-measurable and that $\{e^{-\int_0^{T\wedge T} R_d u_t L_t}\}$ follows a $\mathcal{Q}$-martingale. From theorem 3.22 in Karatzas and Shreve (2000), we thus have that

$$\mathbb{E}^\mathcal{Q}\left[e^{-\int_0^{T\wedge T} R_d u_t L_t}1_{\mathcal{F}_{T\wedge T}}\right] = e^{-\int_0^{T\wedge T} R_d u_t L_t}$$

Hence:

$$BC_0 = o\mathbb{E}^\mathcal{Q}\left[e^{-\int_0^{T\wedge T} R_d u_t e^{\lambda(T-t)}qL_t}1_{\{T\leq T\}}\right]$$

$$= o\mathbb{E}^\mathcal{Q}L_0\mathbb{E}^\mathcal{Q}\left[e^{\lambda(T-t)}1_{\{T\leq T\}}\right]$$

$$= o\mathbb{E}^\mathcal{Q}_0\int_0^T e^{\lambda(T-t)}f^L(t) dt$$

where $f^L$ denotes the density of $\tau$ under $\mathcal{Q}^T$.

From Karatzas and Shreve (2000) (theorem 4.6, page 174), we know that any continuous local martingale can be written as a time-changed Brownian motion. More precisely, there is a $\mathcal{Q}^T$-Brownian motion $B^L$ such that:

$$B^L_{\tau(t)} = \int_0^\tau [\sigma_V - \theta\sigma_B(u,T)]^t dz^L_t$$

We thus rewrite the cumulative default probability as:

$$Q^L(\tau \leq t) = Q^L\left(\min_{\tau \leq T} \frac{e^{\delta t}V_t}{L_t} \leq qe^{\delta T}\right)$$

$$= Q^L\left(\min_{\tau \leq T} \left[B^L_{\tau(t)} - \frac{1}{2}\phi_1(t)\right] \leq \delta T - \ln \frac{V_0}{qL_0}\right)$$

The distribution of the running minimum of a drifted Brownian motion can be found in Karatzas and Shreve (2000). Hence the expression for $Q^L(\tau \leq t)$:

$$Q^L(\tau \leq t) = \mathcal{N}\left(-\ln \frac{V_0 e^{-\delta t}}{qL_0} + \frac{1}{2}\phi_1(t)\right)$$

$$+ \frac{V_0 e^{-\delta t}}{qL_0} \mathcal{N}\left(-\ln \frac{V_0 e^{-\delta t}}{qL_0} - \frac{1}{2}\phi_1(t)\right)$$

where

$$\phi_1(t) = \int_0^t \|\sigma_V - \theta\sigma_B(s,T)\|^2 ds$$

To see that $\phi_1(t)$ is equal to the variance of $\ln(V/L)$ under $\mathcal{Q}$, we use (2.3) and (2.8) to write that:

$$\frac{V_t}{L_t} = \ln \frac{V_0 e^{-\delta t}}{L_0} + \frac{1}{2} \int_0^t (\|\sigma_B(s,T)\|^2 - \|\sigma_V\|^2) ds$$

$$+ \int_0^t [-\theta\sigma_B(s,T)' + \sigma_V'] dz_s$$

which indeed shows that $\phi_1(t) = \mathcal{Q}^P[\ln V_T - \ln L_T]$.

B. Proof of Proposition 2

B.1 Derivation of $Q^L(\tau \leq t)$
We start from the expression of $e^{\delta s}V_s/L_s$ under $\mathcal{Q}^L$:

$$\frac{e^{\delta s}V_s}{L_s} = \frac{V_0}{L_0} \exp\left[\int_0^s [\sigma_V - \theta\sigma_B(u,T)]^t dz^L_t - \frac{1}{2} \int_0^s \|\sigma_V - \theta\sigma_B(u,T)\|^2 dt\right]$$

From Karatzas and Shreve (2000) (theorem 4.6, page 174), we know that any continuous local martingale can be written as a time-changed Brownian motion. More precisely, there is a $\mathcal{Q}^L$-Brownian motion $B^L$ such that:

$$B^L_{\phi(t)} = \int_0^\phi [\sigma_V - \theta\sigma_B(u,T)]^t dz^L_t$$

Let $X$ be the logarithm of the distance-to-default:

$$X_t = \ln \frac{e^{-\delta(t-s)}V_t}{L_t}$$

Under the $T$-forward probability measure, we have that:

$$X_t = -\delta T + \ln \frac{V_0}{L_0} + \int_0^t [\sigma_V - \theta\sigma_B(s,T)]' dz^T_s$$

$$+ \frac{1}{2} [\phi_2(t) - \phi_3(t)]$$

where $z^T$ is a $\mathcal{Q}^T$-Wiener process, which
shows that $X$ is continuous, Gaussian, and has the Markov property. Default is triggered when $X$ crosses the barrier $X = \ln q$ from above. We also let $\pi(X_t, t|X_0, 0)$ be the transition density from $X_0$ at time 0 to $X_t$ at time $t$, and with $g(t|X_0, 0)$ the probability density that $\tau$ is equal to $t$ when the process starts from $X_0$ at time 0, i.e., $g(t|X_0, 0) = \frac{1}{\Delta t} Q^T(t \leq \tau < t + \Delta t | X_0, 0)$. The quantity of interest, $Q^T(\tau > T)$, can be written as:

$$Q^T(\tau > T) = 1 - \int_0^T g(t|X_0, 0) \, dt$$

Hence we need to estimate $g(t|X_0, 0)$ for $t \leq T$. The following equation, due to Fortet (1943), holds for $X_t < X < X_0$:

$$\pi(X_t, t|X_0, 0) = \int_0^t \pi(X_t, t|X_s, s) g(s|X_0, 0) \, ds$$

It states that to be at the level $X_t$ under the barrier at time $t$, the process must have crossed the barrier at some date $s$ between 0 and $t$. Integrating both sides by $\int_{-\infty}^X \, dX_0$, we obtain that:

$$Q^T(X_t < X|X_0, 0) = \int_0^t g(s|X_0, 0) Q^T(X_t < X|X_s, s) \, ds \quad (B.1)$$

We note that, for $s \leq t$:

$$X_t = X_s + \frac{1}{2} [\phi_2(t) - \phi_2(s) + \phi_3(t) - \phi_3(s)] + \int_s^t [\sigma V - \theta \sigma B(u, T)] \, dz_u^T$$

hence the conditional moments, for $s \geq t$:

$$E^{Q^T}[X_t|X_s] = X_s + \frac{1}{2} [\phi_2(t) - \phi_2(s) + \phi_3(t) - \phi_3(s)]$$

$$\nabla^{Q^T}[X_t|X_s] = \phi_1(t) - \phi_1(s)$$

We thus obtain:

$$Q^T(X_t < X|X_s, s) = \mathcal{N} \left( \frac{X - X_s - \frac{1}{2} [\phi_2(t) - \phi_2(s)]}{\sqrt{\phi_3(t)}} \right) = G_1(t)$$

$$Q^T(X_t < X|X_s = X_s, s) = \mathcal{N} \left( -\frac{1}{2} [\phi_2(t) - \phi_2(s) - \phi_3(t) + \phi_3(s)]}{\sqrt{\phi_1(t) - \phi_1(s)}} \right) = G_2(t, s)$$

We now turn to the discretisation of (B.1). We discretise time into $n_T$ intervals and define $\Delta t = T/n_T$ and $t_i = i \Delta t$ for $i = 0, \ldots, n_T$. The discrete version of (B.1) with integrand estimated at midpoints of the interval (as in Collin-Dufresne and Goldstein [2001]) can be written as:

$$G_i(t_i) = \sum_{u=0}^{i-1} G_2 \left( t_i, (2u + 1) \frac{\Delta t}{2} \right) \bar{g} \left( (2u + 1) \frac{\Delta t}{2} \right) \Delta t,$$

where $\bar{g}(s)$ is our estimator of $g(s|X_0, 0)$. It can be checked numerically that the resulting estimator of $Q^T(\tau > T)$ is only not significantly sensitive to the choice of midpoints rather than endpoints in the discretisation, and that a satisfactory convergence is obtained for a discretisation step $T/n_T$ of 0.1.

C. Prices of Risk-Free Bonds in the Presence of Inflation Risk

The prices of the nominal zero-coupon bond paying 1 in nominal terms at time $T$ and of the indexed zero-coupon bond paying $\Phi_T$ are given by:

$$B(t, T) = \mathbb{E}^{Q^T} \left[ e^{-\int_t^T R_u \, ds} \right]$$

and

$$I(t, T) = \mathbb{E}^{Q^T} \left[ e^{-\int_t^T R_u \, ds} \Phi_T \right]$$

From the properties of Ornstein-Uhlenbeck processes, we have that:

$$\int_t^T r_u \, du = b(T - t) - \frac{1}{\kappa} (1 - e^{-\kappa(T - t)}) (b - r_t)$$

$$+ \frac{1}{\kappa} \int_t^T (1 - e^{-\kappa(T - u)}) \sigma_u' \, dz_u$$

$$\int_t^T \pi_u \, du = \beta(T - t) - \frac{1}{\kappa} (1 - e^{-\kappa(T - t)}) (\beta - \pi_t)$$

$$+ \frac{1}{\kappa} \int_t^T (1 - e^{-\kappa(T - u)}) \sigma_u' \, dz_u$$
After some algebra we arrive at:

\[
B(t, T) = \exp \left[ -t - \frac{1 - e^{-a(T-t)}}{a} \right] r_t \left( 1 - e^{-a(T-t)} \right) \pi_t - b \left( T - t - \frac{1 - e^{-a(T-t)}}{a} \right) \]

and:

\[
\beta \left( T - t - \frac{1 - e^{-a(T-t)}}{a} \right) + \sigma \phi \Delta \phi (T - t) + \frac{\sigma_r^2 \sigma \phi}{2a} \left( T - t - \frac{1 - e^{-a(T-t)}}{a} \right) - \frac{2}{2a} \frac{1 - e^{-a(T-t)}}{a} \]

hence:

\[
\sigma_B(t, T) = B(T - t) \sigma_r + C(T - t) \sigma \pi
\]

and:

\[
\sigma_I(t, T) = \sigma \phi + B(T - t) \sigma_r
\]

D. Proof of Proposition 4

We omit the arguments of \( \sigma \) and we present the proof in the case where three classes of debt (fixed-rate, floating-rate, and inflation-indexed) are available. The argument below can be adapted to the case in which only two types of bonds are issued by replacing the volatility vector of \( L \), \( \sigma \), with \( \theta \sigma \). Given that the volatility of \( V/L \) is the same under probability measures \( \mathbb{P} \) and \( \mathbb{Q} \), the variance of the log funding ratio \( \ln(V/L) \) can be indifferently computed under \( \mathbb{P} \) and \( \mathbb{Q} \):

\[
\mathbb{V}^\mathbb{P} \left[ \ln \left( \frac{V_T}{L_T} \right) \right] = \mathbb{V}^\mathbb{Q} \left[ \ln \left( \frac{V_T}{L_T} \right) \right]
\]

From (2.3) and (4.6), the log-funding ratio under \( \mathbb{Q} \) is given by:

\[
\ln \left( \frac{V_T}{L_T} \right) = \ln \left( \frac{V_0 e^{-rT}}{L_0} \right) + \frac{1}{2} \int_0^T \left( \| \sigma_r \theta \| - \| \sigma \phi \|^2 \right) dt + \int_0^T \left( \sigma_r \theta - \sigma \phi \right)' dz_t
\]

Hence, for any strategy \( \theta \):

\[
\mathbb{V}^\mathbb{P} \left[ \ln \left( \frac{V_T}{L_T} \right) \right] = \int_0^T \| \sigma_r \theta - \sigma \|^2 dt
\]

so that (5.2) is equivalent to minimising the following quantity over \( \theta \):

\[
\theta' \left( \int_0^T \sigma_r^2 \sigma \phi \ dt \right) \theta - 2 \theta' \left( \int_0^T \sigma_r^2 \sigma \phi \ dt \right)
\]

Standard quadratic maximisation yields the expression for \( \hat{\theta} \).

E. Proof of Proposition 5

We present the proof in the case in which the firm issues fixed-rate, floating-rate, and indexed bonds. The adaptation to the special case in which it issues only fixed-rate and floating-rate bonds is straightforward, up to some changes in notations. First, we note that from proposition 1, we have that:

\[
BC_0 = \alpha Q(\mathbb{P}) \left[ Q^r(\tau \leq T) + \delta e^{rT} \int_0^T e^{-rT} Q^r(\tau \leq t) dt \right]
\]

and:

\[
v_0 + kL_0 B(0, T)Q_T(\tau > T) = V_0 - (\alpha Q + k) Q^r(\tau \leq T)
\]

\[
- \alpha L_0 \delta e^{rT} \int_0^T e^{-rT} Q^r(\tau \leq t) dt
\]
where $Q^L(\tau \leq t)$ is given by proposition 3 and $\phi_1(t)$ by (2.20). Differentiating $Q^L(\tau \leq t)$ with respect to $\phi_1(t)$ yields:

$$\frac{\partial Q^L(\tau \leq t)}{\partial \phi_1(t)} = \frac{1}{\phi_1(t)} e^{-\frac{1}{2\phi_1(t)} \left[ \ln \frac{V_0 e^{-\delta T}}{qL_0} + \frac{1}{2} \phi_1(t) \right]^2} \frac{V_0 e^{-\delta T}}{qL_0}$$

Since the initial conditions imply that $V_0 > e^{\delta T} qL_0$, this partial derivative is positive, for any $t \in [0, T]$. We now consider the following optimisation programme:

$$\min_{(\theta_s)_{s \in [0,t]}} \phi_1(t) \quad \text{(E.1)}$$

where the optimum is over all deterministic strategies.

Since $\phi_1(t) = \int_0^t \| \sigma_V - \sigma_L(s, T) \theta_s \|^2 ds$, it suffices to minimise each term in the integral. Standard quadratic optimisation shows that the solution to (E.1) is given by:

$$\hat{\theta}_s = [\sigma_L(s, T) \sigma_L(s, T)]^{-1} \sigma_L(s, T)^{\prime} \sigma_V$$

Hence $(\hat{\theta}_s)$ is the solution to (5.6) and (5.7). Obviously, it is also the solution to (5.8).
Appendices

F. Tables

Table 1: Summary statistics on the proportion of long-term floating-rate debt within total long-term debt.

<table>
<thead>
<tr>
<th>Year</th>
<th>1996</th>
<th>2001</th>
<th>2006</th>
</tr>
</thead>
<tbody>
<tr>
<td>Median allocation to floating-rate debt (%)</td>
<td>20</td>
<td>15</td>
<td>16</td>
</tr>
<tr>
<td>Top 25% allocation (%)</td>
<td>93</td>
<td>82</td>
<td>81</td>
</tr>
<tr>
<td>Percentage of firms that issue floating-rate debt (%)</td>
<td>48</td>
<td>38</td>
<td>42</td>
</tr>
</tbody>
</table>

Source: Standard & Poor’s Compustat database, variables DATA9 (Long-term debt - Total) and DATA148 (Long-term debt tied to prime). Only those firms for which both sets of information were available are shown.

Table 2: Base-case parameter values.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>0.261</td>
<td>0.261</td>
<td>1.00</td>
<td>0.2</td>
</tr>
<tr>
<td>$b$</td>
<td>0.05</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>$\lambda_R$</td>
<td>–0.00075</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>$\sigma_R$</td>
<td>0.0224</td>
<td>0.0224</td>
<td>0.0316</td>
<td>0.02</td>
</tr>
<tr>
<td>$\rho_{RV}$</td>
<td>0.05</td>
<td>0.05</td>
<td>–0.25</td>
<td>–0.25</td>
</tr>
<tr>
<td>$\sigma_V$</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>$T$</td>
<td>10</td>
<td>–</td>
<td>[1, 20]</td>
<td>[1, 20]</td>
</tr>
<tr>
<td>$q$</td>
<td>0.7</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>$k$</td>
<td>0.35</td>
<td>0.35</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.5</td>
<td>0.5</td>
<td>–</td>
<td>0.2</td>
</tr>
<tr>
<td>$R_0$</td>
<td>0.05</td>
<td>0.07</td>
<td>–</td>
<td>0.05</td>
</tr>
<tr>
<td>$V_0$</td>
<td>100</td>
<td>100</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

$a$ is the speed of mean reversion in the short-term nominal interest rate, $\lambda_R$ is its long-term mean under the historical probability measure $P$, $b$ is its long-term mean under the risk-neutral probability measure $Q$, and $\sigma_V$ is its volatility; $\delta$ is the payout rate of the firm and $\rho_{RV}$ is the correlation between changes in the asset value and changes in interest rates; $T$ is the maturity of debt; $k$ is the tax rate and $\alpha$ is the rate of bankruptcy costs. The initial unlevered asset value is set at $V_0$ and the initial short-term nominal rate at $R_0$. 

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Table 3: Optimal capital and debt structures with fixed-mix strategies – Impact of the correlation $\rho_{RV}$ between firm’s risk and interest-rate risk.

### Panel A: Optimal debt and capital structures

<table>
<thead>
<tr>
<th>$\rho_{RV}$</th>
<th>-0.99</th>
<th>-0.9</th>
<th>-0.7</th>
<th>-0.5</th>
<th>-0.3</th>
<th>-0.1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed-rate debt $\theta$</td>
<td>3.006</td>
<td>2.708</td>
<td>2.088</td>
<td>1.511</td>
<td>0.937</td>
<td>0.367</td>
<td>0.083</td>
</tr>
<tr>
<td>Floating-rate debt $1 - \theta$</td>
<td>-2.006</td>
<td>-1.709</td>
<td>-1.098</td>
<td>-0.511</td>
<td>0.063</td>
<td>0.633</td>
<td>0.917</td>
</tr>
<tr>
<td>Total value $V_0$</td>
<td>105.925</td>
<td>104.628</td>
<td>103.322</td>
<td>102.727</td>
<td>102.245</td>
<td>102.265</td>
<td>102.280</td>
</tr>
<tr>
<td>Volatility of $\ln V_T$</td>
<td>0.216</td>
<td>0.236</td>
<td>0.280</td>
<td>0.325</td>
<td>0.369</td>
<td>0.413</td>
<td>0.435</td>
</tr>
<tr>
<td>Volatility of $\ln(V_T/L_T)$</td>
<td>0.061</td>
<td>0.120</td>
<td>0.231</td>
<td>0.314</td>
<td>0.369</td>
<td>0.397</td>
<td>0.400</td>
</tr>
<tr>
<td>Correlation of $\ln V_T$ and $\ln L_T$</td>
<td>0.848</td>
<td>0.703</td>
<td>0.423</td>
<td>0.186</td>
<td>0.019</td>
<td>0.201</td>
<td>0.285</td>
</tr>
<tr>
<td>Leverage ratio $D_0/V_0$ (%)</td>
<td>44.920</td>
<td>30.765</td>
<td>28.084</td>
<td>23.928</td>
<td>21.620</td>
<td>20.643</td>
<td>20.524</td>
</tr>
<tr>
<td>Default probability (%)</td>
<td>0.067</td>
<td>0.339</td>
<td>1.012</td>
<td>1.511</td>
<td>1.829</td>
<td>1.991</td>
<td>2.049</td>
</tr>
</tbody>
</table>

### Panel B: Optimal capital structure with fixed-rate debt

<table>
<thead>
<tr>
<th>$\rho_{RV}$</th>
<th>-0.99</th>
<th>-0.9</th>
<th>-0.7</th>
<th>-0.5</th>
<th>-0.3</th>
<th>-0.1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Face value $L_0$</td>
<td>21.863</td>
<td>22.922</td>
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<tr>
<td>Fixed-rate debt $\theta$</td>
<td>-0.202</td>
<td>-0.773</td>
<td>-1.350</td>
<td>-1.941</td>
<td>-2.560</td>
<td>-2.862</td>
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</tr>
<tr>
<td>Floating-rate debt $1 - \theta$</td>
<td>1.202</td>
<td>1.773</td>
<td>2.350</td>
<td>2.941</td>
<td>3.560</td>
<td>3.862</td>
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</tr>
<tr>
<td>Total value $V_0$</td>
<td>102.296</td>
<td>102.429</td>
<td>102.735</td>
<td>103.335</td>
<td>104.062</td>
<td>105.959</td>
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<tr>
<td>Volatility of $\ln V_T$</td>
<td>0.458</td>
<td>0.502</td>
<td>0.546</td>
<td>0.590</td>
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<tr>
<td>Volatility of $\ln(V_T/L_T)$</td>
<td>0.397</td>
<td>0.370</td>
<td>0.315</td>
<td>0.233</td>
<td>0.122</td>
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</tr>
<tr>
<td>Correlation of $\ln V_T$ and $\ln L_T$</td>
<td>0.505</td>
<td>0.615</td>
<td>0.803</td>
<td>0.981</td>
<td>0.991</td>
<td>0.995</td>
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</tr>
<tr>
<td>Leverage ratio $D_0/V_0$ (%)</td>
<td>20.639</td>
<td>21.606</td>
<td>21.801</td>
<td>27.978</td>
<td>36.629</td>
<td>44.674</td>
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<tr>
<td>Default probability (%)</td>
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<td>1.611</td>
<td>1.130</td>
<td>0.426</td>
<td>0.103</td>
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### Relative importance of debt versus capital structure decisions

<table>
<thead>
<tr>
<th>$\rho_{RV}$</th>
<th>-0.99</th>
<th>-0.9</th>
<th>-0.7</th>
<th>-0.5</th>
<th>-0.3</th>
<th>-0.1</th>
<th>0</th>
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<tbody>
<tr>
<td>Relative increase in total value (%)</td>
<td>71.681</td>
<td>41.234</td>
<td>12.919</td>
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<td>3.410</td>
<td>7.195</td>
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<tr>
<td>Relative increase in total value (%)</td>
<td>12.537</td>
<td>29.012</td>
<td>56.876</td>
<td>106.077</td>
<td>208.909</td>
<td>305.336</td>
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</table>

Panel A describes the optimal capital and debt structures when the firm solves the programme (2.14), and panel B shows the optimal capital structure when the firm issues only fixed-rate bonds. $L_T$ is the promised payment to bondholders and $V_T$ is the terminal unlevered value of the firm. The relative increase in total value is the ratio $(V_T - \tilde{V}_0)/(\tilde{V}_0 - V_0)$, where $\tilde{V}_0$ is the maximum total value achieved with fixed-rate debt only and $V^*_0$ is the maximum total value achieved with fixed-rate and floating-rate bonds. Unless otherwise stated, parameters are fixed at their base-case values (see table 2).
Table 4: Optimal capital and debt structures with fixed-mix strategies – Impact of the market price of interest rate risk $\lambda_R$.

### Panel A: Optimal debt and capital structures

<table>
<thead>
<tr>
<th>$\lambda_R$</th>
<th>$-0.99$</th>
<th>$-0.9$</th>
<th>$-0.7$</th>
<th>$-0.5$</th>
<th>$-0.3$</th>
<th>$-0.1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Face value $L_0$</td>
<td>23.478</td>
<td>23.199</td>
<td>22.891</td>
<td>22.548</td>
<td>22.163</td>
<td>21.729</td>
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<tr>
<td>Fixed-rate debt $\theta$</td>
<td>0.002</td>
<td>0.006</td>
<td>0.020</td>
<td>0.074</td>
<td>0.078</td>
<td>0.083</td>
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<tr>
<td>Floating-rate debt $1 - \theta$</td>
<td>0.938</td>
<td>0.994</td>
<td>0.980</td>
<td>0.926</td>
<td>0.922</td>
<td>0.917</td>
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<tr>
<td>Total value $v_0$</td>
<td>103.385</td>
<td>101.183</td>
<td>98.971</td>
<td>97.500</td>
<td>97.500</td>
<td>97.278</td>
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<td>Volatility of $\ln V_T$</td>
<td>0.435</td>
<td>0.435</td>
<td>0.435</td>
<td>0.435</td>
<td>0.435</td>
<td>0.435</td>
</tr>
<tr>
<td>Volatility of $\ln(V_T/L_T)$</td>
<td>0.400</td>
<td>0.400</td>
<td>0.400</td>
<td>0.400</td>
<td>0.400</td>
<td>0.400</td>
</tr>
<tr>
<td>Correlation of $\ln V_T$ and $\ln L_T$</td>
<td>0.265</td>
<td>0.285</td>
<td>0.285</td>
<td>0.285</td>
<td>0.285</td>
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<tr>
<td>Default probability (%)</td>
<td>2.973</td>
<td>2.804</td>
<td>2.625</td>
<td>2.434</td>
<td>2.231</td>
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### Panel B: Optimal capital structure with fixed-rate debt

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<th>$\lambda_R$</th>
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<th>$-0.9$</th>
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</thead>
<tbody>
<tr>
<td>Fixed-rate debt $\theta$</td>
<td>1.000</td>
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<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>Floating-rate debt $1 - \theta$</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Total value $v_0$</td>
<td>103.166</td>
<td>102.978</td>
<td>102.779</td>
<td>102.570</td>
<td>102.352</td>
<td>102.125</td>
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<tr>
<td>Volatility of $\ln V_T$</td>
<td>0.435</td>
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<td>0.435</td>
<td>0.435</td>
<td>0.435</td>
<td>0.435</td>
</tr>
<tr>
<td>Volatility of $\ln(V_T/L_T)$</td>
<td>0.435</td>
<td>0.435</td>
<td>0.435</td>
<td>0.435</td>
<td>0.435</td>
<td>0.435</td>
</tr>
<tr>
<td>Correlation of $\ln V_T$ and $\ln L_T$</td>
<td>0.265</td>
<td>0.285</td>
<td>0.285</td>
<td>0.285</td>
<td>0.285</td>
<td>0.285</td>
</tr>
<tr>
<td>Leverage ratio $D_0/v_0$ (%)</td>
<td>20.590</td>
<td>20.407</td>
<td>20.108</td>
<td>19.691</td>
<td>19.691</td>
<td>19.373</td>
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<tr>
<td>Default probability (%)</td>
<td>6.741</td>
<td>5.436</td>
<td>4.315</td>
<td>3.396</td>
<td>2.574</td>
<td>1.924</td>
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</table>

### Relative importance of debt versus capital structure decisions

<table>
<thead>
<tr>
<th>$\lambda_R$</th>
<th>$-0.99$</th>
<th>$-0.9$</th>
<th>$-0.7$</th>
<th>$-0.5$</th>
<th>$-0.3$</th>
<th>$-0.1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative increase in total value (%)</td>
<td>6.792</td>
<td>6.855</td>
<td>6.924</td>
<td>7.003</td>
<td>7.093</td>
<td>7.196</td>
</tr>
</tbody>
</table>

Panel A describes the optimal capital and debt structures when the firm solves the programme (2.14), and panel B shows the optimal capital structure when the firm issues only fixed-rate bonds. $L_0$ is the promised payment to bondholders and $V_T$ is the terminal unlevered value of the firm. The relative increase in total value is the ratio $(v_T - v_0)/(v_T - v_0)$, where $v_0$ is the maximum total value achieved with fixed-rate debt only and $v_T$ is the maximum total value achieved with fixed-rate and floating-rate bonds. Unless otherwise stated, parameters are fixed at their base-case values (see table 2).
Table 5: Optimal capital and debt structures with fixed-mix strategies – Impact of the volatility $\sigma_R$ of the nominal short-term interest rate.

<table>
<thead>
<tr>
<th>$\sigma_R$</th>
<th>0.01</th>
<th>0.02</th>
<th>0.03</th>
<th>0.04</th>
<th>0.05</th>
<th>0.06</th>
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<tbody>
<tr>
<td>Fixed-rate debt $\theta$</td>
<td>0.081</td>
<td>0.082</td>
<td>0.084</td>
<td>0.087</td>
<td>0.092</td>
<td>0.098</td>
</tr>
<tr>
<td>Floating-rate debt $1 - \theta$</td>
<td>0.919</td>
<td>0.918</td>
<td>0.916</td>
<td>0.913</td>
<td>0.908</td>
<td>0.902</td>
</tr>
<tr>
<td>Total value $v_T$</td>
<td>102.340</td>
<td>102.295</td>
<td>102.219</td>
<td>101.210</td>
<td>100.198</td>
<td>100.070</td>
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<tr>
<td>Volatility of $\ln V_T$</td>
<td>0.407</td>
<td>0.428</td>
<td>0.463</td>
<td>0.513</td>
<td>0.576</td>
<td>0.654</td>
</tr>
<tr>
<td>Volatility of $\ln(V_T/L_T)$</td>
<td>0.400</td>
<td>0.360</td>
<td>0.400</td>
<td>0.401</td>
<td>0.402</td>
<td>0.402</td>
</tr>
<tr>
<td>Correlation of $\ln V_T$ and $\ln L_T$</td>
<td>0.132</td>
<td>0.257</td>
<td>0.370</td>
<td>0.469</td>
<td>0.553</td>
<td>0.623</td>
</tr>
<tr>
<td>Leverage ratio $D_0/v_0$ (%)</td>
<td>20.609</td>
<td>20.549</td>
<td>20.437</td>
<td>20.277</td>
<td>20.057</td>
<td>19.758</td>
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</table>

Panel A: Optimal debt and capital structures

Panel B: Optimal capital structure with fixed-rate debt

Relative importance of debt versus capital structure decisions

Relative increase in total value (%) | 1.411 | 5.713 | 13.121 | 24.045 | 39.176 | 59.636 |

Table 6: Base-case parameters in the presence of inflation risk.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>$a$</td>
<td>0.631</td>
<td>$\delta$</td>
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<tr>
<td>$b$</td>
<td>0.012</td>
<td>$\tau_0$</td>
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</tr>
<tr>
<td>$\kappa$</td>
<td>0.027</td>
<td>$\pi_0$</td>
<td>0.045</td>
</tr>
<tr>
<td>$\beta$</td>
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<td>$V_0$</td>
<td>100</td>
</tr>
<tr>
<td>$\rho_{\pi\pi}$</td>
<td>0.0126</td>
<td>$\Lambda_r$</td>
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</tr>
<tr>
<td>$\rho_{\pi\pi}$</td>
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<tr>
<td>$\rho_{\phi\phi}$</td>
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<td>$\Lambda_{V}$</td>
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<td>$\rho_{\phi\phi}$</td>
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<td>$\sigma_r$</td>
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<td>$\sigma_{\pi}$</td>
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<td>$\sigma_{\phi}$</td>
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</table>

This table lists the base-case parameters in the model with inflation risk. $a$ is the speed of mean reversion for the real rate process, and $b$ is its long-term mean. $\kappa$ and $\beta$ have similar definitions for the expected inflation rate process. The quantities $\rho_{ij}$ for $i$ and $j$ lying within the set of indices $\{\pi, \phi, \Phi, V\}$ are the pairwise correlations between unexpected changes in the state variables; and the quantities $\sigma_i$ are the volatilities of these variables.
### Table 7: Optimal capital and debt structures with fixed-mix strategies – Impacts of the correlation between real interest-rate risk and firm’s risk ($\rho_{RV}$) and of the correlation between price index risk and firm’s risk ($\rho_{\Phi V}$).

#### Panel A: Optimal debt and capital structures

<table>
<thead>
<tr>
<th>$\rho_{RV}$</th>
<th>$\rho_{\Phi V}$</th>
<th>LT</th>
<th>VT</th>
<th>$\Delta V$</th>
<th>$\Delta V_{opt}$</th>
<th>$\Delta V_{fix}$</th>
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</table>

#### Panel B: Optimal capital structure with fixed-rate debt

<table>
<thead>
<tr>
<th>$\rho_{RV}$</th>
<th>$\rho_{\Phi V}$</th>
<th>LT</th>
<th>VT</th>
<th>$\Delta V$</th>
<th>$\Delta V_{opt}$</th>
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</tr>
<tr>
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<td>0.3</td>
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<td>530.25</td>
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<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.4</td>
<td>0.3</td>
<td>530.25</td>
<td>530.25</td>
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<td>0.00</td>
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<tr>
<td>0.5</td>
<td>0.3</td>
<td>530.25</td>
<td>530.25</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Panel A describes the optimal capital and debt structures when the firm solves the programme (4.8), and panel B shows the optimal capital structure when the firm maximises its total value but issues only fixed-rate bonds. $L_r$ denotes the promised payment to bondholders and $V_{opt}$ denotes the terminal unlevered value of the firm. The relative increase in total value is the ratio $(\Delta V_{opt} - \Delta V_{fix})/(\Delta V_{opt} - V_r)$, where $\Delta V_{opt}$ is the maximum total value achieved with two classes of debt (reported in panel A) and $\Delta V_{fix}$ is the maximum total value achieved with fixed-rate debt only (see panel B). Unless otherwise stated, parameters are fixed at their base-case values (see table 6).
Table 8: Risk-minimising capital and debt structures with fixed-mix strategies – Impact of the correlation $\rho_{RV}$ between firm’s risk and interest rate risk.

<table>
<thead>
<tr>
<th>$\rho_{RV}$</th>
<th>−0.99</th>
<th>−0.9</th>
<th>−0.7</th>
<th>−0.5</th>
<th>−0.3</th>
<th>−0.1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed-rate debt</td>
<td>3.800</td>
<td>2.710</td>
<td>2.037</td>
<td>1.512</td>
<td>0.937</td>
<td>0.567</td>
<td>0.082</td>
</tr>
<tr>
<td>Relative increase in total value (%)</td>
<td>71.681</td>
<td>41.234</td>
<td>12.919</td>
<td>2.439</td>
<td>0.085</td>
<td>3.410</td>
<td>7.195</td>
</tr>
<tr>
<td>Volatility of $\ln V_T$</td>
<td>0.465</td>
<td>0.486</td>
<td>0.530</td>
<td>0.570</td>
<td>0.607</td>
<td>0.643</td>
<td>0.660</td>
</tr>
<tr>
<td>Volatility of $\ln V_T - \ln L_T$</td>
<td>0.247</td>
<td>0.246</td>
<td>0.480</td>
<td>0.600</td>
<td>0.607</td>
<td>0.630</td>
<td>0.623</td>
</tr>
<tr>
<td>Correlation of $\ln V_T$ and $\ln L_T$</td>
<td>0.848</td>
<td>0.703</td>
<td>0.423</td>
<td>0.186</td>
<td>0.019</td>
<td>0.201</td>
<td>0.285</td>
</tr>
<tr>
<td>Leverage ratio</td>
<td>0.450</td>
<td>0.365</td>
<td>0.269</td>
<td>0.238</td>
<td>0.216</td>
<td>0.207</td>
<td>0.205</td>
</tr>
</tbody>
</table>

Panel A shows the optimal capital and debt structures when the firm solves programme (2.14), and panel C displays the optimal capital structure when the firm maximises its total value subject to a debt structure of the form (5.4). The relative increase in total value in panel A is the ratio $(v_T - \bar{v})/(\bar{v} - \bar{v}_0)$, where $\bar{v}_0$ is the maximum total value achieved with fixed-rate debt only and $v_T$ is the maximum total value achieved with fixed-rate and floating-rate bonds. The relative increase in total value in panel C is the ratio $(v_T - \bar{v}_0)/(\bar{v}_0 - V_0)$, where $\bar{v}_0$ is the maximum total value achieved with an issuance strategy of the form (5.4). $L_T$ denotes the promised payment to bondholders and $V_T$ denotes the terminal unlevered value of the firm. Unless otherwise stated, parameters are fixed at their base-case values (see table 2).
Table 9: Risk-minimising capital and debt structures with fixed-mix strategies – Impact of the market price of interest rate risk $\lambda_R$.

<table>
<thead>
<tr>
<th>$\lambda_R$</th>
<th>-0.5</th>
<th>-0.4</th>
<th>-0.3</th>
<th>-0.2</th>
<th>-0.1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Face value</td>
<td>23.481</td>
<td>23.200</td>
<td>22.800</td>
<td>22.547</td>
<td>22.162</td>
<td>21.729</td>
</tr>
<tr>
<td>Fixed-rate debt</td>
<td>0.062</td>
<td>0.066</td>
<td>0.069</td>
<td>0.073</td>
<td>0.078</td>
<td>0.083</td>
</tr>
<tr>
<td>Relative increase in total value (%)</td>
<td>6.792</td>
<td>6.855</td>
<td>6.924</td>
<td>7.003</td>
<td>7.093</td>
<td>7.196</td>
</tr>
<tr>
<td>Volatility of $\ln V_T$</td>
<td>0.660</td>
<td>0.660</td>
<td>0.660</td>
<td>0.660</td>
<td>0.660</td>
<td>0.660</td>
</tr>
<tr>
<td>Volatility of $\ln V_T - \ln L_T$</td>
<td>0.633</td>
<td>0.633</td>
<td>0.633</td>
<td>0.633</td>
<td>0.633</td>
<td>0.633</td>
</tr>
<tr>
<td>Correlation of $\ln V_T$ and $\ln L_T$</td>
<td>0.285</td>
<td>0.285</td>
<td>0.285</td>
<td>0.285</td>
<td>0.285</td>
<td>0.285</td>
</tr>
<tr>
<td>Leverage ratio</td>
<td>0.217</td>
<td>0.215</td>
<td>0.213</td>
<td>0.211</td>
<td>0.208</td>
<td>0.205</td>
</tr>
</tbody>
</table>

Panel A shows the optimal capital and debt structures when the firm solves programme (2.14), and panel C displays the optimal capital structure when the firm maximises its total value subject to a debt structure of the form (5.4). The relative increase in total value in panel A is the ratio $\frac{\theta_{\text{max}}}{\theta_{\text{fixed-rate only}}}$, where $\theta_{\text{max}}$ is the maximum total value achieved with fixed-rate debt only and $\theta_{\text{fixed-rate only}}$ is the maximum total value achieved with fixed-rate and floating-rate bonds. The relative increase in total value in panel C is the ratio $\frac{\theta_{\text{max}}}{\theta_{\text{issuance strategy}}}$, where $\theta_{\text{issuance strategy}}$ is the maximum total value achieved with an issuance strategy of the form (5.4). $L_T$ denotes the promised payment to bondholders and $V_T$ denotes the terminal unlevered value of the firm. Unless otherwise stated, parameters are fixed at their base-case values (see table 2).

Table 10: Regression of $\theta^*$ on $\theta_{\text{hed}}$ when observations are jointly driven by $\rho_{RV}$ and $\lambda_R$ and there is an intercept, for different values of the volatility of firm’s value.

<table>
<thead>
<tr>
<th>$\sigma_V$</th>
<th>0.2</th>
<th>0.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_{\text{hed}}$</td>
<td>0.941</td>
<td>0.913</td>
</tr>
<tr>
<td>(717.104)</td>
<td>(646.556)</td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>0.067</td>
<td>0.102</td>
</tr>
<tr>
<td>(26.924)</td>
<td>(25.409)</td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>126</td>
<td>126</td>
</tr>
<tr>
<td>$R^2$</td>
<td>1.000</td>
<td>1.000</td>
</tr>
</tbody>
</table>

This table presents the estimated coefficients of $\theta_{\text{hed}}$ in the regression equation (5.5). t-values are reported between parenthesis. The 126 observations have been generated by letting $\rho_{RV}$ vary over values tested in table 8 and $\lambda_R$ over values tested in table 9. Unless otherwise stated, other parameters are fixed at their base-case values (see table 2).

Table 11: Regression of $\theta^*$ on $\theta_{\text{hed}}$ when observations are jointly driven by $\rho_{RV}$ and $\lambda_R$ and there is an intercept, for different values of the volatility of the short-term interest rate.

<table>
<thead>
<tr>
<th>$\sigma_R$</th>
<th>0.02</th>
<th>0.03</th>
<th>0.04</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_{\text{hed}}$</td>
<td>0.941</td>
<td>0.943</td>
<td>0.940</td>
</tr>
<tr>
<td>(740.852)</td>
<td>(516.363)</td>
<td>(469.544)</td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>0.067</td>
<td>0.063</td>
<td>0.064</td>
</tr>
<tr>
<td>(25.092)</td>
<td>(24.541)</td>
<td>(30.356)</td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>126</td>
<td>126</td>
<td>126</td>
</tr>
<tr>
<td>$R^2$</td>
<td>1.000</td>
<td>1.000</td>
<td>0.999</td>
</tr>
</tbody>
</table>

This table presents the estimated coefficients of $\theta_{\text{hed}}$ in the regression equation (5.5). t-values are reported between parenthesis. The 126 observations have been generated by letting $\rho_{RV}$ vary over values tested in table 8 and $\lambda_R$ over values tested in table 9. Unless otherwise stated, other parameters are fixed at their base-case values (see table 2).
Appendices

Table 12: Optimal capital and debt structures with time-dependent issuance strategies – Impact of the correlation $\rho_{RV}$ between firm’s risk and interest rate risk.

<table>
<thead>
<tr>
<th>$\rho_{RV}$</th>
<th>-0.99</th>
<th>-0.9</th>
<th>-0.7</th>
<th>-0.5</th>
<th>-0.3</th>
<th>-0.1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Optimal capital and debt structures with fixed-mix issuance strategy</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fixed-rate debt $\theta$</td>
<td>3.006</td>
<td>2.710</td>
<td>2.067</td>
<td>1.515</td>
<td>0.937</td>
<td>0.367</td>
<td>0.082</td>
</tr>
<tr>
<td>Relative increase in total value (%)</td>
<td>71.681</td>
<td>41.234</td>
<td>12.919</td>
<td>2.439</td>
<td>0.635</td>
<td>3.419</td>
<td>7.195</td>
</tr>
<tr>
<td>Volatility of ln $V_T$</td>
<td>0.465</td>
<td>0.486</td>
<td>0.530</td>
<td>0.570</td>
<td>0.607</td>
<td>0.643</td>
<td>0.660</td>
</tr>
<tr>
<td>Volatility of ln $V_T / L_T$</td>
<td>0.247</td>
<td>0.346</td>
<td>0.480</td>
<td>0.560</td>
<td>0.607</td>
<td>0.630</td>
<td>0.633</td>
</tr>
<tr>
<td>Correlation of ln $V_T$ and ln $L_T$</td>
<td>0.848</td>
<td>0.703</td>
<td>0.421</td>
<td>0.186</td>
<td>0.019</td>
<td>0.201</td>
<td>0.285</td>
</tr>
<tr>
<td>Leverage ratio $D_0/v_0$</td>
<td>0.450</td>
<td>0.368</td>
<td>0.280</td>
<td>0.238</td>
<td>0.216</td>
<td>0.207</td>
<td>0.205</td>
</tr>
<tr>
<td><strong>Panel B: Optimal capital structure with time-dependent issuance strategy</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Face value $L_0$</td>
<td>21.863</td>
<td>22.924</td>
<td>25.314</td>
<td>28.959</td>
<td>29.546</td>
<td>48.709</td>
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<tr>
<td>Fixed-rate debt $\theta$</td>
<td>-0.202</td>
<td>-0.774</td>
<td>-1.350</td>
<td>-1.941</td>
<td>-2.561</td>
<td>-2.862</td>
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<tr>
<td>Relative increase in total value (%)</td>
<td>12.537</td>
<td>29.012</td>
<td>56.876</td>
<td>106.077</td>
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<td>308.336</td>
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<tr>
<td>Volatility of ln $V_T$</td>
<td>0.676</td>
<td>0.708</td>
<td>0.739</td>
<td>0.768</td>
<td>0.797</td>
<td>0.809</td>
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</tr>
<tr>
<td>Volatility of ln $V_T / L_T$</td>
<td>0.630</td>
<td>0.608</td>
<td>0.561</td>
<td>0.482</td>
<td>0.349</td>
<td>0.250</td>
<td></td>
</tr>
<tr>
<td>Correlation of ln $V_T$ and ln $L_T$</td>
<td>0.365</td>
<td>0.515</td>
<td>0.653</td>
<td>0.781</td>
<td>0.901</td>
<td>0.953</td>
<td></td>
</tr>
<tr>
<td>Leverage ratio $D_0/v_0$</td>
<td>0.207</td>
<td>0.216</td>
<td>0.239</td>
<td>0.281</td>
<td>0.309</td>
<td>0.451</td>
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</tbody>
</table>

Panel A describes the capital and debt structures that maximise firm value when the firm takes a fixed-mix issuance strategy (see objective (2.14)). Panel D shows the capital structure that maximises firm value when the firm takes the time-dependent issuance strategy of proposition 5. The relative increase in total value in panel A is the ratio $\left(\frac{V_T^{\ast} - V_0^{\ast}}{V_0^{\ast} - V_0}\right)$, where $V_0^{\ast}$ is the maximum total value achieved with fixed-rate debt only and $V_T^{\ast}$ is the maximum total value achieved with fixed-rate and floating-rate bonds. The relative increase in total value in panel D is the ratio $\left(\frac{V_T^{\ast} - V_0^{\ast}}{V_0^{\ast} - V_0}\right)$, where $V_T^{\ast}$ is the maximum total value that is achieved with the time-dependent issuance strategy. $L_T$ denotes the promised payment to bondholders, and $V_T$ denotes the terminal unlevered asset value of the firm. Unless otherwise stated, parameters are fixed at their base-case values (see table 2).
Table 13: Optimal capital and debt structures with time-dependent strategies – Impact of the market price of interest rate risk $\lambda_R$

<table>
<thead>
<tr>
<th>$\lambda_R$</th>
<th>-0.5</th>
<th>-0.4</th>
<th>-0.3</th>
<th>-0.2</th>
<th>-0.1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Face value $L_0$</td>
<td>23.481</td>
<td>23.200</td>
<td>22.800</td>
<td>22.547</td>
<td>22.162</td>
<td>21.729</td>
</tr>
<tr>
<td>Fixed-rate debt $\theta$</td>
<td>0.062</td>
<td>0.066</td>
<td>0.069</td>
<td>0.073</td>
<td>0.078</td>
<td>0.083</td>
</tr>
<tr>
<td>Relative increase in total value (%)</td>
<td>6.792</td>
<td>6.855</td>
<td>6.924</td>
<td>7.003</td>
<td>7.063</td>
<td>7.196</td>
</tr>
<tr>
<td>Volatility of $\ln V_T$</td>
<td>0.660</td>
<td>0.660</td>
<td>0.660</td>
<td>0.660</td>
<td>0.660</td>
<td>0.660</td>
</tr>
<tr>
<td>Volatility of $\ln(V_T / L_T)$</td>
<td>0.633</td>
<td>0.633</td>
<td>0.633</td>
<td>0.633</td>
<td>0.633</td>
<td>0.633</td>
</tr>
<tr>
<td>Correlation of $\ln V_T$ and $\ln L_T$</td>
<td>0.285</td>
<td>0.285</td>
<td>0.285</td>
<td>0.285</td>
<td>0.285</td>
<td>0.285</td>
</tr>
<tr>
<td>Leverage ratio $D_0 / \theta$</td>
<td>0.217</td>
<td>0.215</td>
<td>0.213</td>
<td>0.211</td>
<td>0.208</td>
<td>0.205</td>
</tr>
</tbody>
</table>

Panel A describes the capital and debt structures that maximise firm value when the firm takes a fixed-mix issuance strategy (see objective (2.14)). Panel D shows the capital structure that maximises firm value when the firm takes the time-dependent issuance strategy of proposition 5. The relative increase in total value in panel A is the ratio $[\xi_0^* - \xi_0] / (\xi_0 - \xi_0^*)$, where $\xi_0^*$ is the maximum total value achieved with fixed-rate debt only and $\xi_0^*$ is the maximum total value achieved with fixed-rate and floating-rate bonds. The relative increase in total value in panel D is the ratio $[\xi_0^* - \xi_0^*] / (\xi_0 - \xi_0^*)$, where $\xi_0^*$ is the maximum total value that is achieved with the time-dependent issuance strategy. $L_T$ denotes the promised payment to bondholders, and $V_T$ denotes the terminal unlevered asset value of the firm. Unless otherwise stated, parameters are fixed at their base-case values (see table 2).
References


References


References

About EDHEC-Risk Institute
About EDHEC-Risk Institute

The Choice of Asset Allocation and Risk Management
EDHEC-Risk structures all of its research work around asset allocation and risk management. This issue corresponds to a genuine expectation from the market.

On the one hand, the prevailing stock market situation in recent years has shown the limitations of diversification alone as a risk management technique and the usefulness of approaches based on dynamic portfolio allocation.

On the other, the appearance of new asset classes (hedge funds, private equity, real assets), with risk profiles that are very different from those of the traditional investment universe, constitutes a new opportunity and challenge for the implementation of allocation in an asset management or asset-liability management context.

This strategic choice is applied to all of the Institute’s research programmes, whether they involve proposing new methods of strategic allocation, which integrate the alternative class; taking extreme risks into account in portfolio construction; studying the usefulness of derivatives in implementing asset-liability management approaches; or orienting the concept of dynamic “core-satellite” investment management in the framework of absolute return or target-date funds.

An Applied Research Approach
In an attempt to ensure that the research it carries out is truly applicable, EDHEC has implemented a dual validation system for the work of EDHEC-Risk. All research work must be part of a research programme, the relevance and goals of which have been validated from both an academic and a business viewpoint by the centre’s advisory board. This board is made up of internationally recognised researchers, the Institute’s business partners, and representatives of major international institutional investors. Management of the research programmes respects a rigorous validation process, which guarantees the scientific quality and the operational usefulness of the programmes.

Six research programmes have been conducted by the centre to date:
• Asset allocation and alternative diversification
• Style and performance analysis
• Indices and benchmarking
• Operational risks and performance
• Asset allocation and derivative instruments
• ALM and asset management

These programmes receive the support of a large number of financial companies. The results of the research programmes are disseminated through the EDHEC-Risk locations in London, Nice, and Singapore.
In addition, EDHEC-Risk has developed a close partnership with a small number of sponsors within the framework of research chairs or major research projects:

- **Regulation and Institutional Investment, in partnership with AXA Investment Managers**
- **Asset-Liability Management and Institutional Investment Management, in partnership with BNP Paribas Investment Partners**
- **Risk and Regulation in the European Fund Management Industry, in partnership with CACEIS**
- **Structured Products and Derivative Instruments, sponsored by the French Banking Federation (FBF)**
- **Dynamic Allocation Models and New Forms of Target-Date Funds, in partnership with UFG-LFP**
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- **The Case for Inflation-Linked Corporate Bonds: Issuers’ and Investors’ Perspectives, in partnership with Rothschild & Cie**
- **Advanced Investment Solutions for Liability Hedging for Inflation Risk, in partnership with Ontario Teachers’ Pension Plan**
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- **The Benefits of Volatility Derivatives in Equity Portfolio Management, in partnership with Eurex**

The philosophy of the Institute is to validate its work by publication in international journals, as well as to make it available to the sector through its position papers, published studies, and conferences.

Each year, EDHEC-Risk organises a major international conference for institutional investors and investment management professionals with a view to presenting the results of its research: EDHEC-Risk Institutional Days.

EDHEC also provides professionals with access to its website, www.edhec-risk.com, which is entirely devoted to international asset management research. The website, which has more than 42,000 regular visitors, is aimed at professionals who wish to benefit from EDHEC’s analysis and expertise in the area of applied portfolio management research. Its monthly newsletter is distributed to more than 700,000 readers.
About EDHEC-Risk Institute

Research for Business
The Institute’s activities have also given rise to executive education and research service offshoots. EDHEC-Risk’s executive education programmes help investment professionals to upgrade their skills with advanced risk and asset management training across traditional and alternative classes.

The EDHEC-Risk Institute PhD in Finance
www.edhec-risk.com/Aeducation/PhD_Finance
The EDHEC-Risk Institute PhD in Finance is designed for professionals who aspire to higher intellectual levels and aim to redefine the investment banking and asset management industries. It is offered in two tracks: a residential track for high-potential graduate students, who hold part-time positions at EDHEC, and an executive track for practitioners who keep their full-time jobs. Drawing its faculty from the world’s best universities and enjoying the support of the research centre with the greatest impact on the financial industry, the EDHEC-Risk Institute PhD in Finance creates an extraordinary platform for professional development and industry innovation.

FTSE EDHEC-Risk Efficient Indices
www.edhec-risk.com/indexes/efficient
FTSE Group, the award winning global index provider, and EDHEC-Risk Institute launched the first set of FTSE EDHEC-Risk Efficient Indices at the beginning of 2010. Offered for a full global range, including All World, All World ex US, All World ex UK, Developed, Emerging, USA, UK, Eurobloc, Developed Europe, Developed Europe ex UK, Japan, Developed Asia Pacific ex Japan, Asia Pacific, Asia Pacific ex Japan, and Japan, the index series aims to capture equity market returns with an improved risk/reward efficiency compared to cap-weighted indices. The weighting of the portfolio of constituents achieves the highest possible return-to-risk efficiency by maximising the Sharpe ratio (the reward of an investment per unit of risk). These indices provide investors with an enhanced risk-adjusted strategy in comparison to cap-weighted indices, which have been the subject of numerous critiques, both theoretical and practical, over the last few years. The index series is based on all constituent securities in the FTSE All-World Index Series. Constituents are weighted in accordance with EDHEC-Risk’s portfolio optimisation, reflecting their ability to maximise the reward-to-risk ratio for a broad market index. The index series is rebalanced quarterly at the same time as the review of the underlying FTSE All-World Index Series. The performances of the EDHEC-Risk Efficient Indices are published monthly on www.edhec-risk.com.

EDHEC-Risk Alternative Indexes
www.edhec-risk.com/indexes/pure_style
The different hedge fund indexes available on the market are computed from different data, according to diverse fund selection criteria and index construction methods; they unsurprisingly tell very different stories. Challenged by this heterogeneity, investors cannot rely on competing hedge fund indexes to obtain a “true and fair” view of performance and are at a loss when selecting benchmarks. To address this issue, EDHEC Risk was the first to launch composite hedge fund strategy indexes as early as 2003. The thirteen EDHEC-Risk Alternative Indexes are published monthly on www.edhec-risk.com and are freely available to managers and investors.
About Rothschild & Cie
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Rothschild has been involved in investment banking since its beginning over two hundred years ago when Rothschild businesses were established in the principal cities of Europe at the end of the 18th century. Today, the Rothschild Group is one of the world’s leading financial advisory and asset management organisations which is family controlled and independent. It has an established network of offices around the world with more than 2000 people in over 40 countries (including the USA, UK, France, Switzerland, Singapore, China,…).

Rothschild Global Financial Advisory is involved in providing impartial and expert M&A and strategic advice as well as financing and restructuring advice across the range of equity and debt capital markets. Rothschild & Cie Gestion is the asset management arm of the Rothschild Group in France.

Rothschild & Cie Gestion manages EUR 22bn in assets and offers a diversified product range, with expertises in equities (focusing on European equities), bonds (including govies, Euro credit and European convertibles), balanced management, and long-only as well as alternative multi-management.

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Notes