Empirical Properties of Straddle Returns

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Abstract
Recent studies find that a position in at-the-money (ATM) straddles consistently yields losses. This is interpreted as evidence for the non-redundancy of options and as a risk premium for volatility risk. This paper analyses this risk premium in more detail by i) assessing the statistical properties of ATM straddle returns, ii) linking these returns to exogenous factors and iii) analysing the role of straddles in a portfolio context. Our findings show that ATM straddle returns seem to follow a random walk and only a small percentage of their variation can be explained by exogenous factors. In addition, when we include the straddle in a portfolio of the underlying asset and a risk-free asset, the resulting optimal portfolio attributes substantial weight to the straddle position. However, the certainty equivalent gains with respect to the presence of a straddle in a portfolio are small and probably do not compensate for transaction costs. We also find that a high rebalancing frequency is crucial for generating significant negative returns and portfolio benefits. Therefore, from an investor's perspective, straddle trading does not seem to be an attractive way to capture the volatility risk premium.

JEL Classification: G11 - Portfolio Choice; Investment Decisions, G12 - Asset Pricing, G13 - Contingent Pricing

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1. Introduction

A central tenet of the Black-Scholes and Merton option pricing model is that options are redundant assets since they can be perfectly replicated with a dynamic trading strategy in existing assets. Recent research has shown that zero-beta or delta neutral option strategies consistently yield negative returns, though they should yield the risk-free rate if options were redundant (see Coval and Shumway (2001) or Bakshi and Kapadia (2003)).

One case of such a strategy is an option straddle. A straddle combines a long position in a call option with a long position in the put option with the same maturity and strike price. The amounts invested in the put and the call can be chosen so as to obtain an overall beta of zero. Negative returns of such a strategy imply that options are non-redundant assets. From a practical standpoint, this also implies that an investor can generate profits in the option market simply by shorting the straddle strategy. However, the characteristics of straddles as a potential asset have not yet been examined in detail. This paper tries to fill this gap by analysing the return characteristics of at-the-money straddles from an investor's perspective. We construct straddle returns using data for DAX index options. We then study their time series characteristics, their relation to exogenous factors, and their role in asset allocation.

The performance of straddles has received a considerable amount of attention in the literature. While straddles have been used in empirical research on market microstructure (Copeland and Galai (1983), Hansch et al. (1998)), our paper is related to the literature that examines straddles in an asset-pricing context.

The price of an option straddle depends on the volatility of the underlying asset. Hence, buying an option straddle is similar to "investing" in volatility. Several papers use straddle trading strategies to evaluate volatility forecasts. Noh et al. (1994) implement a straddle trading strategy that aims at profiting from volatility forecasts. Engle and Rosenberg (2000) assess the effectiveness of hedging predicted changes in volatility with straddles.

More recently, straddles have been analysed without reference to volatility forecasting. Coval and Shumway (2001) show that an at-the-money straddle position consistently produces negative returns. Holding the straddles yields losses on average, independent of the volatility movements during the holding period. Their interpretation is that straddles hedge against changes in volatility, but this hedge comes at a cost.

Coval and Shumway (2001) interpret the significantly negative straddle returns as the volatility risk premium, or as a premium driven by hedging demands. The hedging demand arises if options are not redundant securities, and instead, prices of options are driven by supply and demand (Bollen and Whaley (2004)), which arise from hedging demands. Such option pricing models predict a dependence of option returns on factors related to partial information, such as dispersion of beliefs (Buraschi and Jiltsov (2006), Guidolin and Timmermann (2003)), and learning uncertainty (David and Veronesi (2002)).

Extending the study of Coval and Shumway (2001), this paper examines in detail the returns to holding at-the-money option straddles. We examine its time series properties and the dependency of the returns on exogenous factors, such as the credit spread and stock market volume. The use of such factors is motivated by the fact that they can be seen as proxies for dispersion of beliefs and learning uncertainty (see David and Veronesi (2002), Massa and Simonov (2005)). To put the return characteristics in a portfolio perspective, we add at-the-money straddles to a portfolio consisting of the underlying asset (the DAX index for the German stock market) and a riskless asset (cash) and assess the resulting portfolio holdings of an expected utility maximiser.

1- Copeland and Galai (1983) use straddles to proxy for the cost of a dealer's bid-ask spread, while Hansch et al. (1998) report that movements between straddle prices, the best ask and the best bid are highly correlated with inventory changes.
We find that straddles exhibit significantly negative and positively skewed returns. Standard tests do not reject the hypothesis that straddle returns follow a random walk. In addition to volatility, the straddle returns are significantly related to the credit spread and the stock market turnover. However, such factors only explain a small fraction of the total variability of straddle returns. Expected utility maximising investors allocate a considerable short position to the straddle strategy and this position leads to certainty equivalent gains, when compared to the case where the investor does not have access to the straddle strategy. However, the magnitude of the holding and the associated utility gains are strongly reduced when lowering the rebalancing frequency of the straddle strategy.

The paper proceeds as follows. Section 2 discusses the dataset and the method for constructing the option straddles, section 3 presents descriptive statistics, a time-series analysis and factor analysis of the straddle returns, section 4 examines the contribution of straddles when included in portfolio choice and section 5 provides conclusions.

2. Data and Methodology

In order to construct the straddle returns, we use data from the German stock and options exchange. To isolate possible effects of stock market returns on the straddle returns, the straddles are constructed to be beta-neutral. Options are bought and the position is rebalanced (to keep it at-the-money and beta-neutral) on a daily, weekly and monthly basis.

2.1 Data

2.1.1 Options and the Underlying Asset

Daily observations of the DAX index options, ticker symbol ODAX, are obtained from Eurex. The data consist of time-stamped intraday transaction prices over a 7-year period, from January 1999 to December 2005. We use prices for European-style put and call options. Similarly, time-stamped intraday prices of the underlying asset (DAX index that has been adjusted for dividends) are also provided by Eurex. These intraday time-stamped data are important, so as to ensure that the straddles are constructed on the same index level around the same time. The EURIBOR (Euro Interbank Offer Rate) interest rate with one month maturity published by the Bundesbank is used as the risk-free rate.

The option universe used in this study is constructed in the following way. First, the option data is screened to delete option prices that are below their intrinsic values. Second, options with Black-Scholes implied volatilities exceeding 100%, or less than 1% are also eliminated.

To reduce the impact of stochastic interest rates, we choose to focus on short-term options: only options that expire in the following calendar month (between 20 and 50 days to expiration) are selected. Following Coval and Shumway (2001), the calculation of daily returns is based on standard returns, instead of logarithmic returns. This is done because options held to maturity often expire worthless (yielding net returns of -1), thus the log-transformation of option returns held to maturity may yield negative infinity.

2.1.2 Factor Analysis

This paper also examines the relation between straddle returns and other variables such as the credit spread, and the turnover (trading volume) of DAX component stocks.

The Lehman Brothers Euro Aggregate Bond index is used to construct the credit spread. The credit spread is taken as the difference in redemption yield between the index for bonds from corporate issuers and the index for government bonds. The turnover of the DAX is taken from the volume of component stocks traded on the Xetra trading system. The above data is obtained from Thomson Datastream, with daily frequency, from January 1999 to December 2005.
2.2 Methodology

2.2.1 Constructing beta-neutral straddles

By virtue of its construction, the payoff of an ATM straddle is insensitive to small fluctuations in the underlying asset's return. To avoid any influence of the DAX returns, the ODAX ATM straddles are constructed to be beta-neutral.

Buying options can be described as a leveraged investment because a DAX option allows an exposure to stock market risk with a small investment amount. This implicit leverage becomes apparent in the option’s beta.

According to Black and Scholes (1973), the beta of a call option can be written as

\[ \beta_c = \frac{s}{C} \left[ \frac{\ln \frac{s}{X} + (r - \lambda + \frac{\sigma^2}{2})t}{\sigma \sqrt{t}} \right] \beta_s \]  

where \( s \) is the price of the underlying asset, \( C \) is price of the call option, \( X \) is the strike price of the option, \( r \) is the risk-free rate, \( \sigma \) is the volatility of the underlying asset, \( t \) is the time to maturity, \( N [\cdot] \) is the cumulative normal distribution, \( \beta_s \) is the underlying asset’s beta and \( \lambda \) is the dividend yield of the underlying asset.

Similarly, the beta of a put can be computed as

\[ \beta_p = \frac{s}{P} \left( N \left[ \frac{\ln \frac{s}{X} + (r - \lambda + \frac{\sigma^2}{2})t}{\sigma \sqrt{t}} \right] - 1 \right) \beta_s \]  

where \( s \) is the price of the underlying asset, \( P \) is price of the put option, \( X \) is the strike price of the option, \( r \) is the risk-free rate, \( \sigma \) is the volatility of the underlying asset, \( t \) is the time to maturity, \( N [\cdot] \) is the cumulative normal distribution, \( \beta_s \) is the underlying asset’s beta and \( \lambda \) is the dividend yield of the underlying asset.

It can be shown from the above equations that the beta of a call is positive, while the beta of a put is negative (if the asset beta \( \beta_s \) is positive).

Beta-neutral straddle positions can now be formed by buying puts and calls with the same strike price and maturity in proportion to their betas. Since the beta from the call position cancels the beta from the put position, the straddles are isolated from movements in the underlying asset.

Specifically, as shown in Coval and Shumway (2001), the zero-beta straddles are constructed by solving the following equations

\[ r_v = \theta r_c + (1 - \theta) r_p \]
\[ \theta \beta_c + (1 - \theta) \beta_p = 0 \]  

where \( r_v \) is the straddle returns, \( \theta \) is the fraction of the straddle's value in call options, \( \beta_c \) and \( \beta_p \) are the betas of the call and put and \( r_c \) and \( r_p \) are the returns of call and put, respectively. This problem is solved by the weight function

\[ \theta = \frac{-\beta_p}{\beta_c - \beta_p} \]  

where \( \beta_c \) and \( \beta_p \) are the betas of the call and put.
Substituting equation 4 into equation 3, the straddle returns can be computed as:

$$r_v = \frac{-\beta_p}{\beta_e - \beta_p} r_c + \frac{\beta_c}{\beta_e - \beta_p} r_p$$

(5)

where the call options beta, $\beta_c$ and the put options beta $\beta_p$ will be computed as the Black-Scholes beta given by equation 1 and 2.

In the Coval and Shumway (2001) paper, based on the assumption of put-call parity, they computed the weights for their straddles using only the call option betas. However, under actual market trading conditions, put-call parity does not always hold. As such, deviating from their study, we have chosen to use both put and call option betas to construct our beta-neutral straddles.

2.2.2 Straddle groups and rebalancing frequency

Our beta-neutral straddles are constructed using matching pairs of put-call options that satisfy the following criteria:

1. **At-the-money** — To study the properties of the volatility premium, the straddles should be constructed at-the-money. At the moment when the straddle is to be constructed, only the options with strike prices close to the DAX index level at that point in time are selected.

2. **Strike price and maturity date** — The option pairs used in the beta-neutral straddles will have the same strike price and maturity date.

3. **Traded “simultaneously”** — To construct a straddle where the resulting beta is close to zero, the put and call options must be bought at the same DAX index level. To do that, the put and call options should be traded at the same time. As such, in this study, the put and call option pairs chosen for the straddles are traded within the same hour, with a maximum of 55 minutes of delay.

To construct at-the-money straddles, we choose options with strike prices $K$ within 150 points of the current level of the DAX index $S$, that is $-75 < S - K < 75$. Since strike prices for available options vary in intervals of 50 DAX points, at most 3 option pairs satisfy this condition at each point in time. If we identify more than one option pair in the permissible range of strike prices, we use the pair with the strike price that is closest to the current DAX index level.

These straddles are then rebalanced on a daily, weekly and monthly basis. Daily rebalancing allows maintaining the desired moneyness and the beta-neutrality. Each day, a pair of matching put-call options is identified and bought at the same time (or within the hour). On the following trading day, if the DAX index level deviates more than 50 points from the straddle strike price at the market opening, the straddle is sold, and a new straddle is constructed based on the new DAX index level. Using this rebalancing method, the new straddle is constructed only after the previous straddle is sold, so that the investor cannot hold more than one straddle at any point in time. If we cannot sell the straddle on its due date (if either one of the options is not traded, or the trades cannot be carried out at the same time), the straddle is held for the day, and sold on the next trading day. On the other hand, after selling the previous straddle, if the new straddle cannot be constructed under the required criteria, it will be constructed on the next trading day. In this case, the investor will hold no option straddle for that day, so the straddle return is zero on the day that the straddle cannot be constructed.

The lower rebalancing frequencies may lead to straddle strategies that depart from their initial characteristics. They may not be at-the-money and beta-neutral in between two rebalancing dates. For weekly rebalancing strategies, the straddles are rebalanced every Tuesday. If the Tuesday happens to be a public holiday, the straddle is evaluated and rebalanced on Monday or the previous
trading day. Like the daily rebalanced straddles, if the straddle cannot be sold on a Tuesday, it will be sold on Wednesday instead. In this case, the weekly return is evaluated on Wednesday. However, if the new straddle cannot be constructed on a Tuesday, the weekly return is still evaluated on Tuesday, while the new straddle will be constructed on Wednesday. The monthly rebalanced straddles are constructed according to the same rules with rebalancing taking place on the second business day of each month.

3. Statistical Properties

In this section, we examine the statistical properties of the derived straddle returns in detail. First, the average betas of the straddles with respect to the underlying asset are reported. Then we look at the return distribution using the descriptive statistics of the straddle returns. Following that, we examine the time-series properties of the straddle returns and finally, the dependency of straddle returns on exogenous factors is presented.

3.1 Beta-Neutrality

As described in the previous section (section 2), the three types of at-the-money beta-neutral straddles examined in this study differ in their rebalancing frequencies: they are rebalanced daily, weekly and monthly. Although these straddles are constructed to be beta-neutral, the betas may not remain zero throughout the holding period, especially when the straddles are rebalanced less often to offset the beta, or when the underlying asset is very volatile. If the resulting betas of the straddles are not neutral or kept reasonably low throughout the holding period, the straddle returns could be influenced by the underlying asset’s returns. This effect (of the underlying asset’s returns on the straddles) will render it more difficult to study straddle returns as the premium for volatility of the underlying asset.

To check for the beta-neutrality of the straddles, we compute the daily betas of the straddles using the formula below:

$$\beta_{s,t} = \theta_T \beta_{c,t} + (1 - \theta_T) \beta_{p,t}$$

(6)

where $\beta_{s,t}$ is the beta of the straddle at time $t$, $\theta_T$ is the weight allocated to the call option at the time $t = T$ when the straddle is constructed/rebalanced, $\beta_{c,t}$ is the beta of the call option at time $t$, $\beta_{p,t}$ is the beta of the put option at time $t$. $\beta_{c,t}$ and $\beta_{p,t}$ are computed as the Black-Scholes betas (see equations 1 and 2), and $\theta_T$ is computed using equation 4, as reproduced below:

$$\theta_T = \frac{-\beta_{p,T}}{\beta_{c,T} - \beta_{p,T}}$$

It is necessary for $\beta_{s,t} = 0$ to produce beta neutral straddles. However, as $\beta_{c,t}$ and $\beta_{p,t}$ may fluctuate due to changes in the value of the underlying asset and time, it is necessary to adjust $\theta_T$ to keep the straddle beta $\beta_s$ close to zero. For straddles where the $\theta_T$ is adjusted less often, it is more likely that the resulting straddle beta will deviate further away from zero.

Table 1 reports the mean of the the daily betas for the three different re-balancing frequencies. We use the t-test to verify our null hypothesis that the average daily beta is zero in each case, hence the t-statistics and their corresponding p-values are also reported in the same table. As expected, the average beta of the daily rebalanced straddles is very close to zero, since the straddles are rebalanced daily. The t-statistic is computed to be 0:44, which confirms that we cannot reject the null hypothesis that the average beta of the daily straddles is zero.

On the other hand, as the weekly straddles are not adjusted daily, the average daily beta is computed to be -0:18, suggesting that the DAX returns may have some influence on the weekly
straddle returns. However, when subjected to the t-test, the average beta yields a p-value of 0.31, so we can still safely assume that the average daily beta of the weekly rebalanced straddles is almost zero.

Similarly, the beta neutrality of the monthly straddles is not as obvious as the daily straddles, since they are rebalanced only once a month. The average daily beta is -0.33 with a p-value 0.09 for the t-test, making it a borderline case (if we assume a 10% confidence interval) for rejecting the null hypothesis. Overall, irrespective of the rebalancing frequency, we cannot reject that the null hypothesis that the average daily betas of the straddles are zero with a 5% confidence interval.

To verify the beta-neutrality of the straddles, figure 1 provides scatter plots of the straddle returns versus the DAX index returns. From the figure, we can see that the daily straddle returns are the least correlated with the DAX index returns, while the weekly and monthly straddles exhibits some non-linear dependence with the DAX index returns.

While we cannot reject the hypothesis that the Black-Scholes betas are equal to zero, the scatter plots reveal non-linear dependence of straddle returns on DAX returns for the weekly and monthly rebalancing frequency. This fact should be kept in mind when analysing the results below. In particular, only the straddle strategy with daily rebalancing can be considered to be truly beta neutral.

Table 1: Descriptive statistics of straddle betas

<table>
<thead>
<tr>
<th></th>
<th>Daily</th>
<th>Weekly</th>
<th>Monthly</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>3.44 x 10^{-17}</td>
<td>-0.183</td>
<td>-0.331</td>
</tr>
<tr>
<td>t-statistic</td>
<td>0.77</td>
<td>-1.01</td>
<td>-1.69</td>
</tr>
<tr>
<td>p-value</td>
<td>0.44</td>
<td>0.31</td>
<td>0.09</td>
</tr>
</tbody>
</table>

Figure 1: Straddle returns vs DAX index returns

3.2 Descriptive statistics
The straddle returns are evaluated when the straddle is rebalanced. That is, we compute daily returns for daily rebalanced straddles, weekly returns for weekly rebalanced straddles and monthly returns for monthly rebalanced straddles. We also report average daily returns for all three strategies to allow a direct comparison.
Table 2 presents descriptive statistics for the three at-the-money beta-neutral straddle returns. Panel A reports summary statistics. Panel B reports normality test statistics as well as their critical values at the 5% level. Panel C reports results for the Augmented Dickey Fuller test for unit root nonstationarity.

Table 2: Descriptive statistics of straddle returns

<table>
<thead>
<tr>
<th>Rebalancing Frequency</th>
<th>Daily</th>
<th>Weekly</th>
<th>Monthly</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean (%)</td>
<td>-1.257</td>
<td>-1.790</td>
<td>-6.074</td>
</tr>
<tr>
<td>Daily average (%)</td>
<td>-1.257</td>
<td>-0.358</td>
<td>-0.289</td>
</tr>
<tr>
<td>t-statistic</td>
<td>-9.366</td>
<td>-2.070</td>
<td>-1.398</td>
</tr>
<tr>
<td>Std. Dev</td>
<td>0.057</td>
<td>0.165</td>
<td>0.398</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.190</td>
<td>1.898</td>
<td>1.111</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>18.33</td>
<td>9.069</td>
<td>3.800</td>
</tr>
<tr>
<td>Max.</td>
<td>0.475</td>
<td>0.953</td>
<td>1.293</td>
</tr>
<tr>
<td>Median</td>
<td>-0.013</td>
<td>-0.054</td>
<td>-0.188</td>
</tr>
<tr>
<td>Min.</td>
<td>-0.462</td>
<td>-0.395</td>
<td>-0.784</td>
</tr>
<tr>
<td>Lilliefors Stat.</td>
<td>0.110</td>
<td>0.159</td>
<td>0.170</td>
</tr>
<tr>
<td>5% crit. val.</td>
<td>0.021</td>
<td>0.047</td>
<td>0.097</td>
</tr>
<tr>
<td>Jarque-Bera Stat.</td>
<td>17393</td>
<td>768.94</td>
<td>18.44</td>
</tr>
<tr>
<td>5% crit. val.</td>
<td>5.96</td>
<td>5.81</td>
<td>5.34</td>
</tr>
</tbody>
</table>

Similar to Coval and Shumway (2001), we find that for all rebalancing frequencies, straddle returns are negative on average. The t-statistics show that the mean returns are significantly different from zero at the 5% level for the weekly and daily strategies, but not for the monthly strategy. The average daily return decreases as the straddle is rebalanced less often: the strategy with daily rebalancing looses 1.26% per day, the weekly strategy looses 0.36% per day, and the monthly strategy looses 0.29% per day. The negative returns of these straddles are the cost of holding at-the-money straddles. This cost is independent of the returns of the underlying asset since the straddles are beta neutral. The losses of the straddles can be interpreted as the cost to hedge against changes in the volatility of the underlying asset. Conversely, investors willing to sell straddles could obtain positive returns in exchange for their exposure to volatility risk.

In addition, all straddles exhibit excess kurtosis and are positively skewed, suggesting that the returns are not normally distributed. This is confirmed by the results of the Lilliefors and Jarque-Bera tests. Both tests lead to a rejection of the hypothesis of a normal distribution for all rebalancing frequencies. The hypothesis of a unit root is clearly rejected by the Augmented Dickey Fuller test, providing evidence for the stationary of the time series.

3.3 Time Series Analysis

Researchers have employed option straddles to test the informational efficiency of the options market. For example, Harvey and Whaley (1992) and Noh et al. (1994) employ various volatility forecasts to formulate straddle trading strategies that generate abnormal returns. The idea is that if volatility forecasts may be used in option strategies to “beat” the market, then option markets are not efficient.

While forecasting volatility is not in the scope of our study, a minimum requirement for weak-form market efficiency, is that these straddle returns cannot be forecasted based on past returns.
In order to test this weak-form efficiency, we subject the time series of our straddle returns to the standard tests for a random walk, namely, the Ljung-Box test and the variance ratio test (see Campbell et al. (1997), Chapter 2).

Time-series dependence can be assessed by looking at the significance and sign of autocorrelation coefficients of the return series. However, even in the absence of time-series dependence, certain autocorrelation coefficients might still be significantly different from zero. Therefore, we want to test the hypothesis that the autocorrelation coefficients up to a certain order are jointly zero. The Ljung-Box test (also known as the portmanteau test) for time-series dependence provides such a test since it is based on the sum of squares of a sequence of autocorrelations.\(^2\)

Another standard test in finance is based on the variance ratio. This test statistic is based on the fact that the variance of the sum of a series of returns (i.e., the lower frequency returns) must be equal to the sum of the variances of the higher frequency returns if the series follows a random walk.

More formally, let \( r_t(q) \) denote a vector of returns, sampled every \( q \) days, i.e.,
\[
 r_t(q) \equiv r_t + r_{t-1} + \cdots + r_{t-q+1}. 
\]
Then, denoting the daily returns as \( r_t \), the variance ratio is
\[
 VR(q) = \frac{Var(r_t(q))}{q \times Var(r_t)} 
\]

It can be shown that for a series of uncorrelated increments the variance ratio must approach unity in large samples, even in the presence of heteroscedasticity, which is present in most financial return series. Lo and Mackinlay (1988) compute the asymptotic variance of the test statistic \( VR \) and derive a standardized test statistic based on the variance ratio. This test statistic makes it possible to compute p-values for the null hypothesis that the variance ratio is equal to one.

Table 3 reports the results for the Ljung-Box Q-statistic and the variance ratio test statistic.

It becomes apparent from table 3 that, for most cases, the null hypothesis of no time-series dependence of the straddle returns cannot be rejected at conventional levels of significance, implying that straddle returns follow a random walk.

The only exception are the weekly straddle returns, where the Ljung-Box statistic is significant at the 10% level and even at the 5% level in one case. In addition, the variance ratio for \( p = 2 \) is significant at the 10% level in one case and at the 5% level in another case. However, the null-hypothesis of a unit variance ratio cannot be rejected at any of the remaining intervals \( p \) for the weekly straddles. The significant results are driven by the dependence observed over the very short interval of two days. Likewise, the null hypothesis of the variance ratio test cannot be rejected for all daily and monthly straddle returns, providing evidence that they follow a random walk. Overall, we may conclude that option markets are efficient in the sense of weak-form efficiency. From an investment perspective, timing strategies using the information on past returns data, can therefore be excluded.

### 3.4 Factor Analysis

To further understand the properties of the straddle returns, we proceed to examine their link with exogenous factors. To do so, we deviate from the pure time-series approach taken in the previous tests, and look at linear regression specifications including exogenous variables.

It is obvious that returns of beta-neutral straddles should heavily depend on the volatility of the underlying asset. In addition, the significant negative straddle returns we observe may be explained as the volatility risk premium (Coval and Shumway (2001)), or as a premium driven by

\(^2\) The recommended number of lags to check for autocorrelations can be computed as \( N = 2 \sqrt{T} \) where \( N \) is the number of lags and \( T \) is the time period. Here we set the number of lags to 20 for all series.
hedging demands. Therefore, it seems to be interesting to see if we can explain the time variation in straddle returns by exogenous factors linked to such hedging demands.

Table 3: Random Walk Tests for Straddle Returns
This table reports results of the random walk tests on the daily straddle returns. The period of study is from January 1999 to December 2005. Straddle returns are calculated according to equation 5. The Ljung-Box Q statistics tests the null hypothesis that daily straddle returns are not autocorrelated up to 20 lags. The variance ratio tests the null hypothesis that variance ratio of lag N (2 to 20) is one.

<table>
<thead>
<tr>
<th></th>
<th>Daily</th>
<th>Weekly</th>
<th>Monthly</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Ljung-Box Test</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ljung Box Q stat.</td>
<td>21.41</td>
<td>38.71</td>
<td>23.54</td>
</tr>
<tr>
<td>p-value</td>
<td>0.37</td>
<td>0.01</td>
<td>0.26</td>
</tr>
<tr>
<td><strong>Panel B: Variance Ratio Test</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VR(2)</td>
<td>1.01</td>
<td>0.84</td>
<td>0.98</td>
</tr>
<tr>
<td>p-value</td>
<td>0.68</td>
<td>0.02</td>
<td>0.91</td>
</tr>
<tr>
<td>VR(4)</td>
<td>1.06</td>
<td>0.94</td>
<td>0.87</td>
</tr>
<tr>
<td>p-value</td>
<td>0.19</td>
<td>0.58</td>
<td>0.59</td>
</tr>
<tr>
<td>VR(6)</td>
<td>1.09</td>
<td>0.98</td>
<td>0.71</td>
</tr>
<tr>
<td>p-value</td>
<td>0.10</td>
<td>0.91</td>
<td>0.32</td>
</tr>
<tr>
<td>VR(8)</td>
<td>1.10</td>
<td>1.02</td>
<td>0.73</td>
</tr>
<tr>
<td>p-value</td>
<td>0.12</td>
<td>0.90</td>
<td>0.41</td>
</tr>
<tr>
<td>VR(10)</td>
<td>1.12</td>
<td>1.07</td>
<td>0.70</td>
</tr>
<tr>
<td>p-value</td>
<td>0.12</td>
<td>0.71</td>
<td>0.41</td>
</tr>
<tr>
<td>VR(12)</td>
<td>1.14</td>
<td>1.07</td>
<td>0.58</td>
</tr>
<tr>
<td>p-value</td>
<td>0.10</td>
<td>0.72</td>
<td>0.29</td>
</tr>
<tr>
<td>VR(14)</td>
<td>1.15</td>
<td>1.08</td>
<td>0.47</td>
</tr>
<tr>
<td>p-value</td>
<td>0.10</td>
<td>0.72</td>
<td>0.23</td>
</tr>
<tr>
<td>VR(16)</td>
<td>1.16</td>
<td>1.07</td>
<td>0.30</td>
</tr>
<tr>
<td>p-value</td>
<td>0.10</td>
<td>0.76</td>
<td>0.17</td>
</tr>
<tr>
<td>VR(18)</td>
<td>1.15</td>
<td>1.05</td>
<td>0.51</td>
</tr>
<tr>
<td>p-value</td>
<td>0.13</td>
<td>0.83</td>
<td>0.32</td>
</tr>
<tr>
<td>VR(20)</td>
<td>1.15</td>
<td>1.02</td>
<td>0.29</td>
</tr>
<tr>
<td>p-value</td>
<td>0.17</td>
<td>0.93</td>
<td>0.22</td>
</tr>
</tbody>
</table>

The current literature on hedging demand focuses on the role of partial information. The idea is that options are not redundant securities, instead, prices of options are driven by supply and demand (Bollen and Whaley (2004)), which itself depends on such hedging demands. Such option pricing models predict a dependence of option returns on factors such as dispersion of beliefs (Buraschi and Jiltsov (2006), Guidolin and Timmermann (2003)) or learning uncertainty (David and Veronesi (2002)).

Finding empirical proxies for such factors is challenging, as investor’s beliefs and uncertainty are not observable. We decide to use the credit spread between bonds issued by corporations and governments as a proxy for uncertainty. This can be justified by the link that has been established between uncertainty and credit spread (see David and Veronesi (2002)). In order to proxy for dispersion of beliefs, we follow Massa and Simonov (2005), and use stock market volume as a proxy for disagreement between investors. This can be justified by the fact that trading is mainly driven by heterogeneity of investor’s beliefs (see citations in Massa and Simonov (2005)).

Instead of using the credit spread as it is, the change in credit spread is used in our regressions. Change in credit spread is computed as

$$\Delta C_{rt} = C_{rt} - C_{r_{t-1}}$$

where \( C_r \) is the credit spread between the Lehmann Euro Aggregate government and corporate bond indices. In addition, we use the average of the sum of squared daily returns over the corresponding
period as a proxy for volatility. Volatility is added to the regressions to ensure that the factor loadings with the credit spread and the turnover are not merely due to stock market volatility.

Table 4 shows the regression results for the different straddle strategies with three different strike prices and the three different rebalancing frequencies. The variables we included have positive coefficients for all rebalancing frequencies. However, the regression coefficients are only significantly different from zero for daily rebalancing, as indicated by the t-statistics. This indicates that daily rebalanced straddle returns are positively related to volatility, credit spread and turnover. Interestingly, all coefficients become insignificant as the holding period for the straddle increases. From R²s, we can see that the explanatory power of the independent variables for the straddle returns is very low.

<table>
<thead>
<tr>
<th></th>
<th>Daily</th>
<th>Weekly</th>
<th>Monthly</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-0.019</td>
<td>-0.052</td>
<td>-0.098</td>
</tr>
<tr>
<td>t-statistic</td>
<td>-1.647</td>
<td>-1.580</td>
<td>-0.725</td>
</tr>
<tr>
<td>Chg Credit Spread</td>
<td>0.081</td>
<td>0.003</td>
<td>0.346</td>
</tr>
<tr>
<td>t-statistic</td>
<td>2.652</td>
<td>0.036</td>
<td>1.162</td>
</tr>
<tr>
<td>Turnover (x10⁻⁴)</td>
<td>0.083</td>
<td>0.141</td>
<td>0.828</td>
</tr>
<tr>
<td>t-statistic</td>
<td>1.525</td>
<td>0.734</td>
<td>1.775</td>
</tr>
<tr>
<td>Chg Volatility</td>
<td>0.022</td>
<td>0.107</td>
<td>0.104</td>
</tr>
<tr>
<td>t-statistic</td>
<td>2.353</td>
<td>1.066</td>
<td>0.255</td>
</tr>
<tr>
<td>R²</td>
<td>0.091</td>
<td>0.052</td>
<td>0.082</td>
</tr>
</tbody>
</table>

Overall, we interpret these results as evidence that apart from volatility, straddle returns with daily rebalancing are also linked to factors that proxy for dispersions in beliefs and learning uncertainty. Having said that, it should be noted that, even collectively, these factors can only explain a small fraction of the variation in daily rebalanced straddle returns. Finally, it is interesting that the regression coefficients become insignificant for longer rebalancing horizons. This suggests that maintaining a high rebalancing frequency is essential in order to maintain the desired characteristic of the straddle returns. In addition to being truly beta neutral, daily rebalanced straddles provide factor exposures which are coherent with interpreting their returns as a risk premium.

4. Portfolio Perspective

While the time-series properties and factor exposures of any asset are of interest from an investment perspective, the ultimate question is that of the optimal fraction allocated to this asset. Here we address this question and ask "what is the fraction that a risk-averse investor would optimally hold in the straddle strategies?"

We consider an expected utility maximiser with a power utility functions. In order to compare the case where the investor has access to the straddles to the case where he does not have this access, we first specify a vector of ten reference weights for a portfolio without straddles (allocation between a riskless asset and a single risky asset). The reference weights we consider are 1.4, 1.25, 1.1, 0.95, 0.8, 0.65, 0.50, 0.35, 0.20, and 0.05 in the risky asset.

We then consider that these are the optimal weights of the utility maximiser and solve for the ten corresponding risk aversion coefficients. The resulting risk aversion coefficients are then used in the portfolio problem where the straddles are added to the asset menu. Solving with these risk aversion coefficients, we then obtain ten sets of portfolio weights with the straddle, risky and riskless asset. Thus we can directly compare these weights to the case where there is no access to straddles, based on the same risk aversion coefficient. We now turn to the presentation of the utility specification and the respective results.
We have seen in the descriptive statistics for the straddle returns, that we can reject the null hypothesis of a normal distribution very clearly. Straddle returns are positively skewed and display high excess kurtosis. Therefore, holding a short position in the straddles exposes the investor to negative skewness and high excess kurtosis.

In order to allow for an impact of higher moments on the investor’s portfolio choice, we first express the utility over final wealth $U(W)$ through a Taylor series expansion around the expected value as

$$U(W) = \sum_{k=0}^{\infty} \frac{U^{(k)}(E(W))}{k!} (W - E(W))^k$$  \(8\)

where $U^{(k)}$ denotes the $k$-th derivative of the utility function.

Following the approach of Dittmar (2002), who proposes a Taylor series expansion of the utility function up to the order of four, we use the following approximation.

$$U(w) \approx U(E(W)) + \sum_{k=1}^{4} \frac{U^{(k)}(E(W))}{k!} (W - E(W))^k$$  \(9\)

In contrast to Dittmar (2002), we take a stand on the form of the agent’s utility function and specify the agent’s objective as maximising expected power utility, i.e., $E[U(W)]$, where $U(W) = \frac{W^{1-\gamma}}{1-\gamma}$ and $W$ is a random variable representing terminal wealth.

The use of this objective will ensure that portfolio moments of higher order, such as the third central moment $(W - E(W))^3$ and the fourth central moment $(W - E(W))^4$ are taken into account in the optimisation. Since the second and fourth derivatives of the power utility function are negative, while the third one is positive, we obtain the well documented preference for positive skewness (see Kraus and Litzenberger (1976)) and aversion to high kurtosis.

A problem with our specification may be that the approximation may be poor. An alternative to this utility specification is to evaluate expected utility by assuming that portfolio returns follow a certain probability law. Given that we include straddle returns, whose distributional properties are not well known, there is not much guidance which type of distribution to choose. In addition, fitting a distribution to the straddle returns would introduce a high amount of model risk. An alternative would be to use the empirical distribution directly or to form bootstrap samples based on this distribution. This approach would introduce considerable sample risk, since the empirical distribution may not be a good description of the true distribution and all its moments. Our approach constitutes a trade off of model risk and sample risk. We do not assume a particular distribution function nor do we use the empirical distribution directly to evaluate the expected utility. With the chosen approach, we obtain portfolio moments from the empirical distribution, and use these in evaluating expected utility.

Figure 2: Power Utility: Optimal Daily Portfolios with and without Straddles
The results indicated in Figures 2, 3 and 4 show that the negative returns of the straddle lead to short positions in this asset in the optimal portfolios. While the holding in the stock market index are relatively little affected by the introduction of the straddle, the position in the short-term interest rate increases, as the proceeds from shorting the straddles are invested in the riskless asset. The magnitude of the short position in the straddle becomes much less pronounced for the weekly and daily straddle trading strategies. Moreover, the straddles' high excess kurtosis and the negative skewness associated with the short straddle position mean that the short position allocated to straddles remains limited.3

Figure 3: Power Utility: Optimal Weekly Portfolios without and with Straddles

Figure 4: Power Utility: Optimal Monthly Portfolios without and with Straddles

3 - At the same time, one must bear in mind that co-skewness, respectively co-kurtosis with the stock market return and the interest rate also matter, in addition to the moments of the straddle return distribution.
The portfolio weights show a marked difference between daily straddles and those constructed with lower rebalancing frequencies. The short position in the case of daily rebalancing is a multiple of the initial portfolio wealth for most risk aversion levels. Even for the lower rebalancing frequency, we can conclude that risk-averse investors should optimal allocations imply an important short position in straddles.

So far, we have only looked at the size of the optimal weight in the straddle trading strategy. A logical next step is to assess how much better off the investor is with the inclusion of the straddle in his portfolio. In order to assess this question, we compute the certainty equivalents of investing in stocks and the short-term interest rate (cash), and the corresponding certainty equivalent of investing into stocks, cash, and the straddles. We then take the difference between the two certainty equivalents. This certainty equivalent gain in terms of return per rebalancing period is shown in table 5.

Table 5: Certainty Equivalent Gain for Daily, Weekly, And Monthly Straddle Trading

<table>
<thead>
<tr>
<th>Reference Stock Weight</th>
<th>Daily</th>
<th>Weekly</th>
<th>Monthly</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.40000</td>
<td>0.07696</td>
<td>0.02853</td>
<td>0.01434</td>
</tr>
<tr>
<td>1.25000</td>
<td>0.07153</td>
<td>0.02585</td>
<td>0.01304</td>
</tr>
<tr>
<td>1.10000</td>
<td>0.06574</td>
<td>0.02310</td>
<td>0.01170</td>
</tr>
<tr>
<td>0.95000</td>
<td>0.05951</td>
<td>0.02027</td>
<td>0.01031</td>
</tr>
<tr>
<td>0.80000</td>
<td>0.05276</td>
<td>0.01734</td>
<td>0.00886</td>
</tr>
<tr>
<td>0.65000</td>
<td>0.04535</td>
<td>0.01433</td>
<td>0.00735</td>
</tr>
<tr>
<td>0.50000</td>
<td>0.03712</td>
<td>0.01121</td>
<td>0.00577</td>
</tr>
<tr>
<td>0.35000</td>
<td>0.02784</td>
<td>0.00798</td>
<td>0.00413</td>
</tr>
<tr>
<td>0.20000</td>
<td>0.01716</td>
<td>0.00464</td>
<td>0.00241</td>
</tr>
<tr>
<td>0.05000</td>
<td>0.00463</td>
<td>0.00118</td>
<td>0.00062</td>
</tr>
</tbody>
</table>

It can be seen from table 5 that the certainty equivalent gains from investing in the straddles are very high for the daily rebalancing strategy, ranging from 0.5% to more than 7% per day depending on risk aversion. For weekly rebalancing, certainty equivalent gains range from 12 basis points to more than 2% per week. For monthly rebalancing, certainty equivalent gains range from six basis points to more than 1%. Do these certainty equivalent gains provide evidence that a risk-averse investor should include such straddle strategies in her portfolio? To answer this question, one has to consider the magnitude of the optimal straddle weights. For investors with low risk aversion, the optimal weight for the daily straddle strategy ranges from 500% to 1000% of portfolio wealth. Even in the presence of relatively low transaction costs, say a round trip cost of 1%, the utility gains in the table will not be sufficient to compensate these costs. Put differently, certainty equivalent gains will probably not survive in the presence of transaction costs.

Taking the investor with a reference weight of 50% in the risky asset, the difference in utility gains across rebalancing frequencies is striking. Daily rebalancing leads to monetary utility gains of 3.7% per day, while weekly rebalancing only leads to a 1.1% gain per week and monthly rebalancing to 0.6% per month. This difference clearly underlines the necessity of costly rebalancing to maintain the attractiveness of the straddle.
5. Conclusion
This paper assesses the returns to holding periodically rebalanced straddle positions in the DAX options market. We analyse straddles with daily, weekly and monthly rebalancing. Each period, we construct beta-neutral positions using a pair of matching call and put contracts and find that such a strategy yields significantly negative returns for the cases of daily and weekly rebalancing frequencies.

The empirical properties of straddle returns have been analysed in more detail. Time series analysis finds that straddle returns seem to follow random walks, thus excluding any possibilities to predict future prices from past prices for such option positions. In addition, factor analysis shows that other than the volatility of the underlying asset, daily straddle returns are also linked to factors that proxy for dispersion of beliefs and investors' learning uncertainty. However, though significant, these factors can only explain a small fraction of the variation in daily straddle returns, and they are not significantly different from zero at lower rebalancing frequencies.

The final point analysed in the paper is the portfolio perspective. We consider utility maximising investors and ask what the optimal position in our straddle trading strategies would be for such investors. We find that the negative returns in straddles lead to large short positions of the straddle trading strategies in the optimal portfolios. When analysing the added value that straddles bring to an investor who previously held the stock market index and cash, we find that utility gains decrease with rebalancing frequency. However, even for the daily rebalanced straddles, utility would not cover transaction costs in option markets.

This paper sheds some doubt over the practical relevance of option strategies put forward in the academic literature as proxies for risk premia. In principle, if risk premia other than the typical stock market risk, interest rate risk, and credit risk exist, investors should consider how to gain exposure to such risk premia when constructing optimal portfolios. However, the volatility risk premium obtained from options trading, is not exploitable in practice. The negative risk premium from straddle trading is only significantly different from zero at a frequency of at least weekly rebalancing. Factor exposures consistent with interpreting straddle returns as a risk premium are only significant with at least daily data. Finally, certainty equivalent gains for including straddles in a portfolio in addition to a stock market index and cash are high only for daily rebalanced straddles. However, for these daily rebalanced straddles, the transaction costs arise every day and would cancel the utility gains.

From a practical perspective, it can be concluded that the empirical evidence on a risk premium inherent in straddle returns does not lead to any realistic investment opportunities, unless the investor does not face transaction costs.

References


