Asset Allocation with Shadow Assets

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Bernd Scherer
Professor of Finance, EDHEC Business School

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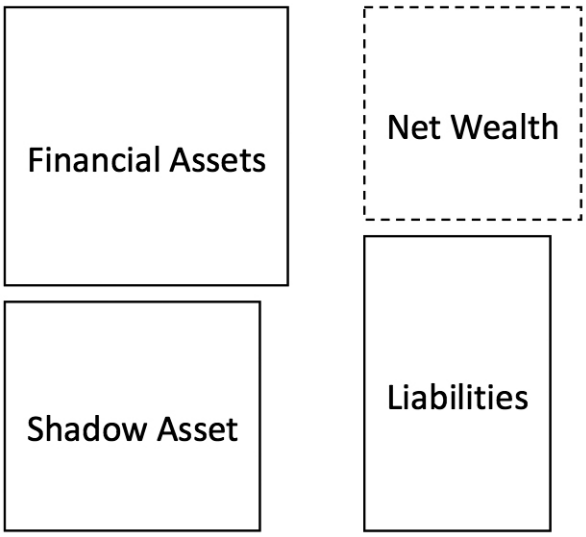
Introduction

The wealth of most investors contains both financial assets as well as non-financial assets. I define shadow assets as (mostly) nonfinancial and non-tradable assets that are exogenous to the investor's asset allocation decision. Examples for shadow assets are human capital, nonfinancial sovereign assets (e.g. underground oil reserves) the present value of future alumni contributions for university endowments or the non-listed family business for the client of a family office. Allocations to these shadow assets can hardly be changed and yet their existence will change the investor's perspective on total wealth at risk.

Exhibit 1 provides a simplified economic balance sheet for a hypothetical investor.[1] While the inclusion of financial assets, liabilities and net wealth has been standard in asset (liability) management, the existence of shadow assets is not accounted for in the portfolio management literature.[2] Ignoring shadow assets is unfortunate as it is the existence and nature of shadow assets that distinguishes private investors, university endowments, sovereign wealth funds and family offices and hence leads to different demands for risky securities. Shadow assets influence the outcome of asset allocation decisions via their covariance with financial assets and their effect on total wealth. They create a unique hedging demand (the need to manage covariance risks in non-traded shadow assets via tradable financial assets) that leave standard (financial asset centric) asset allocation advice inadequate at best.

Exhibit 1: Stylized economic balance sheet with shadow assets. The left side of the economic balance sheet contains total wealth (both financial assets and shadow assets) while the right side contains investors liabilities. The remaining net wealth position is sometimes called surplus, equity or discretionary wealth. Optimally investors manage their net wealth position. Less ideally they manage total wealth while managing financial portfolio wealth alone is least optimal.



What are examples for shadow assets? Let us start with *individual investors*. Young investors are typically richer than their financial assets suggest as they own substantial amounts of shadow assets that should have a major influence on their asset allocation. I define human capital as the present value of future savings, which (given slavery is abolished) is large relative to financial wealth for those that start their career and small for those that are about to retire. Human capital is neither tradable nor can its nature (i.e. the profession and industry the investor works in) be easily changed, i.e. it is exogenous for all practical purposes. Equally, the nature of human capital differs from professions to profession. While civil servants and academics exhibit bond like human capital (their pay usually does not depend on the state of the economy and their career path is highly projectable) the human capital of investment bankers will have a high sensitivity to stock markets. In other words, investment bankers already own stocks via their human capital. Investors working in the automobile industry would be well advised not to buy other automobile stocks, let alone their own company stock. However even experienced market professionals tend to commit

1 - This essay mostly ignores corporate pension funds which should be instead dealt with in a corporate finance framework.

2 - We implicitly assume that shadow assets do not come with shadow liabilities. For a sovereign wealth fund it would, for example, be wrong to activate the present value of future tax receipts as shadow assets without acknowledging the present value of future government spending. FABOZZI/WILCOX (2009) analyze private investors without shadow assets and with essentially non-stochastic liabilities, while SHARPE/TINT (1990) analyze pension funds not including shadow assets (like the plan sponsor guarantee). CAMPBELL/VICEIRA (2002) allow for human capital in investment problems but ignore liabilities altogether.

these mistakes as the bankruptcy of Lehman Brothers revealed where many employees held on to their stocks to the very end. Corporate pension promises (typically non-tradable) belong in the same category. Equally expected inheritance (real estate, i.e. the parent's home, security portfolio) qualifies as shadow assets.

Family offices are a related category of private investors. For these type of investors, the shadow asset is made up by the family business of those they manage money for. Decisions on the family business can only be made by the owner, not by the family office manger to whom this remains exogenous. Even for the owner of typically non-listed businesses, the decision making process is usually not free but constrained by family tradition and personal ego.

Sovereign wealth funds also need to include shadow assets into their implicit balance sheets. Underground oil is a typical example of a shadow asset. Oil reserves are not tradable (limited to current production), exogenously given and vast in size. Their covariance with sovereign wealth fund financial assets has been left unmanaged. Many sovereign wealth funds held diversifying (recession hedging) fixed income investments that pay off if oil prices fall.[3] Equally a foreign exchange (excess reserves) based sovereign wealth fund also needs to account for shadow assets defined as the present value of the fiscal surplus (present value of future tax revenues minus present value of future government expenditures). In the case of China for example this shadow asset exhibits a strong exposure to global commodity prices as well as the US consumer. Exaggerating our point one might want to say a Chinese Sovereign Wealth fund might want to short Wall Mart stocks.

University endowments also hold shadow assets on their balance sheet in the form of the present value of future contributions from their alumni. If the economy performs well, the Alumni will be in a better position to contribute than otherwise. Equally if the Alumni is concentrated in a given sector of the economy (technology and engineering or finance), these contributions also contain sector risks.

The few examples above show that the existence of shadow assets is omnipresent in an investor's investment problems. Realistic and fiduciary investment advice requires the inclusion of shadow assets. Only then will the scope of investment advice reflect the domain of investor differences.

Optimal Asset Allocation with Shadow Assets

We now formally investigate the influence of shadow assets on asset allocation decisions.[4] Let us define *q* as the fraction of financial wealth relative to total aggregate wealth (sum of financial wealth and shadow assets)

$$\theta = \frac{financial\ \ assets}{financial\ \ assets\ \ + shadow\ \ asset}$$

We can show (see Appendix A), that the optimal demand for a set of risky assets is a weighted mixture of traditional mean variance speculative demand and shadow asset beta (scaled covariance between financial assets and shadow assets) as provided in equation (1).

$$\begin{pmatrix} asset \\ demand \end{pmatrix} = \left(\frac{1}{\theta}\right)\begin{pmatrix} speculative \\ demand \end{pmatrix} + \left(1 - \frac{1}{\theta}\right)\begin{pmatrix} shadow\ \ asset \\ beta \end{pmatrix} \tag{1}$$

The existence of shadow assets has two consequences. First, investors are generally richer than their financial wealth suggests. Hence speculative demand needs to be larger than in the case in a financial asset only optimization.[5] Relative to the asset centric (or stand alone case), speculative demand will be increased. The direct effect of including shadow assets is that it makes an

3 - SESTER/ZIEMBA (2009) document the equity orientated asset allocation for most sovereign wealth funds while SCHERER (2011) provides normative advice on how to include oil reserves into asset allocation problems.
4 - This section focuses on asset allocation (non-stochastic, cash like liabilities) ignoring stochastic liabilities. We leave asset liability management to the next section.
5 - Assume an investor (with wealth independent risk aversion, i.e. CRRA) and a 50:50 split between financial and shadow assets (i.e., $q = \frac{1}{2}$) that finds it optimal to hold 40% of its (total) wealth in risky assets. He we will then need to invest 80% of his financial wealth into risky assets in order to achieve an allocation of 40% risky assets on total wealth (that is in this example twice as big as financial wealth).

investor more aggressive. A second (indirect) effect on total demand will depend on the shadow asset covariance with financial assets. If financial assets co-vary strongly with shadow assets they become unattractive as the volatility of aggregate wealth increases for a non-diversifying financial asset.

Let us provide an example for (1). Suppose our investor is a young investment banker. In this case he will be richer than his bank account (financial wealth) tells us, as the present value of his future savings (his shadow asset) is likely to be substantial. Moreover his shadow asset will contain a strong equity component as his investment banking bonus is largely dependent on the performance of equity markets. To give numbers to our prescriptions we set $q = \frac{1}{3}$ and assume shadow asset beta of 0.5 (human capital has an implicit 50% equity allocation). We also assume our investor exhibits (stand alone) speculative demand for equities of 40%. In this setting the young investment banker would instead not invest 40% but only 20% in equities when young, as the 120% speculative demand (3 times 40%) is offset by a -100% (0.5 times 2) in hedge demand.

$$\left(\frac{1}{\frac{1}{3}}\right)(40\%) + \left(1 - \frac{1}{\frac{1}{3}}\right)(50\%) = 3 \cdot 40\% - 2 \cdot 50\% = 20\% \tag{2}$$

Over time q will converge to zero. At the day of his retirement we find q = 1 by definition (the shadow asset is depleted) and the (now ex) investment banker will hold 40% of his financial wealth in equities. This simple example shows that the demand for risky assets does not necessarily decline over the life cycle and it is the characteristic of the shadow asset and its depletion that will define the optimal path.[6] Standard target date funds are more likely to be suitable for university professors than for investment bankers or workers in the construction industry.

Asset Liability Management with Shadow Asset

So far we have been looking exclusively at the asset side of Exhibit 1. However it is unrealistic to ignore liabilities. First all investors have explicit or implicit liabilities. Even private investors exhibit (real) liabilities in the form of bequest motives or minimum wealth level requirements (to fund a desired level of living after retirement). Second, we could always include a benchmark asset as a pseudo liability asset (i.e. a short position of 100% of aggregate wealth). The most natural benchmark asset (in the absence of defined liabilities) is a holding of inflation linked bonds. How does this change our analysis? Fully modelling financial assets, shadow assets and liabilities for a given funding ratio (financial assets plus shadow assets divided by liabilities), f, yields

$$\begin{pmatrix} asset \\ demand \end{pmatrix} = \left(\frac{1}{\theta}\right)\begin{pmatrix} speculative \\ demand \end{pmatrix} + \left(1 - \frac{1}{\theta}\right)\begin{pmatrix} shadow\ asset \\ beta \end{pmatrix} + \left(\frac{1}{\theta}\right)\left(\frac{1}{f}\right)\begin{pmatrix} liability \\ beta \end{pmatrix} \tag{3}$$

In addition to (1) we now see that asset demand is a weighted combination of speculative demand, shadow asset hedging and liability hedging demand (three fund separation). The more leveraged an "entity" (smaller f), the less aggressively should the "entity" invest as the volatility of net wealth increases and hence liability hedging demand grows. For f becoming large, liabilities play a minor role. More generally financial assets are held if they show attractive (stand alone) risk reward tradeoffs, if they help to reduce fluctuations in shadow assets (negative covariance with shadow assets) or if they help hedge liability related risks (positive covariance with liabilities).

We continue with our example of the young investment banker and introduce bonds as a second risky asset assuming that the investment banker's speculative demand for 5 year duration bonds is 20%. We further assume that the investment banker wants to reserve one third of his total wealth for retirement purposes (implicit liability) such that f = 3. What would his optimal

6 - Not that our analysis does not rule out negative values for shadow assets. This could occur in the case of an unreliable wife that triggers divorce if financial wealth becomes large or in the implicit put of a sovereign provided to the sovereign's banking system.

portfolio look like? Inserting our assumptions into (3) yields

$$\begin{pmatrix} 1 \\ \frac{1}{3} \end{pmatrix}\begin{pmatrix} 40\% \\ 10\% \end{pmatrix} + \begin{pmatrix} 1 - \frac{1}{\frac{1}{3}} \end{pmatrix}\begin{pmatrix} 50\% \\ 0 \end{pmatrix} + \begin{pmatrix} \frac{1}{\frac{1}{3}} \end{pmatrix}\begin{pmatrix} \frac{1}{3} \end{pmatrix}\begin{pmatrix} 0\% \\ 100 \end{pmatrix} = \begin{pmatrix} 120\% \\ 30\% \end{pmatrix} + \begin{pmatrix} -100\% \\ 0\% \end{pmatrix} + \begin{pmatrix} 0\% \\ 100\% \end{pmatrix} = \begin{pmatrix} 20\% \\ 130\% \end{pmatrix} \tag{4}$$

Equation (4) results in a total demand of 20% equities and 130% bonds in the investor's financial wealth. Our investor looks leveraged, but this form of leverage poses little problem in reality. He could invest 20% in equities and the remaining 80% in 8.125 duration bonds to arrive at a synthetic leverage of 130% ($80\%\frac{8.125}{5} = 130\%$) for fixed income and 50% synthetic leverage in total. Note that this would not require physical leverage. Also note that the new solution differs considerably from an asset centric allocation of 40% equities and 10% bonds (with the reminder in cash).

The Costs of Ignoring Advice

So far we saw that adding shadow assets has two effects on investors. First, it will make investors more aggressive. Speculative demand increases as more risk taking in financial assets is required to end up with the desired allocation of risky assets on total wealth. The second effect is more ambiguous. Depending whether the covariance of risky financial assets with shadow assets is positive or negative, hedging demand will be negative (or positive). We can derive the following expression (see Appendix C) for the utility advantage of accounting for covariance between shadow and financial assets (versus ignoring the existence of shadow assets).

$$\frac{risk - aversion}{2}(\theta - 1)^2 \begin{pmatrix} speculative & - & shadow\ asset \\ demand & & beta \end{pmatrix}^T \begin{pmatrix} Covariance \\ matrix \end{pmatrix}\begin{pmatrix} speculative & - & shadow\ asset \\ demand & & beta \end{pmatrix} \tag{5}$$

This equation is straightforward to interpret. First, it is always non-negative.[7] In other words the expected utility for the correct solution always exceeds the asset centric (asset only) solution. The difference increases with risk aversion as ignoring shadow assets will leave our investor with too much volatility of total wealth. Second, for q = 1 (all assets are financial assets) there is no difference in *ex ante* expected utility between both approaches. This is not surprising as we removed shadow assets from the asset allocation problem. Third, the same cannot be said for zero co variances between shadow assets and financial assets, i.e. a zero shadow asset beta. This result comes as a surprise. Wouldn't we expect, that a zero covariance between shadow assets and financial assets is sufficient to ignore the existence of shadow assets? Our initial intuition is wrong as the existence of positive (negative) shadow assets increases (decreases) the aggressiveness of an investor irrespective of their covariance with financial assets.

Conclusion

Asset allocation advice for private investors, sovereign wealth funds, endowments or family offices is yet asset centric. This is consistent with the practitioner's implementation of the capital asset pricing model. While all risky assets should be included in defining the market portfolio, only listed financial assets are used in practice. This has made it easier for many practitioners to ignore the impact of shadow assets on portfolio choice. However, we believe this approach is hardly fiduciary. Investment advice in the client's best interest needs to incorporate financial assets, shadow assets and liabilities as optimal allocations differ considerably when shadow assets are properly accounted for.

7 - This is true as long as the covariance matrix of financial asset returns is positive semi-definite.

Appendix A: Asset Allocation with Shadow Assets

Assume the return on assets (financial assets plus shadow assets) is given by

$$r = \left[\theta w^{T} r_a + (1-\theta) r_s\right] \tag{A1}$$

where r_a denotes a n x 1 vector of (financial asset returns) and r_a denotes the return of a shadow asset. Moreover 1 - q defines the percentage weight of shadow assets as a fraction of total assets and q stands for the weight of financial assets relative to total The choice variable for our investor is w, the n x 1 vector of financial asset weights. We write the problem of trading of returns on total wealth versus the volatility of total wealth

$$m - \frac{L}{2} s^2 \tag{A2}$$

To keep track of what is going on we make use of matrix algebra and define both return and risk of total wealth from

$$\begin{aligned} \sigma^2 &= \begin{bmatrix} w\theta \\ 1-\theta \end{bmatrix}^T \begin{bmatrix} \Omega_{aa} & \Omega_{as} \\ \Omega_{sa} & \Omega_{ss} \end{bmatrix} \begin{bmatrix} w\theta \\ 1-\theta \end{bmatrix} \\ \mu &= \begin{bmatrix} w\theta \\ 1-\theta \end{bmatrix}^T \begin{bmatrix} \mu_a \\ \mu_s \end{bmatrix} \end{aligned} \tag{A3}$$

Here $m_a = E(r_a)$, $m_s = E(r_s)$, W_{aa} denotes a $n ¥ n$ matrix of asset returns, W_{as} a $n ¥ 1$ vector of co-variances between financial assets and a single shadow asset with volatility W_{ss}. Collecting the terms for only such expressions that involve w, we can write the objective function as

$$w^* = \arg\max_{w} w^T m_a - \frac{L}{2}\left(q^2 w^T \mathrm{W}_{aa} w + 2q(1- q)\mathrm{W}_{as} w\right) \tag{A4}$$

The solution to (A4) is given by

$$w^* = \left(\tfrac{1}{q}\right) l^{-1} \mathrm{W}_{aa}^{1} m_a + \left(1 - \tfrac{1}{q}\right) \mathrm{W}_{aa}^{1} \mathrm{W}_{as} \tag{A5}$$

This equation corresponds to equation (1) in the text where $l^{-1}\mathrm{W}_{aa}^{1} m_a$ denotes speculative demand and $\mathrm{W}_{aa}^{1}\mathrm{W}_{ao}$ stands for the shadow asset beta. We can also derive implied returns from the first-order condition:

$$q m_a - l\, q^2 \mathrm{W}_{aa} w - l\,(1- q) q \mathrm{W}_{as} = 0 \tag{B5}$$

$$m_a^{impl} = l\left(q \mathrm{W}_{aa} w + (1- q)\mathrm{W}_{as}\right) \tag{B6}$$

An asset will demand a higher return in an (optimal) investor portfolio if it exhibits a high marginal contribution to asset portfolio risk and if it co-varies positively with shadow assets. For q = 1 we are back to the usual expression for implied returns.

Appendix B: Asset Liability Management with Shadow Assets

Assume now we model the return on net wealth (financial assets plus shadow assets minus liabilities) as given by

$$r_{nw} = \left[\theta w^{T} r_a + (1-\theta) r_s\right] - \left(\tfrac{1}{f}\right) r_l \tag{B1}$$

Additional to Appendix A we define r_{nw} as the return on net wealth and r_l as the return on liabilities. The variable f defines the funding ratio (the ratio of total assets to liabilities). The choice variable for our investor is w. We write the problem of trading of returns on net wealth versus the volatility of net wealth as

$$m_{nw} - \frac{L}{2} s^2_{nw} \tag{B2}$$

To keep track of what is going on we again make use of matrix algebra and define both return and risk of net wealth from

$$\sigma_{nw}^2 = \begin{bmatrix} w\theta \\ 1-\theta \\ -\frac{1}{f} \end{bmatrix}^T \begin{bmatrix} \Omega_{aa} & \Omega_{as} & \Omega_{al} \\ \Omega_{sa} & \Omega_{ss} & \Omega_{sl} \\ \Omega_{la} & \Omega_{ls} & \Omega_{ll} \end{bmatrix} \begin{bmatrix} w\theta \\ 1-\theta \\ -\frac{1}{f} \end{bmatrix}$$

$$\mu_{nw} = \begin{bmatrix} w\theta \\ 1-\theta \\ -\frac{1}{f} \end{bmatrix}^T \begin{bmatrix} \mu_a \\ \mu_o \\ \mu_l \end{bmatrix} \qquad \text{(B3)}$$

Equation (B3) involves three new covariance terms, where W_{al} denotes a $n¥1$ vector of covariance between assets and liabilities, W_{sl} defines the covariance between shadow assets and liabilities, while W_{ll} is variance of liabilities. Collecting the terms for only such expressions that involve w, we can write the objective function as

$$w^* = \arg\max_w w^T m_a - \frac{L}{2}\left(q^2 w^T W_{aa} w + 2q(1- q)W_{ao} w - 2q\left(\frac{1}{f}\right)W_{al} w\right) \qquad \text{(B3)}$$

The solution to (B3) is given by

$$w^* = \left(\frac{1}{q}\right) l^{-1} W_{aa}^{1} m_a + \left(1- \frac{1}{q}\right) W_{aa}^{1} W_{as} + \left(\frac{1}{q}\right)\left(\frac{1}{f}\right) W_{aa}^{1} W_{al} \qquad \text{(B4)}$$

This replicates equation (3) from the main text. As before, we can also derive implied returns from the first-order condition:

$$q m_a - l q^2 W_{aa} w - l (1- q) q W_{as} + l q \frac{1}{f} W_{al} = 0 \qquad \text{(B5)}$$

$$m_a^{impl} = l\left(q W_{aa} w + (1- q) W_{as} - \frac{1}{f} W_{al}\right) \qquad \text{(B6)}$$

An asset will demand a higher return in an (optimal) investor portfolio if it exhibits a high marginal contribution to asset portfolio risk, if it co-varies positively with shadow assets, or if it co-varies negatively with liabilities.

Appendix C: Difference in Utility

The cost of ignoring optimal advice is given by the difference in expected utility generated from the two different solutions (with and without shadow assets). Using the optimal solution with shadow assets, $w^*(q, W_{as})$ as given in (A5) as well as the expression for pure speculative demand, i.e. $w^* = l^{-1} W_{aa}^{1} m$, we arrive (after some tedious algebra) at

$$\begin{aligned} D u &= u(w(q, W_{as})) - u(w) \\ &= \frac{L}{2}\left(\frac{W_{aa}^{1} m}{l} - W_{aa}^{1} W_{as}\right) W \left(\frac{W_{aa}^{1} m}{l} - W_{aa}^{1} W_{as}\right)^T (q - 1)^2 \end{aligned} \qquad \text{(C1)}$$

This reproduces equation (5) in the main text.

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For more information, please contact:
Carolyn Essid on +33 493 187 824
or by e-mail to: carolyn.essid@edhec-risk.com

EDHEC-Risk Institute
393-400 promenade des Anglais
BP 3116
06202 Nice Cedex 3 - France

EDHEC Risk Institute—Europe
10 Fleet Place - Ludgate
London EC4M 7RB - United Kingdom

EDHEC Risk Institute—Asia
1 George Street - #07-02
Singapore 049145

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