Improved Risk Reporting with Factor-Based Diversification Measures

February 2014
# Table of Contents

Executive Summary ........................................................................................................... 5

1. Introduction .................................................................................................................. 17

2. Portfolio Diversification Measures ............................................................................ 21

3. Empirical Analysis for Equity Indices ....................................................................... 29

4. Empirical Analysis for Pension Funds ....................................................................... 39

5. Conclusion .................................................................................................................... 53

Appendices ....................................................................................................................... 55

References ......................................................................................................................... 87

About CACEIS .................................................................................................................. 91

About EDHEC-Risk Institute ............................................................................................. 93

Foreword

The present publication, “Improved Risk Reporting with Factor-Based Diversification Measures,” is drawn from the CACEIS research chair on “New Frontiers in Risk Assessment and Performance Reporting” at EDHEC-Risk Institute.

This chair looks at improved risk reporting, integrating the shift from asset allocation to factor allocation, improved geographic segmentation for equity investing, and improved risk measurement for diversified equity portfolios.

Before the financial crisis, pension funds were insufficiently diversified, with concentration in a small number of asset categories. Since the crisis of 2007, there has been a genuine trend towards investment in new asset classes and categories in order to diversify, but that does not mean that the diversification is effective. As we see in the current publication, what is important is the “effective number of bets” (ENB) in a portfolio, not the effective number of constituents (ENC). Only ENB delivers superior performance. Increasing the number of asset classes or categories without taking the inter-relations between their risks into account does not provide any real gain in terms of performance.

Investors are right to wonder about the excessive concentration of their cap-weighted benchmarks, because the excessive concentration has a negative impact both on performance and on the capacity to weather bear markets. However they should be even more concerned about the real risk concentration of these indices. ENB is clearly a more statistically significant indicator that ENC in appreciating the quality of portfolio diversification and protection against market shocks. Here too, the results show that risk allocation and risk-based weighting make sense when constructing well-diversified indices.

I would like to thank Lionel Martellini and his co-authors, Tiffanie Carli and Romain Deguest, for the quality of this path-breaking publication. We would also like to extend our warmest thanks to our partners at CACEIS for their insights into the issues discussed and their commitment to the research chair.

We wish you a useful and informative read.

Noël Amenc
Professor of Finance
Director of EDHEC-Risk Institute
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Executive Summary
Introducing an Improved Measure of Diversification

Risk reporting is increasingly regarded by sophisticated investors as an important ingredient in their decision-making process, and a large number of indicators are now available to help them assess the risks of their portfolio. The most commonly used risk measures such as volatility (a measure of average risk), Value-at-Risk (a measure of extreme risk) or tracking error (a measure of relative risk), however, are typically backward-looking risk measures computed over one historical scenario. As a result, they provide very little information, if any, regarding the possible causes of the portfolio riskiness, the probability of a severe outcome in the future, or the reward that an investor can expect in exchange for bearing those risks. In this context, it appears to be of critical importance for investors and asset managers to also be able to rely on forward-looking risk indicators for their portfolios.

Common intuition and portfolio theory both suggest that the degree of diversification of a portfolio is a key indicator when assessing its ability to generate attractive risk-adjusted performance across various market conditions. The benefits of diversification are intuitively clear: efficient diversification generates a reduction of unrewarded risks that leads to an enhancement of the portfolio risk-adjusted performance. On the other hand, in the absence of a formal definition for diversification, it is not as straightforward a task as it might seem to provide a quantitative measure of how well or poorly diversified a portfolio is. The usual definition of diversification is that it is the practice of not “putting all your eggs in one basket”. Having eggs (dollars) spread across many baskets is, however, a rather loose prescription in the absence of a formal definition for the true meaning of “many” and “baskets”.

An initial approach to measuring portfolio diversification would consist of a simple count of the number of constituents the portfolio is invested in. One key problem with this approach is that what matters from a risk perspective is not the nominal number of constituents in a portfolio, but instead its effective number of constituents (ENC). To understand the nuance, let us consider the example of a fictitious equity portfolio that would allocate 99% of the wealth to one stock and spread the remaining 1% of the wealth to the 499 remaining stocks within the S&P 500 index universe. While the nominal number of stocks in that portfolio (defined as the number of stocks that receive some non zero allocation) is 500, it is clear that the effective number of stocks in the portfolio is hardly greater than one, and that this poorly diversified portfolio will behave essentially like a highly concentrated one-stock portfolio from a risk perspective. In this context, it appears that a natural and meaningful measure of the effective number of constituents (ENC) in a portfolio is given by the entropy of the portfolio weight distribution. This quantity, a dispersion measure for probability distributions commonly used in statistics and information theory, is indeed equal to the nominal number N for a well-balanced equally-weighted portfolio, but would converge to 1 if the allocation to all assets but one converges to zero as in the example above, thus confirming the extreme concentration in this portfolio.
On the other hand, if one is indeed entitled to considering that a well-balanced allocation of dollars (eggs) to identical securities (baskets) may be regarded as a well-diversified allocation, the existence of differences in risks across securities would require some adjustment to the proposed measure of sound diversification. In other words, what needs to be well-balanced is not the number of eggs in each basket per se, but rather the risk contribution of each basket. In this context, a well-diversified portfolio would seek to have more eggs in more robust baskets, and fewer eggs in frailer baskets.

At this stage, the need remains for a critical assessment of what should be the proper interpretation for the “baskets” in this proverbial definition of diversification. The straightforward approach, which suggests that baskets are asset classes in an asset allocation context, or securities for a portfolio constructed within a given asset class, is in fact misleading or at least severely incomplete. Indeed recent research (e.g. Ang et al. (2009)) has highlighted that risk and allocation decisions could be best expressed in terms of rewarded risk factors, as opposed to standard asset class decompositions, which can be somewhat arbitrary. For example, a seemingly well-diversified allocation to many asset classes that essentially load on the same risk factor (e.g., equity risk) can eventually generate a portfolio with a very concentrated set of risk exposures. Going back to the eggs-and-baskets analogy, having a well-balanced allocation of eggs across many different baskets that would be tied together can hardly be regarded as an astute way to ensure a proper diversification of the risks involved in carrying eggs to the market. In other words, baskets should be interpreted as uncorrelated risk factors, as opposed to correlated asset classes, and it is only if the distribution of the contributions of various factors to the risk of the portfolio is well-balanced that the investor’s portfolio can truly be regarded as well-diversified.

Putting all these elements together, we propose using the effective number of bets (ENB) in our empirical analysis, which would serve as a meaningful measure of diversification for investors’ portfolios (see Meucci (2009) and Deguest, Martellini and Meucci (2013) for more details). 1

One natural way to turn correlated asset returns into uncorrelated factor returns is to use principal component analysis (PCA). While useful in other contexts, the PCA approach suffers from a number of shortcomings when estimating the effective number of bets. The first shortcoming is the difficulty in interpreting the factors, which are pure statistical artefacts. The second shortcoming, particularly severe in the context of the design of a diversification measure, is that by construction, principal components are defined so as to achieve the highest possible explanatory power. As a result, the contribution of the first few factors is often overwhelmingly large with respect to the contribution of other factors, and the portfolio diversification measure empirically tends to be biased towards low values. 2 A competing approach to extracting uncorrelated factors from a basket of correlated constituents, which we use in the analysis that follows, is the minimal linear torsion (MLT) approach, which focuses on extracted uncorrelated factors that are as close as possible to the original constituents, in the sense that they have the same volatility as the original constituents.

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1 - ENB is formally defined as the entropy of the distribution of contributions of uncorrelated factors to the risk of the portfolio.
2 - For example, the ENB measure is equal to 1 for an equally-weighted portfolio based on a universe of assets with equal volatility and pairwise correlation values, regardless of the correlation value, while the intuition would suggest that for a vanishing constant correlation value such a portfolio should have a number of bets equal to the number of assets. This counter-intuitive full-concentration effect follows because the equally-weighted portfolio is in this case fully exposed to the first principal component and not exposed to any other principal component (see Meucci, Santangelo and Deguest (2013)).
constituents and achieve the lowest average tracking error with the initial constituents. By construction, the obtained factors are the closest uncorrelated representations of the original constituents, which alleviates the concern over interpretation, and the explanatory power of the factors are not biased in favour of some particular factors.

Analysing the Relationship between Portfolio Diversification and Portfolio Performance in Various Market Conditions

Our main objective is to analyse the diversification of a portfolio, measured either in terms in effective number of constituents (ENC) or (more appropriately) in terms of effective number of bets (ENB), and its relationship with subsequent portfolio performance. We provide an empirical application of this measure for intra-class and inter-class diversification. For intra-class diversification, we cast the empirical analysis in the context of various popular equity indices, with a particular emphasis on the S&P 500 index. For inter-class diversification, we analyse policy portfolios for the 1,000 largest US pension funds.

Diversification Measures for International Equity Indices

We first compute the ENC and ENB measures for the S&P 500 index, and test for their predictive power using weekly total return data over a sample period extending from 4 January 1957 to 31 December 2012. For both diversification measures, we actually perform six linear regression analyses, with each linear regression testing the relationship between the diversification measure at a given week and the annualised performance for each of the following six different lengths of the predictive period: the following quarter; the following semester; the following year; the following two years; the following five years; and the following 10 years. In Table 1, we show the results obtained from the six linear regressions for the S&P500 index. The predictive power of the diversification measure is statistically significant for both ENC and ENB measures as we obtain a p-value indistinguishable from 0 (at two decimal points) for every recording period chosen for the measure of subsequent performance.

These results suggest that there is a positive relationship between the level of diversification as measured via the ENC or ENB indicator and the subsequent performance of the S&P 500 index whatever its period of analysis. It should be noted, however, that the coefficients of proportionality remain relatively low (between 0.21 and 0.42).

In addition, we find for both diversification measures that the R-squared and the t-stats of the linear regressions increase with the length of the period of annualised performance computation, which shows that the diversification measures have better forecasting power over long horizons. Lastly, if we only focus on the quarterly and the semi-annual performance computation, we notice that t-statistics are higher for the ENB compared to the ENC, suggesting a stronger relationship between diversification and subsequent performance for the former measure compared to the latter. This result confirms that the entropy of the distribution of risk contributions to the portfolio from uncorrelated factors is a
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Improved Risk Reporting with Factor-Based Diversification Measures — February 2014

more meaningful measure of diversification compared to the entropy of the distribution of the dollar contributions to the portfolio from correlated assets.

After having approached the problem from a time-series perspective, we then test the link between diversification measures and the performance of equity indices from a cross-sectional perspective. To do so, we conduct an analysis of the relationship between the performance of the 14 equity indices during the sub-prime crisis and their diversification measures computed at some point before the crisis started. The indices we consider in this analysis are the S&P 500 and 13 other popular equity indices, namely the CAC 40 index, the DAX 30 index, the Dow Jones 30 index, the Euro Stoxx 50 index, the Euro Stoxx 300 index, the FTSE 100 index, the FTSE All World index, the Hang Seng index, the Nasdaq 100 index, the SPI index, the Stoxx Europe 200 index, the Stoxx Europe 600 index and the Topix 100 index. For each of these indices, we compute the ENC measure and the ENB measure on the longest available time period.

We seek to perform the analysis on a period of particularly severe market correction in order to test whether the indices that were the best diversified in terms of the effective number of uncorrelated risk contributions (that is to say with the highest ENB) at some date prior to the start of the crisis tend to perform the best during the subsequent bear market period. Our choice for a sample period including a recent severe bear market is the period ranging from the beginning of September 2008 until the end of February 2009 – a period starting when the US subprime crisis propagated to the banking sector and turned into a global financial crisis, and finishing when a first relief was obtained as the United States and other developed countries issued their first economic rescue plans.

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Executive Summary

Table 1: Time-Series Analysis of the Relationship Performances/Diversification for the S&P500

These two tables display the diagnostics of the linear regression between the annualised performance of the S&P500 computed on different periods and its ENC and ENB. Each diversification measure is computed weekly over the whole historical data period of the S&P500. Annualised performances of the S&P500 are calculated at a weekly frequency on each quarter, each semester, each year, each 2-year period, each 5-year period and each 10-year period immediately following the dates of computation of each diversification measure.

<table>
<thead>
<tr>
<th>(a) ENC</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>∆t Following</td>
<td>Quarter</td>
<td>Following</td>
<td>Semester</td>
<td>Following</td>
<td>Year</td>
<td>Following</td>
</tr>
<tr>
<td>Coefficients</td>
<td>0.22</td>
<td>0.22</td>
<td>0.28</td>
<td>0.33</td>
<td>0.28</td>
<td>0.27</td>
</tr>
<tr>
<td>R-Squared</td>
<td>0.36%</td>
<td>0.70%</td>
<td>2.37%</td>
<td>6.88%</td>
<td>12.52%</td>
<td>22.13%</td>
</tr>
<tr>
<td>t-stat</td>
<td>3.20</td>
<td>4.47</td>
<td>8.28</td>
<td>14.30</td>
<td>19.32</td>
<td>25.84</td>
</tr>
<tr>
<td>p-value</td>
<td>0.14%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(b) ENB</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>∆t Following</td>
<td>Quarter</td>
<td>Following</td>
<td>Semester</td>
<td>Following</td>
<td>Year</td>
<td>Following</td>
</tr>
<tr>
<td>Coefficients</td>
<td>0.39</td>
<td>0.30</td>
<td>0.21</td>
<td>0.25</td>
<td>0.33</td>
<td>0.42</td>
</tr>
<tr>
<td>R-Squared</td>
<td>0.86%</td>
<td>0.96%</td>
<td>0.99%</td>
<td>2.86%</td>
<td>11.02%</td>
<td>32.00%</td>
</tr>
<tr>
<td>t-stat</td>
<td>4.98</td>
<td>5.25</td>
<td>5.31</td>
<td>9.02</td>
<td>17.98</td>
<td>33.25</td>
</tr>
<tr>
<td>p-value</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
</tbody>
</table>
We compare the annualised performances of the selected equity indices over the period starting at the beginning of September 2008 and ending at the end of February 2009, and their average diversification measures computed across six different periods. The average diversification measures are computed on periods immediately preceding the calculation of the index performance. In Figure 1, we plot the annualised performances of the 14 equity indices between September 2008 and February 2009 with respect to each diversification measure computed at the date immediately preceding the bear market period at the end of August 2008. We perform linear regressions in order to test the robustness of the relationship between performance and diversification.

Figure 1: Performances of 14 Equity Indices with respect to their Diversification Measures during the Subprime Crisis

These figures display the annualised performances of the 14 equity indices during the worst of the subprime crisis (between the beginning of September 2008 and the end of February 2009) with respect to their respective effective number of constituents (ENC) and their effective number of uncorrelated bets (ENB) computed at the end of August 2008. These figures display the outlier (the FTSE 100), but the slope of the linear regression is computed without this outlier.

4 - These periods being at the end of August 2008, during August 2008, during the quarter preceding September 2008, during the semester preceding September 2008, during the year starting on September 2007 and ending at the end of August 2008 and during the two-year period starting on September 2006 and ending at the end of August 2008.
measures. For each diversification measure (ENC and ENB), we thus obtain six sets of statistics, corresponding to the six periods of calculation of the average diversification measures. Since the FTSE 100 index appeared as a clear outlier, we compute these linear regressions without this outlier. Hence, the straight line drawn on Figure 1 corresponds to the coefficients calculated from the linear regressions without the outlier. We analyse in more detail the results for diversification measures computed at the end of August 2008; however, the results obtained on the other periods of computation of the diversification measures follow the same trend.

The statistical analysis shows a clear positive linear relationship between the performance of the index on the period starting on September 2008 and ending at the end of February 2009, and each one of the two diversification measures computed at the end of August 2008. We observe that the positive relationship is statistically more significant for the ENB measure than for the ENC measure. Indeed, the regression based on ENB measures has a 92% confidence level and a 24.7% R-squared compared to the regression based on ENC measures, which only have an 84% confidence level and a 17.12% R-squared. In addition, the slope for the performance-to-ENB relationship is steeper (almost twice as steep) than the slope for the performance-to-ENC relationship. Overall, our results suggest that the higher the ENB of an index prior to the worst of the crisis, the more likely it was to perform better during September 2008-February 2009 compared to an index that had a lower ENB at the same date. This is again consistent with the interpretation of the ENB as a meaningful diversification measure. Therefore, we conclude from this cross-sectional analysis that in a period of severe bear markets, equity indices that were the best diversified in terms of uncorrelated sources of risks (i.e. high ENB) prior to the period of market downturn exhibited better resistance than equity indices that enjoyed a lower degree of diversification.

Diversification Measures for US Pension Funds
We use the P&I Top 1,000 database to obtain information on the asset allocation of each of the 1,000 largest US pension funds as of 30 September 2002, 30 September 2007 and 30 September 2012. We exclusively focus on the portion allocated to their defined-benefit plan; if they also have a defined-contribution plan, we do not analyse the amount they allocate to this plan. In order to represent the different asset classes pension fund assets are invested in, we consider the following (arguably arbitrary) partition of the asset allocation: domestic fixed income; international fixed income; High-yield bond; inflation-linked bond; domestic equity; international equity; global equity; private equity; real-estate; commodity; mortgage; and cash. Once the partition is completed, we choose appropriate benchmarks for each asset class and use the MLT approach (Meucci et al. (2013)) to turn correlated asset class returns into uncorrelated factor returns.

We estimate the ENB diversification measure for each pension fund in the database as of 30 September 2002, 30 September 2007 and 30 September 2012. We also compute the ENC, defined as the entropy of the asset class exposure, as of the same dates. This definition, which is maximised
for the equally-weighted portfolio, is a naive diversification measure that does not account for the presence of differences in risk and correlation levels within the set of asset classes. As recalled above, this is in contrast with the ENB measure, which is based on normalised uncorrelated factors. On the other hand, the ENB measure is an instantaneous observable quantity, while the ENC measure requires an estimate for the covariance matrix of asset returns so as to apply the minimum torsion methodology. In order to estimate the covariance matrix needed to compute the ENB measure, we use five years of historical weekly returns before the date at which we perform the computation.

In Figure 2, we display the distributions of the ENC and ENB measures. When looking at the evolution of each diversification measure, it seems that a change occurred between 2007 and 2012, as most US pension funds seem to have increased the diversification level in their portfolio between these two dates. For instance, between 2002 and 2007, the mean of the distribution of the ENCs increases by 1.3%, while between 2007

Figure 2: Distribution of Diversification Measures of US Pension Funds
These figures display the distribution of the effective number of constituents (ENC) and the effective number of bets (ENB) for the US pension funds of the P&I database in 2002, 2007 and 2012
and 2012, it increases by 40.7%. Therefore, it seems that US pension funds dedicated some effort between 2007 and 2012 to improving their level of diversification. However, we note that while US pension funds increased their ENC by 40.7% in five years, they only increased their ENB by 14.4% over the same time period.

We then analyse whether the diversification measures computed over these pension funds at the end of September 2007 can give insights on the returns of US pension funds performance in subsequent months. In our test, we compute the fund returns over two different periods: over the year directly following the date of computation of the diversification measures (from 28/09/2007 to 26/09/2008), and over the worst period of the subprime crisis for the financial sector (from 05/09/2008 to 27/02/2009). For each diversification measure, we first plot the relationship between the US pension funds' annualised performances at date t+n months according to their level of diversification measure at date t (end of September 2007). Then, we statistically test the degree of significance of our results. We replicate this test for each diversification measure and for the two periods of time considered, and report the results in Figure 3.

It is first striking to see that the relationship between US pension fund performances and their level of ENB is positive, and this relationship is statistically significant.

This result holds true for the two periods of performance computation. Overall, these results mean that, at the end of September 2007, a pension fund that had a higher ENB (hence holding a better diversified portfolio) was more likely to reach higher performances (lower loss levels) during 28/09/2007-26/09/2008 and during 05/09/2008-27/02/2009 than a pension fund that had a lower ENB, assuming the policy portfolio weights remaining constant. On the other hand, higher levels of ENC for a pension fund at the end of September are likely to have no impact, if not negative effects, on its performances during 28/09/2007-26/09/2008 and during 05/09/2008-27/02/2009 than a pension fund that had a lower ENB, assuming the policy portfolio weights remaining constant. The opportunity cost of this exceedingly cautious strategy is of course prohibitive in terms of renouncement to the access of the risk premia on risky asset classes that is allowed by a well-diversified portfolio.

Executive Summary

5 - We actually do not use pension fund actual performance in our analysis and assume instead that the fund asset allocation remains constant over the months following the computation of the diversification measures at date t. We use this methodology for two main reasons. First, we have information about pension fund allocation only at the end of calendar years or at the end of fiscal years (end of June). Secondly, this approach allows us to preserve a stronger link between diversification measures computed at a date t and pension funds' performances at t+n months.

6 - It should be noted that not all pension fund managers seek to hold a well-diversified portfolio. In particular, the liability-driven investing paradigm implies that pension fund managers interested in minimising the volatility of their funding ratio would hold a concentrated fixed-income portfolio with interest rate exposures similar to the interest rate risk exposures in the pension liabilities. Intuitively, expect such an extremely safe strategy to offer by construction good downside protection in bear equity markets, and we did find that, in spite of the positive relationship between ENB and performance in 2008, the very top performers were the pension funds holding only sovereign bonds. The opportunity cost of this exceedingly cautious strategy is of course prohibitive in terms of renouncement to the access of the risk premia on risky asset classes that is allowed by a well-diversified portfolio.
Executive Summary

Figure 3: Performances of US Pension Funds with respect to their Diversification Measures at the End of September 2007

These figures display the annualised performances of the US pension funds of the P&I database computed on two different periods with respect to their diversification measures at the end of September 2007. The annualised performances are calculated on the year immediately following the date of computation of the diversification measures (from 28/09/2007 to 26/09/2008) and during the worse of the subprime crisis (from 05/09/2008 to 27/02/2009). We consider that pension funds’ asset allocation has not changed since the end of September 2007, therefore, the performances displayed here are only estimates.
The Effective Number of Bets as a Useful New Risk Indicator

Overall our analysis suggests that a better assessment of the degree of diversification of a portfolio in terms of its effective number of bets (ENB) would provide useful insights regarding the risk and return profile of the portfolio in various market conditions. The ENB measure appears to be a useful risk indicator not only across, but also within asset classes. In particular, we find statistical evidence of a positive time-series and cross-sectional relationship between this diversification measure and portfolio performance in bear markets. As such, it appears that the ENB measure could be a useful addition to the list of risk indicators for equity and policy portfolios, in addition to standard measures such as Value-at-Risk for example.
Executive Summary
1. Introduction
1. Introduction

Risk reporting is increasingly regarded by sophisticated investors as an important ingredient in their decision making process. The most commonly used risk measures such as volatility (a measure of average risk), Value-at-Risk (a measure of extreme risk) or tracking error (a measure of relative risk), however, are typically backward-looking risk measures computed over one historical scenario. As a result, they provide very little information, if any, regarding the possible causes of the portfolio riskiness and the probability of a severe outcome in the future, and their usefulness in a decision making context remains limited. For example, an extremely risky portfolio such as a leveraged long position in far out-of-the-money put options may well appear extremely safe in terms of the historical values of these risk measures, that is until a severe market correction takes place (Goetzmann et al. (2005)). In this context, it is of critical importance for investors and asset managers to be able to rely on more forward-looking estimates of loss potential for their portfolios. The main focus of this paper is on analysing meaningful measures of how well, or poorly diversified, a portfolio is, exploring the implication in terms of advanced risk reporting techniques, and assessing whether a relationship exists between a suitable measure of the degree of diversification of a portfolio and its performance in various market conditions.

While the benefits of diversification are intuitively clear, the proverbial recommendation of “spreading eggs across many different baskets” is relatively vague, and what exactly a well-diversified portfolio is remains somewhat ambiguous in the absence of a formal quantitative framework for analysing such questions. Fortunately, recent advances in financial engineering have paved the way for a better understanding of the true meaning of diversification. In particular, academic research (e.g., Ang et al. (2009)) has highlighted that risk and allocation decisions could be best expressed in terms of rewarded risk factors, as opposed to standard asset class decompositions, which can be somewhat arbitrary. For example, convertible bond returns are subject to equity risk, volatility risk, interest rate risk and credit risk. As a consequence, analysing the optimal allocation to such hybrid securities as part of a broad bond portfolio is not likely to lead to particularly useful insights. Conversely, a seemingly well-diversified allocation to many asset classes that essentially load on the same risk factor (e.g., equity risk) can eventually generate a portfolio with very concentrated risk exposure. More generally, given that security and asset class returns can be explained by their exposure to pervasive systematic risk factors, looking through the asset class decomposition level to focus on the underlying factor decomposition level appears to be a perfectly legitimate approach, which is also supported by standard asset pricing models such as the intertemporal CAPM (Merton (1973)) or the arbitrage pricing theory (Ross (1976)). Two main benefits can be expected from shifting to a representation expressed in terms of risk factors, as opposed to asset classes. On the one hand, allocating to risk factors may provide a cheaper, as well as more liquid and transparent, access to underlying sources of returns in markets where the value added by existing active
Improved Risk Reporting with Factor-Based Diversification Measures — February 2014

1. Introduction

In this paper, we first review a number of weight-based measures of (naive) diversification as well as risk-based measures of (scientific) diversification that have been introduced in the academic and practitioner literatures, and analyse the shortcomings associated with these measures. We then argue that the effective number of (uncorrelated) bets (ENB), formally defined in Meucci (2009a) as the dispersion of the factor exposure distribution, provides a more meaningful assessment of how well-balanced is an investor’s dollar (egg) allocation to various baskets (factors). We also provide an empirical illustration of the usefulness of this measure for intra-class and inter-class diversification. For intra-class diversification, we cast the empirical analysis in the context of various popular equity indices, with a particular emphasis on the S&P500 index. For inter-class diversification, we analyse policy portfolios for two sets of pension funds, the first set being a large sample of the world’s 10 largest pension funds. In a first application to international equity indices, we use the minimal linear torsion approach (Meucci et al. (2013)) to turn correlated constituents into uncorrelated factors, and find statistical evidence of a positive (negative) time-series and cross-sectional relationship between the ENB risk diversification measure and performance in bear (bull) markets. We find a weaker relationship when using other diversification measures such as the effective number of constituents (ENC), thus confirming the relevance of the effective number of bets on uncorrelated risk factors as a meaningful measure of diversification. Finally, we find the predictive power of the effective number of bets diversification measure for equity market performance to be statistically and economically significant, comparable to predictive power of the dividend yield for example (Cochrane (1997)), with an explanatory power that increases with the holding period. In a second application to US pension fund policy portfolios, we find that better diversified policy portfolios in the sense of a higher number of uncorrelated bets tend to perform better on average in bear markets, even though top performers are, as expected, policy portfolios highly concentrated in the best performing asset class for the sample period under consideration. Overall, our results suggest that the effective number of (uncorrelated) bets could be a useful risk indicator to be added to risk reports for equity and policy portfolios.

The rest of the paper is organised as follows. In Section 2, we review various measures of portfolio diversification, and argue in favour of risk-
1. Introduction

factor-based measures. We conduct an empirical analysis of these measures for international equity index indices in Section 3, and consider an application to pension fund policy portfolios in Section 4. Section 5 concludes. Technical details are relegated to a dedicated Appendix.
2. Portfolio Diversification Measures
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In this section, we present a comparative analysis of various measures of portfolio diversification/concentration, with a discussion of their respective merits and shortcomings.

2.1 Weight-Based Measures of Portfolio Diversification

A key distinction exists between weight-based measures of portfolio concentration, which are based on the analysis of the portfolio weight distribution independently of the risk characteristics of the constituents of the portfolio, and risk-based measures of portfolio concentration, which incorporate information about the correlation and volatility structure of the return on the portfolio constituents. In a nutshell, weight-based measures can be regarded as measures of naive diversification, while risk-based measures can be regarded as measures of scientific diversification.

Most weight-based measures of portfolio concentration provide a quantitative estimate of the effective number of constituents (ENC) in a portfolio, in an attempt to alleviate the problems related to the use of the nominal number of constituents in a portfolio which can be very misleading, in particular in case of a very ill-balanced allocation of the portfolio to the various constituents (e.g., one security makes up for 99% of the portfolio while other securities make up collectively for the remaining 1%). We denote with \( w \) the weight vector representing the percentage invested in each asset of a given portfolio, and we define the following class of diversification/concentration measures:

\[
\text{ENC}_\alpha(w) = \|w\|^{\frac{1}{1-\alpha}} = \left( \sum_{k=1}^{N} w_k^\alpha \right)^{\frac{1}{1-\alpha}},
\]

\( \alpha \geq 0, \alpha \neq 1. \)  \hspace{1cm} (2.1)

Taking \( \alpha = 2 \) leads to a diversification measure defined as the inverse of the Herfindahl Index, which is itself a well-known measure of portfolio concentration, or \( \text{ENC}_2(w) = \frac{1}{\sum_{k=1}^{N} w_k^2} \).

Portfolio diversification (respectively, concentration) is increasing (respectively, decreasing) in the ENC measure. Note that this measure is directly proportional to the inverse of the variance of the portfolio weights, as can be seen from the following lemma.

**Lemma 1** If a portfolio contains \( N \) constituents, then the \( \text{ENC}_2 \) measure of the portfolio can be expressed in terms of the variance of the weight distribution as:

\[
\text{ENC}_2(w) = \frac{1}{N \text{Var}(w) + \frac{1}{N}}. \hspace{1cm} (2.2)
\]

**Proof.** See Appendix B.1.

It can be shown that when \( \alpha \) converges to 1, then \( \text{ENC}_\alpha \) converges to the entropy of the distribution of the portfolio weights:

\[
\text{ENC}_1(w) = \exp \left( -\sum_{k=1}^{N} w_k \ln(w_k) \right) \hspace{1cm} (2.3)
\]

It is straightforward to check that, for positive weights, \( \text{ENC}_\alpha \) reaches a minimum equal to 1 if the portfolio is fully concentrated in a single constituent, and a maximum equal to \( N \), the nominal number of constituents, achieved for the equally-weighted portfolio. These properties justify using this family of measures to compute the effective number of constituents.
of constituents in a portfolio. However, the question of which $\alpha$ to use remains. When dealing with longshort portfolios, it seems natural to use the ENC$_2$ measure since ENC$_1$ is not defined for negative weights due to the logarithmic function. The presence of negative weights, and the resulting leverage, penalises the concentration measure as we can see on the following simple example. Consider a portfolio with $w_1 = -\frac{1}{2}$, and $w_2 = w_3 = w_4 = \frac{1}{2}$. This leads to $\text{ENC}_2(\mathbf{w}) = 1$ which is the highest degree of concentration achieved by a long-only portfolio (corresponding to $w_1 = 1$, and $w_2 = w_3 = w_4 = 0$). However, if we increase the size of the short position, and consider the portfolio given by: $w_1 = -1$, and $w_2 = w_3 = w_4 = 0$. This shows that ENC$_2$ penalises short positions, and values between 0 and 1 can be achieved, but only when portfolio contain large short positions. However, when we consider long-only portfolios, both the ENC$_1$ and ENC$_2$ measures are well-defined and can be therefore be used. Note that since we will only deal with long-only portfolios in the following, our empirical analysis will be done using ENC$_1$ as a measure of the effective number of constituents. A robustness check will be presented in the Appendix to show that our main results remain qualitatively valid when ENC$_2$ is used instead.

In spite of their intuitive appeal, these weight-based measures suffer from a number of major shortcomings. In particular, ENC measures can be deceiving when applied to assets with non homogenous risks. Consider for example a position invested for 50% in a 1% volatility bond, and the other 50% in a 30% volatility stock, and assume for simplicity that the stock and bond returns are uncorrelated. The weights are perfectly distributed, but the risk is highly concentrated. This is due to differences in the total variance of each constituent, with $(50\%)^2 \times (30\%)^2$ being much larger than $(50\%)^2 \times (1\%)^2$, thus implying that the equity allocation has a much larger contribution to portfolio risk compared to the bond allocation. On the other hand, ENC measures can be deceiving when applied to assets with correlated risks. For instance, consider a portfolio with equal weights invested in two bonds with similar duration and volatility. Despite the fact that both dollar contributions and risk contributions are homogeneously distributed within the portfolio, risk is still very concentrated because of the high correlation between the two bonds. In other words, the main shortcoming of the ENC measure as a measure of portfolio diversification comes from the fact that it does not use information about differences in volatility and pairs of correlations across assets.

### 2.2 Risk-Based Measures of Portfolio Diversification

To account for information in the covariance matrix, a number of risk-based measures of diversification have also been introduced by various authors. Before introducing them, we want to stress the fact that a low number of observations with a large number of constituents in the portfolios may lead to non-robust sample covariance estimates. Hence, we first robustify the sample covariance matrix $\Sigma_{\text{mp}}$ by identifying implicit factors using principal component analysis (PCA), and proceeding as follows:

1. First, we compute the sample correlation matrix...
2. Portfolio Diversification Measures

matrix, \( \Omega_{\text{amp}} \), and the diagonal matrix of constituents’ volatilities, \( D \);
2. Then, we diagonalise the correlation matrix as \( \Omega_{\text{amp}} = R E^2_{\text{amp}} R' \).  
   \( E^2_{\text{amp}} \) is the diagonal matrix of eigenvalues sorted by decreasing order and \( R \) is a matrix of normalised eigenvectors;
3. Then, we identify \( k \) systematic factors, which correspond to the largest \( k \) eigenvalues of \( \Omega_{\text{amp}} \). The remaining eigenvalues are set to 0, leading to a new diagonal matrix of eigenvalues \( E^2 \);
4. The resulting robustified correlation \( \Omega \) matrix is obtained by computing \( R E^2 R' \), and replacing the diagonal elements with 1 (otherwise it may not be a true correlation matrix because its diagonal elements may be different from 1);
5. Finally, the robustified covariance matrix is equal to: \( \Sigma = D \Omega D \).

It is therefore the robustified covariance matrix \( \Sigma \) that is used to derive risk-based measures of portfolio diversification. In order to take into account the covariance matrix, Goetzmann et al. (2005) use the ratio of the variance of the portfolio to the weighted average variance of the portfolio constituents:

\[
\sigma_P^2 = w' \Sigma w = \sum_{k=1}^{N} w_k [\Sigma w]_k
\]

\[
= \sum_{k=1}^{N} w_k \left[ w_k \sigma_k^2 + \sigma_k \sum_{j=1, j \neq k}^{N} \rho_{kj} w_j \sigma_j \right]
\]

where \( [X]_k \) denotes the \( k \)th element of vector \( X \). This leads to the following scaled contributions:

\[
q_k = \frac{w_k [\Sigma w]_k}{w' \Sigma w}, \text{ where } \sum_{k=1}^{N} q_k = 1
\]

Note the portfolios such that the contribution \( q_k \) of each constituent to the variance are all equal is named risk parity portfolio (see Roncalli (2013) for conditions of existence and unicity of the risk parity portfolio). To account for the presence of cross-sectional dispersion in the correlation matrix, one can apply the naive measure of concentration ENC introduced above to the contributions to portfolio risk. This allows us to define the effective number of correlated bets in a portfolio as the dispersion of the variance contributions of its constituents:

### 2.2.1 Measures of the Effective Number of Correlated Bets (ENCB)

To try and identify a meaningful measure of the number of bets (baskets) to which investors’ dollars (eggs) are allocated, one can first define the contribution of each constituent to the overall variance of the portfolio \( \sigma_P^2 \) as Roncalli (2013)

\[
\sigma_P^2 = w' \Sigma w = \sum_{k=1}^{N} w_k [\Sigma w]_k
\]

\[
= \sum_{k=1}^{N} w_k \left[ w_k \sigma_k^2 + \sigma_k \sum_{j=1, j \neq k}^{N} \rho_{kj} w_j \sigma_j \right]
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This family of measures, indexed with the free parameter \( \alpha \), takes into account not only the number of available assets but also the correlation properties between them. More specifically, a constituent that is highly positively correlated with all the other constituents will tend to have a higher contribution to the variance (considering long-only portfolio for simplicity), leading to a lower effective number of correlated bets since most of the portfolio risk is concentrated in that constituent. However, the measure of the contribution of each constituent to the variance is somehow arbitrary in how the overlapping correlated terms are affected to the various constituents. Indeed, the approach consists in adding to the variance of the position in constituent \( k \), equal to \( w_k^2 \sigma_k^2 \), which is unambiguously related to constituent \( k \), all covariance terms \( w_k \sigma_k \sum_{j=1,j \neq k}^{N} p_{kj} w_j \sigma_j \) emanating from the overlap between constituent \( k \) and the other constituents, and which could also be affected to the other constituents. More generally, having dollars (eggs) evenly spread across bets (baskets) is not necessarily a sufficient condition for proper diversification if the bets are correlated (baskets are tied together). For these reasons, we now turn to the analysis of uncorrelated bets.

### 2.2.2 Measures of the Effective Number of Uncorrelated Bets (ENB)

To alleviate the concern over an arbitrary affectation of the correlated components to the various portfolio constituents, Meucci (2009a) proposes to decompose the portfolio return \( r_P \) as the sum of \( N \) uncorrelated factors \( F_1, ..., F_N \) (see also Deguest et al. (2013) or Meucci et al. (2013)). We thus have:

\[
r_P = w' r = w' F r_F,
\]

where \( r \) denotes the vector of returns of the original constituents, and \( r_F \) the vector of uncorrelated factors’ returns. The main challenge with this approach is to turn correlated asset returns into uncorrelated factor returns. The factor returns can typically be expressed as a linear transformation of the original returns: \( r_F = A' r \) for some well chosen transformation \( A \) guaranteeing that the covariance matrix of the factors \( \Sigma_F = A \Sigma A' \) is a diagonal matrix. \( A \) is an \( N \times N \) transition matrix from the assets to the factors and it is therefore critical that it be invertible since \( w_F = A^{-1} w \).

Then, we define the contribution of each factor to the overall variance of the portfolio \( \sigma_P^2 \) as:

\[
\sigma_P^2 = w' \Sigma w = w_F' \Sigma_F w_F = \sum_{k=1}^{N} w_{F_k}^2 \sigma_{F_k}^2.
\]

which leads to the following percentage contributions for each factor:

\[
p_k = \frac{w_{F_k}^2 \sigma_{F_k}^2}{w' \Sigma w} \quad \text{where} \quad \sum_{k=1}^{N} p_k = 1, \quad p_k \geq 0.
\]

The portfolios named factor risk parity portfolios in Deguest et al. (2013) are built such that the contribution \( p_k \) of each factor to the variance are all equal.\(^6\)

Mimicking the definition of ENC and ENCB, we now introduce the effective number of “uncorrelated” bets of portfolio \( w \), or ENB in short, as:

\[
\text{ENB}_\alpha (w, A) = ||p||^{\frac{1}{1-\alpha}} = \left( \sum_{k=1}^{N} p_k^\alpha \right)^{\frac{1}{1-\alpha}}
\]
Improved Risk Reporting with Factor-Based Diversification Measures — February 2014

2. Portfolio Diversification Measures

\[ \alpha \geq 0, \alpha \neq 1. \] (2.5)

Note that Meucci (2009a) defines the effective number of uncorrelated bets using the entropy metric for the dispersion of the factor contributions:

\[ \text{ENB}_1(\mathbf{w}, \mathbf{A}) = \exp \left( - \sum_{k=1}^{N} p_k \ln(p_k) \right) \] (2.6)

In order to remain coherent with the choice made for the ENC measure, we will follow Meucci (2009a) and use the entropy metric denoted as \( \text{ENB}_1 \). There are many ways to decompose the portfolio into \( N \) contributions of uncorrelated factors, i.e., to compute a valid matrix \( \mathbf{A} \). In the following, we will explore two competing approaches to extract \( N \) implicit uncorrelated factors from the constituents. We also consider an approach based on an explicit factor model obtained by orthogonalisation of the Fama–French market, value and size factors (Fama and French (1992)) together with the momentum factor (Carhart (1997)).

**Extracting Implicit Factors via Principal Component Analysis (PCA)**

Principal co-moment analysis is a standard procedure for extracting uncorrelated factors from a basket of correlated constituents (see for instance Meucci (2009a), Frahm and Wiechers (2011), Lohre et al. (2011), and Deguest et al. (2013)). Formally, it is based on the diagonalisation of the covariance matrix \( \Sigma \) of asset returns, \( \Sigma = \mathbf{P} \Lambda \mathbf{P}' \), with \( \Lambda^2 \) being the diagonal matrix of eigenvalues \( \lambda_1^2, \ldots, \lambda_N^2 \) of \( \Sigma \) and \( \mathbf{P} \) the matrix of eigenvectors. Each eigenvector can be interpreted as the vector of weights for a "principal factor portfolio", while each eigenvalue represents the variance of a factor. In that case, we simply set \( \mathbf{A} = \mathbf{P} \).

Hence, an allocation to the \( N \) constituents can be regarded as an allocation to the principal factor portfolios, with weights \( \mathbf{w}_f = \mathbf{A}^{-1} \mathbf{w} = \mathbf{P}' \mathbf{w} \) and covariance matrix \( \Sigma_f = \Lambda^2 \). This leads to the following variance contributions:

\[ p_k = \frac{[\mathbf{P}' \mathbf{w}]_k^2 \lambda_k^2}{\mathbf{w}' \Sigma \mathbf{w}} \] (2.7)

There are a number of shortcomings, however, with the PCA approach. The first shortcoming is the difficulty in interpreting the factors, which are pure statistical artifacts. The second shortcoming, particularly severe in the context of the design of a diversification measure, is that by construction principal components are defined so as to achieve the highest possible explanatory power. As a result, the contribution of the first few factors is often overwhelmingly large with respect to the contribution of other factors, and the portfolio diversification measure empirically tends to be biased towards low values (see Section 3). This approach may lead to counter-intuitive results. In particular, Meucci et al. (2013) show that the ENB measure is equal to 1 for an equally-weighted portfolio based on a universe of assets with equal volatility and pairwise correlation values, regardless of the correlation value, while the intuition would suggest that for a vanishing constant correlation value such a portfolio should have a number of bets equal to the number of assets. This counter-intuitive full-concentration effect follows because the equally-weighted portfolio is in this case fully exposed to the first principal component and not exposed to any other principal component (for more details, see Appendix A2 in Meucci et al. (2013)).
Extracting Implicit Factors via Minimal Linear Torsion (MLT) A competing approach to extracting uncorrelated factors from a basket of correlated constituents has been introduced in Meucci et al. (2013), who propose to look for the minimal uncorrelated linear transformation $A$ in the sense that:
• the factors are as close as possible to the original constituents, i.e., $\sum_{k=1}^{N} \text{Var}(r_k - r_{F_k})$ is minimal;
• the factors have the same variances as the original constituents, i.e., $\sigma_{F_k} = \sigma_k$ for all $k = 1, \ldots, N$.

By construction, the obtained factors are the closest uncorrelated representations of the original constituents. When the constituents are highly correlated, the factors can prove to be substantially distorted transformations of the original constituents, but at least they enjoy the desirable property of emanating from the least distorted transformation. We show in Appendix B.2 that the formal solution to this problem is to take $A = PA^{-1}UV'D$, where $D$ is the diagonal matrix containing the original constituents’ volatilities, $P$ and $\Lambda$ the outputs of the PCA run on $\Sigma$, and $U$ and $V$ the matrices obtained from the singular value decomposition of $\Lambda P'U' = USV$ (where $S$ is the diagonal matrix containing the singular values of $\Lambda P'U'$).

One can easily check that starting with a set of $N$ assets with the same volatility and a zero value for all pairwise correlations, the minimal linear torsion bets will be equal to the original assets (in other words, the minimal linear transformation is a no-transformation in this case since the assets already were uncorrelated). In this case, the ENB will be equal to $N$, as the intuition suggests, as opposed to 1, which would be the misleading value obtained with the PCA approach.

Using Explicit Factors – The Case of Fama-French-Carhart Four Factor Model In this case, we propose to consider explicit factors in order to perform our factor-based portfolio diversification analysis. In the following, we consider Fama-French-Carhart four factor model (FF in brief) and give a new framework to compute the variance contributions. We denote by $r_{F}$ the vector of the four-factor returns, and with $r_{F\perp}$, the vector of the orthogonalised four-factor returns. First, we build the orthogonalised factors from the minimal linear transformation $A_F$ such that:
• the orthogonalised factors are as close as possible to the original factors, i.e. $\sum_{k=1}^{N} \text{Var}(r_{F_k} - r_{F\perp_k})$ is minimal;
• the orthogonalised factors have the same variances as the original factors, i.e. $\sigma_{F\perp_k} = \sigma_{F_k}$ for all $k = 1, \ldots, 4$.

Once we have the orthogonalised version of the four factors, we compute the exposure of any portfolio $w$ to each of the factors as:

$$r_{F} = w' \Lambda \sigma^{2}_{F} + \epsilon, \quad (2.9)$$

where $\epsilon$ is a vector of residual that is uncorrelated with the orthogonalised four factors. Finally, since the factors are uncorrelated by construction, we can write the portfolio variance as:

$$\sigma_{F}^{2} = w' \Sigma w = \sum_{k=1}^{4} w_{F\perp_k}^{2} a_{F\perp_k}^{2} + \sigma_{\epsilon}^{2},$$
2. Portfolio Diversification Measures

which leads to the following percentage variance contributions:

$$p_k = \frac{w_{FF}^2 F_{\text{orth}}^k \sigma_{FF}^2}{\sigma_p^2 - \sigma^2},$$

where \(\sum_{k=1}^{4} p_k = 1, \quad p_k \geq 0.\)

Note that Equation (2.9) and the properties of \(\epsilon\) given above show that vector \(w_{FF}^\perp\) simply contains the betas of the linear regression of \(r_P\) with respect to the orthogonalised four factors. Since the factors are uncorrelated by construction, this leads to:

$$p_k = \frac{w_{FF}^2 F_{\text{orth}}^k \sigma_{FF}^2}{\sigma_p^2 - \sigma^2},$$

for all \(k = 1, ..., 4,\)

where \(\rho_{FF}^k, r_P\) denotes the correlation between the portfolio of the \(k^{th}\) orthogonalised factor. Then, replacing the values obtained for \(w_{FF}^\perp_k\) in the variance contributions leads to the following simplification:

$$p_k = \frac{\sigma_p^2}{\sigma_p^2 - \sigma^2} \rho_{FF}^k, r_P \propto \rho_{FF}^k, r_P^\perp. \quad (2.10)$$

In the next section, we analyse how these factor-based measures of portfolio concentration can be used within the context of improved reporting techniques that allow for a better measurement of portfolio risk exposures.
3. Empirical Analysis for Equity Indices
In this section, we perform an empirical illustration of the benefits of using the ENB measure of portfolio diversification in the context of an improved risk reporting methodology. We compare the ENB measure to the more naive ENC measure and present an application to the measure of diversification for 14 seemingly well-diversified equity portfolios, with a more detailed analysis of the S&P500. We first present some descriptive statistics for 14 equity indices, and then analyse the link between diversification measures and equity index performance. For these empirical studies, we consider a set of popular indices representing each particular universe, and we estimate their diversification measures. The list of equity universes that we consider is given as follows:

**US universe:**
- Large cap stock index: S&P 500
- Technology stock index: NASDAQ 100
- Industrial stock index: DOW JONES 30

**Global European universe:**
- Large cap stock index: STOXX Europe 200
- Broader stock index: STOXX Europe 600

**Global Eurozone universe:**
- Very large cap stock index: EURO STOXX 50
- Broader stock index: EURO STOXX 300

**European countries:**
- UK stock index: FTSE 100
- France stock index: CAC 40
- Germany stock index: DAX
- Switzerland stock index: SPI

**Asia:**
- Japan stock index: TOPIX 100
- Hong-Kong stock index: HANG SENG

We collect data for the indices listed above from Datastream, except for the S&P500 for which we collect the historical data from CRSP. For each equity index, we first extract the list of equity constituents that have at least been once in the sample period part of the constituent list of the index. Then, for each equity constituent, we upload its historical total return series (with reinvested dividend) at the weekly frequency, as well as historical market values, i.e., share prices multiplied by the number of ordinary shares in issue. Finally, we also extract the historical returns with reinvested dividend for each aggregated index. As a base case, we consider the S&P500 index, and collect return data at a weekly frequency on the period starting on 4 January 1957 until 31 December 2012. We use a one-year period in order to estimate the sample covariance matrix of the index constituents, then we robustify our estimator using Section 2.2, and we roll over this one-year window without overlap to generate annual estimates for the diversification measures. We use the same methodology for the 13 other indices for which we estimate the covariance matrices on the maximum historical data available. Eventually, we calculated the ENC and the ENB using the entropy measure, see Equations (2.3), and (2.6). In particular, we calculated the ENB using three methods: principal component analysis (PCA), minimum linear torsion (MLT) model and the Fama-French-Carhart four factor (FF) model and presented a detailed comparison of these three approaches in Section 2.2.
for equity indices and their diversification measures. We first focus the analysis on the S&P500 universe, which we use as a base case since a long sample of returns is available for this index before turning in the next section to results on cross-sectional tests among 14 regional equity indices. We study several diversification measures for the S&P500: the ENC and three versions of the ENB (PCA, MLT and FF), as explained before. First, we observe in Figure 1 that the ENC measure is about 2.5 times smaller than the nominal number of constituents, which shows a substantial level of concentration since the portfolio is on average effectively invested in roughly 200 stocks instead of 500. This finding is consistent with the results in Haugen and Baker (1991), who among others have highlighted the poor efficiency score of capitalisation-weighted indices. Intuitively, the mediocre risk-adjusted performance of cap-weighted portfolios may not be surprising, given that the weighting method automatically gives very high weights to some very large cap stocks and therefore leads to concentrated portfolios. Usually, the 10-30 largest stocks make up the majority of the weighting in the index. Put differently, even if an index has more than 500 components, 90% of the components make up an almost negligible part of the index weights.

Even though the ENC is a natural measure to estimate the level of concentration of a given portfolio, it does not provide insights into the level of diversification in terms of uncorrelated sources of risk. In order to do so, we plot on Figure 1 the ENB measure using three different approaches, PCA, MLT and FF. We observe that the ENB of the S&P500 index computed using a PCA approach has very small values (between 1 and 2). This means that the S&P500 is exposed to at most a couple of uncorrelated sources of risk. Remember that when the ENB is equal to one, it means that the index is exposed to a single risk factor, which is the market factor since it is commonly acknowledged that the first factor of the PCA is close to an equally weighted portfolio of the market constituents. As outlined in the previous discussion on the shortcomings of the PCA approach, the methodology is not well-suited for estimating the ENB since by construction it allocates to uncorrelated factors with decreasing powers of explanation. Consequently, the first risk factor is the one that has the highest explanatory power while the last factor has the smallest one. Therefore, the first factor found when computing a PCA should be overweighted when calculating ENB, which would explain why we have low ENB results. We test this assumption and compute the ENB using a PCA approach on the equally-weighted S&P500. We see on Figure 17 that from January 1959 to December 2012, the values of the ENB are approximately equal to 1 (up to a maximum of 2) meaning that a single factor essentially explains the whole exposure to risk of the S&P500 index when the index is equally-weighted. This supports our previous assumption that the first risk factor determined from the PCA approach explains almost all the S&P500 index risk exposure.

The ENB calculated using the four Fama-French orthogonal factors is also very low (between 1 and 1.5 during the whole period). Note that in this case, since we only consider four factors and not N factors, where N is the nominal number of constituents, the maximum value that the ENB can achieve is equal to 4. Observing...
ENB values around 1 or 2 means that, as for the PCA approach, only one factor explains almost all the exposure to risk of the S&P500. In our context, similarly to the PCA method, it is very likely that the dominant market factor that has the highest explanatory power, and in that sense, both the PCA and four Fama-French factor approaches generate downward-biased estimates of the effective number of independent bets.

In order to avoid overloading a single risk factor in the orthogonal risks decomposition, a more reliable approach consists in using an MLT of the original constituents in order to extract risk factors that are close to the original constituents, and compute ENB measures from these risk factors regarded as orthogonal versions of the original constituents. In the graph representing the evolution of the ENB of the S&P500 index using an MLT approach on Figure 1, we note that the ENB reaches more reasonable values ranging between 150 and 350. We complement the above analysis by displaying, in Table 1 the correlations between the different diversification measures computed on the period starting in December 1959 and ending in December 2012. We have argued before that the ENB computed using a principal component analysis and the ENB computed using a Fama-French factor model do not show satisfying results in terms of magnitude. We also find that the correlations obtained between the different ENB models are extremely low and sometimes even negative, and the one that is the most positively correlated with the ENC measure, which reflects a reasonable albeit naive diversification measure, is the one computed with the MLT approach (11.82%). On the other hand, the correlation between the ENB using an MLT model and the ENB using a PCA is equal to 1.58%, the correlation between the ENB using an MLT model and ENB using a FF model is -29.10% and the correlation between ENB using a Fama-French factor model and ENB using a PCA approach is 10.43%. Note that the ENB using a Fama-French factor model is also positively correlated to the ENC (1.64%) but very close to 0.

Since the MLT seems to be the most reliable approach to computing the effective number of (uncorrelated) bets, we choose to focus on this approach in the remainder of the paper. We next try and analyse whether an equally-weighted portfolio achieves a higher level of diversification compared to a cap-weighted portfolio (see for example DeMiguel et al. (2009) for evidence that equally-weighted portfolios dominate their cap-weighted counterpart in terms of Sharpe ratio). We can observe the results of the ENC and ENB diversification measures in Figure 2, each figure displaying a diversification measure computed with the equally-weighted S&P500 (in green) and with the cap-weighted S&P500 (in blue). We note that the ENC computed from the equally-weighted S&P500 is approximately equal to 500 during the whole period. This is consistent with the definition of the Effective Number of Constituents, which reaches a maximum equal to the total number of constituents in the index when all constituents have the same weight in the portfolio, which is exactly the definition of an equally-weighted portfolio. If the ENC is not always exactly equal to 500 it is because the S&P500 index does not always exactly contains 500 constituents or because at some dates data on some
3. Empirical Analysis for Equity Indices

constituents of the index were missing in the database. (The number of missing constituents is illustrated in Figure 3.) However, we can see in Figure 2 that the ENB computed on the equally-weighted portfolio is very close to the ENB computed on the cap-weighted portfolio all along the period. This result is consistent as the equally-weighted scheme of diversification is in fact a naive way to diversify a portfolio in terms of risk. From these results, we see that ENC does not provide good measures for risk diversification, but is only good to reflect the level of concentration in the constituents of a given portfolio. It does not account for the presence of differences in volatility and correlation levels amongst constituents, as does the ENB measure. Overall, we find that the equally-weighted version of the S&P 500 index is indeed better diversified in terms of effective number of constituents, but they are not better diversified in terms of effective number of uncorrelated bets. While this finding may be perhaps somewhat surprising, it can intuitively be explained by the fact that equal-weighting is a naive form of portfolio diversification which does not utilise the information in the covariance matrix.

Finally, we try to relate the evolutions of the ENC and ENB (based on MLT approach) diversification measures to changes in economic conditions. In order to do so, we support our analysis by computing the correlations of the diversification measures and several economic indicators: the returns of the cap-weighted S&P500 (with reinvested dividends), the variations of the fitted GARCH Volatility, the term spread, the credit spread and the interest rate, and the returns of the four Fama-French factors. We compute in Table 3 the correlation of each measure of diversification with each economic indicator on the period ranging from 1959 until the end of 2012. The table exhibits very low levels of correlations between the diversification measures and the economic factors. Indeed, all correlations are less than 10% in absolute values, except for the one between the ENC measure and the S&P500 volatility. The ENC measure appears to be negatively correlated (-10.97%) with the index volatility, which means that when the volatility increases then the ENC decreases, leading to increasing levels of concentration in the index. On the other hand, the ENB seems to be uncorrelated to the volatility of the index (-0.72%). Overall, diversification measures seems to be very partially explained by standard state variables, which suggests that they contain some original meaningful information.

Finally, we move beyond a contemporaneous analysis and test for the predictive power of diversification measures for the S&P500 index over the sample period, that is between 4 January 1957 and 31 December 2012. We actually compute six linear regressions, each linear regression testing the relationship between the diversification measure at a given week t and the annualised performance on a given period starting at date week t + 1. We compute six different time-series of annualised performances for six different lengths of the predictive period: the following quarter, the following semester, the following year, the following two years, the following five years and the following ten years. We compute the diversification measures and the six time-series of annualised performances at a weekly frequency.
Table 6 displays the results obtained from the six linear regressions for the S&P500 index. These predictive powers of the diversification measures are statistically significant for both ENC and ENB measures as they have a p-value close to 0 for every period of computation of the annualised performance. These results suggest that there is a positive relationship between the level of diversification (measured via the ENC or ENB indicator and the subsequence performance of the S&P500 index, whatever its period of computation. It should be noted, however, that the coefficients of proportionality remain low (between 0.21 and 0.42). In addition, we find for both diversification measures that the R-squared and the t-stats of the linear regressions increase with the length of the period of annualised performance computation, which shows that the diversification measures have better forecasting power over long horizons. Lastly, if we only focus on the quarterly and the semi-annually performance computation, we notice that the t-stat of the ENB is higher than the one of the ENC with a higher level of significance. From the point of view of an investor who seeks to use diversification measures for short-term tactical purposes, it is therefore more interesting to look at the evolution of the ENB measure rather than focusing on the ENC measure.

3.2 Cross-Sectional Analysis of Diversification Measures

In the previous section, we analysed the diversification measures from a time-series analysis based on long-term historical return data for the S&P500 index. In this section, we test the link between diversification measures of risk and the performance of equity indices from a cross-sectional perspective instead. To do so, we conduct an analysis of the relationship between the performance of the 14 equity indices during the sub-prime crisis and their diversification measures computed at some point before the crisis started.

The indices we consider in this analysis are the S&P500 and 13 other equity indices mentioned before: the CAC 40 index, the DAX 30 index, the Dow Jones 30 index, the Euro Stoxx 50 index, the Euro Stoxx 300 index, the FTSE 100 index, the FTSE All World index, the Hang Seng index, the NASDAQ 100 index, the SPI index, the Stoxx Europe 200 index, the Stoxx Europe 600 index and the Topix 100 index. Therefore, we consider equity indices of different sizes and different universes. For each of them, we compute the ENC measure and the ENB measure using an MLT approach on the largest time period available. These time periods vary from 17 years of historical data for the longest to six years of historical data for the shortest. Figures 5 and 6 respectively display the ENC and the ENB using an MLT approach for the CAC40 index and for the Stoxx Europe 600 index computed following the same protocol than for the S&P500. We choose to present two indices with different sizes on purpose. We notice that the figures we obtain for these two indices are very close to those of the S&P500 (except for a shorter time period), which suggests that the conclusion we have obtained so far are relatively robust with respect to the choice of the index.

We then analyse in Figure 4 the relationship between the average ENB computed using an MLT approach and the average ENC computed on the whole historical period.
for each index and their facial number of constituents. Note that in Figure 4, we do not include the FTSE All World because it has too many constituents (around 3,000) which does not allow for a clear comparison with the other indices. Also, the number of constituents of some indices may vary over time, so we estimate that the SPI 200 constituents and that the Hangseng has 45 constituents, which corresponds to the average value over their respective historical periods. Unsurprisingly, we find in Table 2 that there is a strong linear relationship between the number of constituents of an index and its average ENB measure. This means that there is a positive proportional relationship between the level of diversification of an index in terms of uncorrelated risk factors and the number of constituents that this index contains. However, if we consider the relationship between the ENC of an index and its number of constituents, we see that the relationship is not as linear, since it seems to grow sublinearly. Intuitively, this is explained by the fact that cap-weighted indices are highly concentrated. Therefore, increasing the number of constituents will not decrease the concentration of cap-weighted in a linear way.

Next, we test whether a link exists between the performance of the 14 equity indices in bear markets and their respective diversification measures at a given date before the start of the bear market. As each of the 14 equity indices does not have the same length of historical data, we need to find a period that is common to every index in order to cross-compare their performance. In addition, we want to compute our analysis on a period of particularly severe market correction in order to test if the indices that were the best diversified in terms of uncorrelated risks (that is to say with the highest ENB) at a date prior to the start of the crisis are the ones that performed the best during this period of bear market. To find a period of harsh market downturn that was common to every equity index, we naturally focus on the period ranging from the beginning of September 2008 to the end of February 2009. The reasons for this choice are the following. If the 2007-2009 crisis has started on July 2007 with the massive default on subprime mortgages in the United States, it is in the autumn of 2008 that it started to spread to the banking sector and turned into a global financial crisis. Indeed, autumn of 2008 witnessed a rise of the confidence crisis, the discovery of toxic assets in bank accounts, a rise of the interbank interest rates, and credit constrictions to firms and households which exacerbated the global economic recession. In this context, the crisis hit stock exchanges all over the world, and financials were particularly badly hit with several banks being acquired by competitors, saved by the US Federal Reserve, or simply declared bankrupt. For instance, on 6 September 2008, the US government nationalised Freddie Mac and Fannie Mae and on 15 September 2008, Lehman Brothers collapsed triggering a massive drop in stock prices for many financial institutions. It is also during this period that Bernard Madoff, former non-executive chairman of the NASDAQ, was arrested for a large scale Ponzi scheme (on 12 December 2008) and that Iceland and Ukraine became insolvent. We chose to end the period at the end of February 2009, when the worst of the shock had passed and when governments (United States, Great Britain, etc.) issued
their first economic rescue plans. Therefore, we compare the annualised performances of the indices on the period starting at the beginning of September 2008 and ending at the end of February 2009 and their average diversification measures computed on six different periods. The average diversification measures are computed on periods immediately preceding the calculation of the index performance. In Figure 7, we plot the annualised performances of the 14 equity indices between September 2008 and February 2009 with respect to each diversification measure computed at the date immediately preceding the period of bear market at the end of August 2008. We perform linear regressions in order to test the robustness of the relationship between performance and diversification measures. For each diversification measure (ENC and ENB), we obtain six sets of statistics, corresponding to the six periods of calculation of the average diversification measures. Since the FTSE 100 index appeared as a clear outlier in Figure 7, we compute these linear regressions without this outlier. Our results are displayed in Table 4. In addition, the straight line drawn on Figure 7 corresponds to the coefficients calculated from the linear regressions without the outlier. We analyse in more details the results for diversification measures computed at the end of August 2008; however, the results obtained on the other periods of computation of the diversification measures follow the same trend. The statistical analysis displays a positive linear relationship between the performance of the index on the period starting on September 2008 and ending at the end of February 2009 and each one of the two diversification measures computed at the end of August 2008. We observe that the positive relationship is statistically more significant for the ENB measure than for the ENC measure. Indeed, the regression based on ENB measures has a 92% confidence level and a 24.7% R-squared compared to the regression based on ENC measures, which only have a 84% confidence level and a 17.12% R-squared. In addition, the slope for the performance-to-ENB relationship is steeper (almost twice as steep) than the slope for the performance-to-ENC relationship. Overall, our results suggests that the higher was the ENB of an index prior to the worst of the crisis, the more likely it was to perform better during September 2008–February 2009 compared to an index that had a lower ENB at the same date. This is consistent with the interpretation of the ENB as a meaningful diversification measure. Therefore, we conclude from this cross-sectional analysis that in a period of severe bear markets indices that were the most diversified in terms of uncorrelated sources of risks (i.e., high ENB) prior to the period of market downturn performed better than the others. However, even though there seems to be a positive relationship, it is not statistically proven that indices that were the best diversified in terms of constituents (i.e., high ENC) prior to the period of market downturn, performed better than the other equity indices. Hence, diversifying in terms of constituents seem to be less rewarded than diversifying in terms of uncorrelated bets during bear market.

For comparison purposes, we conduct the same analysis during a bull market period, focussing on the recovery period that started in the beginning of March 2009.
and ended at the end of February 2010. Figure 8 shows that the linear relationship between diversification measures and performances is now negative for both the ENC and the ENB. This is confirmed in Table 5, where we look at the same period to estimate the performance during a bull market period, but where instead of looking at the diversification measure at one particular date (end of February 2009), we look at diversification measures over several extending windows (previous month, previous, quarter, previous semester, previous year and previous two years). In most regressions, we observe low p-value, which tend to confirm the negative slope with a reasonable level of confidence. This shows that, during the recovery of the subprime crisis (bull market), indices with lower levels of diversification performed better than indices with higher levels of diversification (both weight and factor based). This can be explained by the fact that when an equity index loads on more uncorrelated factors it may not increase as quickly as when the equity portfolio shows a more pronounced concentration. The stock index with the highest performance could be a portfolio fully concentrated in the best performing stock.
3. Empirical Analysis for Equity Indices
4. Empirical Analysis for Pension Funds
In this section, we compute the diversification measures at the level of pension fund policy portfolios to assess whether or not pension funds achieve reasonable degrees of diversification. Then, we study the relationship between the characteristics of the funds, e.g., size and type of fund (private versus public) and the diversification measures. Finally, we analyse the relationship between the performance of the pension funds during both bull or bear markets and their level of diversification.

4.1 Data Collection Methodology
We consider two different universes to perform empirical tests on pension funds. In this section, we present a detailed description of both data sets used to run tests on pension funds. The first universe is made of the 1,000 largest US pension funds. We use the P&I Top 1,000 database to get the asset allocation of each of these pension funds as of 30 September 2002, 30 September 2007 and 30 September 2012. We exclusively focus on the portion allocated to their defined benefit plan; if they also have a defined contribution plan, we do not analyse the amount they allocate to this plan. We are left with 750 pensions funds in 2002, 780 in 2007 and 320 in 2012 (the last figure is quite low because less than half of the 1,000 pension funds in the database filled in P&I’s survey). The second universe is made of the world’s 10 largest pension funds. We hand collect data on asset allocation for the years 2007 and 2012 using public information gathered from their official website, from their financial statements and from their comprehensive annual reports.

In order to represent the different asset classes each pension fund is invested in, we consider the following arguably somewhat arbitrary partition of the asset allocation: domestic fixed income, international/global fixed income, high-yield bond, inflation-linked (IL) bond, domestic equity, international equity, global equity, private equity, real-estate, commodity, mortgage, and cash. One would note that the international fixed income and the global fixed income asset classes are grouped into a single asset class. “Global” means that the investment portfolio is a mixed-strategy made of domestic and foreign assets; “International” means that the investment portfolio is made of non-US assets only. The choice to merge these two kinds of asset classes was made by the P&I Research Center because US pension funds tend to invest a very small amount of their assets in non-US fixed incomes. Indeed, on average on the three years, 22 out of 1,000 pension funds had invested 10% or more of its total asset allocation in international/global fixed incomes. However, the distinction between international and global holds more clearly for equities, and this has to be stressed because it implies that the different asset classes are not all linearly independent: US and international equities can replicate the global equities. In particular, it means that it is not possible to reach (in theory) the highest ENB measure equal to the number of asset classes, and that one of the three asset classes should be ignored to compute the ENB. However, in practice, these three asset classes are not perfectly collinear, so the computation of the ENB measure can still be carried out.

In order to obtain the above partition, we had to slightly modify the original databases.
and make several assumptions. Firstly, in the 2002 original database, we gathered the portion allocated to “Sponsoring Companies Stock” and to “Other Domestic Equity” into a single asset class: “Domestic equity”. Secondly, we did not always have the same partition of asset allocation for the three years. Therefore when one of the asset classes from our list was missing in one database, we added it and filled it with zeros. This is why we have 0% asset allocation for the “Global equity” asset class for the years 2002 and 2007, 0% asset allocation for the “Mortgage” asset class for the year 2012 and 0% asset allocation for the “Commodity” asset class for the year 2002. Thirdly, in each of the three original databases we did not have access to the percentage allocated to high-yield bonds and to inflation-linked bonds. However, we had their amount in dollars, and consequently we managed to deduct their percentage asset allocation. Then, in order to remain at 100% total allocation, we removed the calculated portions from the “Domestic fixed income” percentage allocation. By doing so, we assumed that the high-yield bonds and the inflation-linked bonds in which the US pension funds invest are all domestic bonds. We applied this methodology for most of the pension funds but there were some exceptions. When it was specified that high-yield bonds and inflation-linked bonds had been counted in the portion allocated to “Other investments”, we withdrew the portion of high-yield bonds and inflation-linked bonds from “Other investments”. Fourthly, as for the high-yield bond and the inflation-linked bond asset classes, the portion allocated to commodities was never specified. We just had its amount in dollars for the years 2007 and 2012 and knew that this asset class was part of the “Alternative investments” asset class for the year 2012. Therefore, using the same method as for the high-yield bond and the inflation-linked bond asset classes, we deducted the portion allocated to commodities and subtracted it from the “Alternative investments” asset class in 2012 or from “Other investments” for the years 2007 and 2012 when it was specified. Finally, we removed from our partition the portion allocated to “Other investments” (and to “Alternative investments” for the year 2012) and we normalise the percentages in order to obtain a total of 100% allocation. These “Other” or “Alternative” investments that we discard represent in average 4.50% of the total investment in 2002, 11.33% in 2007, and 8.65% in 2012.

Once the three databases were homogenised, we still had some small remaining discrepancies. First, for some pension funds we had negative percentages allocated to the domestic fixed income asset class, to the “Other investments” asset class and to the “Alternative investments” asset class. This happened because these pension funds did not carefully fill the original databases. In some cases, they classified all their bonds under “International/global fixed income” whereas they should have been part of the “Domestic fixed income”; in some other cases they classified all their other investments (and/or alternative investments) under the “Private equity” asset class. However these discrepancies happened to be a negligible portion of the universe (around three out of 1,000 pension funds for each year). Secondly, the sum of the percentage allocated to each asset class did not always equal 100%. This was due to a wrong approximation of the amount
allocated to the asset classes when the pension funds filled the original forms. As a consequence, we had totals amounting 100.1% or 99.9% for several pension funds. Hence we have rescaled the weights in order to guarantee that they all sum up to 100%.

Once the partition is completed, we choose appropriate benchmarks for each asset class. We make the following selection:

**Domestic fixed income:** For the domestic fixed income asset class, we choose the Bank of America Merrill Lynch US Corporate & Government Index (B0A0). This index includes US Treasury, US agency, foreign government, supranational and corporate securities. Inflation-linked debts are excluded from this index;

**International/Global fixed income:** For the international/global fixed income asset class, we choose the JP Morgan Global Aggregate Bond Index (JPM GABI). We deliberately choose a global fixed income market index for this asset class in order to stick to the assumption made by the P&I Research Center when building its databases;

**High-yield bond:** For the high-yield bond asset class, we choose the Bank of America Merrill Lynch US High-yield Index (H0A0). This index tracks the corporate bonds publicly issued in the US domestic market that have a rating below investment grades and that is based on an average Moody's, S&P and Fitch. We chose a US market index for this asset class according to the assumption we have done when modifying the P&I databases. As we assumed that the high-yield corporate bonds (and the inflation-linked bonds) were belonging to the total domestic fixed income asset class, we made the same assumption here and chose a US domestic market index tracker;

**Inflation-linked bond:** Concerning the inflation-linked bond asset class, we choose the Bank of America Merrill Lynch US Inflation-linked Treasury Index (G0QI). It tracks inflation-linked sovereign debt that are denominated in US dollars and issued by the US government in its domestic market. We chose this index among the US market indices for the same reasons as for the high-yield bond index. This index excludes STRIPS but takes into account the originally-issued zero-coupon bonds;

**Domestic equity:** We choose the S&P500 composite index to track the domestic equity asset class as it a broad and popular index;

**International equity:** We choose the FTSE World Ex-US as a benchmark for the international equity asset class;

**Global equity:** We choose the FTSE All World as a benchmark for the global equity asset class;

**Private equity:** For the private equity asset class, we choose the S&P 600 small-cap benchmark;

**Real-estate:** For the real estate asset class, we choose the MSCI REIT as a benchmark. This index most includes equity REITs from the MSCI US Investable Market 2500 Index;

**Commodity:** For the commodity asset class, we choose the S&P GSCI commodity;

**Mortgage:** For the mortgage asset class, we choose the Bank of America Merrill Lynch US Mortgage Backed Securities Index (M0A0);

**Cash:** We choose the three-month US Treasury-Bill as a benchmark to track the cash asset class.

We collect weekly returns for each benchmark (with reinvested dividends) for the period ranging from 30 September, 1997 to 30 September 2012 and display their descriptive
4. Empirical Analysis for Pension Funds

statistics in Table 7 over the entire period. We notice that the US bonds, US IL bonds, and the mortgage backed securities display on the sample period the highest Sharpe ratios among all the asset classes. This comes from a lower volatility compared to the other asset classes. On the other hand, the asset classes with the lowest Sharpe ratios over the sample period are commodities (close to 0) and equities (their Sharpe ratios is close to 0.13 for all types of equities: US, international, or global). In panel (b) of Table 7, we compute the correlations between each asset class over the entire period of time, and observe, as expected, that the correlations among similar asset classes remain high: the three equity benchmarks are highly correlated, and so are the four bond benchmarks. However, across very different asset classes, correlations can be quite low.

From the covariance matrix estimated over the entire sample period (we do not robustify the covariance estimator here since there are only a few constituents), we compute the factor exposures for both the PCA and the MLT methods, and report these numbers in Tables 8 and 9. From Table 8, one can provide some interpretation for the factors obtained through the PCA approach: F1 seems to be a factor related to equities, F2 to commodities, F3 to real estate, F4 to ex-US equities, F5 to bonds, F6 to large cap equities (vs.) small caps, F7 to high-yields, etc. As often, it becomes increasingly difficult to interpret factors that start to have an exceedingly low explanation power. Similarly, we display in Table 9 factor exposures for the MLT approach. A quick look at the diagonal of the table displaying the asset classes exposure to the different factors confirms that each factor seems to be related to one particular asset class, which is consistent with the focus of the MLT approach. For reasons outlined in the section dedicated to equity indices, we focus in what follows on the minimum linear torsion approach, which is better suited for our purpose than the principal component analysis.

In Section 4.3, we use another, smaller, set of pension funds, which is a selection of the ten largest pension funds in the world, according to Towers Watson’s 2012 ranking. For our selection, we exclude the 8th largest pension fund, the Central Provident Fund (Singapore) is a comprehensive compulsory saving plan for Singapore citizens and permanent residents which is essentially dedicated to domestic fixed income investments, thus making it an outlier in the analysis. We also exclude the 10th largest pension fund, which is the Employees Provident Fund (Malaysia) because of the difficulty to find relevant domestic fixed-income benchmark with sufficient history. Instead, we include PFZW (Netherlands) and California State Teachers (U.S.A.), which are ranked 11th and 12th, respectively. These funds are ranked by decreasing order of assets under management (amount evaluated at year-end 2011) in the list that follows:

1. Government Pension Investment Fund (Japan);
2. Government Pension Fund (Norway);
3. Stichting Pensoenfonds ABP (Netherlands);
4. Korea National Pension Service (Korea);
5. Federal Retirement Thrift Investment Board (U.S.A.);
6. California Public Employees (U.S.A.);
7. Chikyoren Local Government Officials (Japan);


The benchmarks we use are similar to the benchmarks used in the analysis of US pension funds, except for two main differences (see the list below for more details). First, they are specific to the country of origin of each pension plan in the case of domestic asset classes (fixed-income and equities). Secondly, we consider for each index the return series expressed in the six different currencies corresponding to the countries of origin for the pension funds in our sample, with a hedge against foreign exchange fluctuations. More specifically, we hedge each benchmark expressed in foreign currency by selling foreign currency exposure against the domestic currency at forward price. On the last business day of each month, the exposure in each foreign currency is sold forward with a one month maturity. When the forward contracts mature, the resulting net cash flow is reinvested in the unhedged benchmark.

Domestic fixed income: We use the JP Morgan Government Bond Total Return Index for each of the following domestic markets: US, Canada, Europe, Japan, South Korea, and Norway;
International/Global fixed income: We use the JP Morgan Global Aggregate Bond Index (JPM GABI);
High-yield bond: We use the Bank of America Merrill Lynch Global High-yield Index (HW00); this index tracks the performance of USD, CAD, GBP and EUR denominated below investment-grade debt publicly issued in the major domestic or eurobond markets;
Inflation-linked bond: We use the Bank of America Merrill Lynch Global Government Inflation Linked Index; this index is a broad, market value weighted, capped total return index designed to measure the performance of inflation-linked sovereign debt that is publicly issued and denominated in the issuer’s own domestic market and currency;
Domestic equity: We use the MSCI equity indices for each of the markets required in the analysis;
International & Global equity: We use the MSCI All Country World where the aggregate return is the combination of each constituent’s return;
Private equity: We use the S&P 600 small-cap benchmark;
Real-estate: We use the MSCI All Country World Index Real Estate where the aggregate return is the combination of each constituent’s return;
Commodity: We use the S&P GSCI Commodity Index;
Mortgage: We choose the Bank of America Merrill Lynch US Mortgage Backed Securities Index (M0A0).

Daily returns on all of these indices are collected over the 10 year period beginning on 28 June 2002, and ending on 28 June 2012 (see Section 4.3 for the detailed analysis).

4.2 Diversification Measures for the 1,000 Largest US Pension Funds
We compute ENB using a PCA and ENB using an MLT, at the end of September 2002, at the end of September 2007 and at the end of September 2012. In order to estimate the covariance matrix between the different asset classes, we use five years of
historical weekly returns before the date at which we perform the computation. We also compute the ENC measure at the same dates. As our universe is made of roughly 1,000 pension funds (it is less than 1,000 because some funds did not fill out the P&I form, or did so but not with enough details to be included in our analysis), we obtain 1,000 diversification measures at each date. In Figure 9, we display the distributions of the ENC, and ENB computed using a PCA and ENB computed with an MLT approach. We first notice that the distribution of the ENB computed through a PCA is on average close to one for all pension funds and for the three years, just as was already the case for the equity indices. We thus decide to focus on the ENB computed with an MLT approach in order to cross-compare the diversification measures. When looking at the evolution of each diversification measure, it seems that a change occurred between 2007 and 2012, as most US pension funds seem to have increased the diversification level in their portfolio between these two dates. For instance, between 2002 and 2007, the mean of the distribution of the ENCs increases by 1.3% while between 2007 and 2012, it increases by 40.7%. Therefore, it seems that US pension funds dedicated some effort between 2007 and 2012 to improve their level of diversification. However, we note that when US pension funds increase their ENC by 40.7% in five years, they only increase their ENB by 14.4% between 2007 and 2012.

We then test whether the distributions displayed in Figure 9 depend on the US pension funds’ characteristics such as the amount of assets under management or their status (public or private). In Figure 10, we look at the distribution of the ENC and the ENB computed using a PCA approach and the ENB computed using an MLT approach according to the amount of assets under management of the pension funds. For each diversification measure and for each date, we compute two distributions: one for the pension funds managing the lowest 30% of assets under management (in red in the figure), and one for the pension funds managing the highest 30% of assets under management (in blue in the figure). We note that the ENC of the 30% pension funds managing the lowest amount of assets under management are on average higher than the ENC of the 30% pension funds managing the highest amount of assets under management at the end of September 2002, 2007 and 2012. We test the level of significance of the average of the two distributions of ENC for the three years using a two-sample t-test and report the results in Table 10. We find that the mean of the distribution of the pension funds managing the lowest amount of assets is always significantly different (higher) at a 95% confidence interval from the mean of the distribution of the pension funds managing the highest amount of assets (except in 2012 for the ENB computed with the MLT, where we find that bigger funds are slightly better diversified). This means that smaller funds tend to be less concentrated than larger funds, which could be explained by liquidity constraints preventing some of the largest fund to invest significant fraction of their assets in some alternative asset classes.

In Figure 11, we look at the distribution of public versus corporate pension funds’ diversification measures. It is interesting to notice that public pension funds always have,
on average, a higher ENC than corporate pension funds. This is confirmed by looking at panel (b) of Table 10, where we see that the t-test of the equal mean hypothesis is rejected with a 95% confidence for each of the three years. However, this tends to be the opposite when we look at the ENB measure. Indeed, it is statistically impossible to identify in 2002 and 2007 whether the public or corporate pension funds had the higher ENB measure. Moreover, the t-test of panel (b) in Table 10 shows that in 2012 corporate pension funds had, on average, higher ENB measures than public funds. This shows that the 2007-2009 subprime crisis led corporate funds to increase their level of diversification in terms of risk factors, which made them more comparable if not better diversified than public funds.

In the empirical analysis for equity indices which we performed previously, we tested for the cross-sectional relationship between diversification measures and subsequent performance.

In this section, we replicate the same analysis on the 1,000 largest US pension funds, and we analyse whether the diversification measures computed over these pension funds at the end of September 2007 can give insights on the returns of US pension funds performance several months after. In order to perform our test, we need to make several assumptions. We actually do not use pension fund actual performance in our analysis and assume instead that the fund asset allocation remains constant over the months following the computation of the diversification measures at date $t$. Then, we use the performances of each benchmark representing an asset class in our universe in order to estimate these pension fund performances at any future date following the date $t$ of diversification measurement. We use this methodology for two main reasons. The first reason for our choice is more practical. We want to look at performances at different points in time. However, most of the time, we cannot get this information from pension funds’ releases as they display their returns at the end of calendar years or at the end of fiscal years (end of June). Secondly, this approach allows us to preserve a stronger link between diversification measures computed at a date $t$ and pension funds’ performances at $t + n$ months. Indeed, pension funds are free to change their asset allocation after the date of computation of the diversification measures. In the end, depending on the degree of modification of these allocations, the diversification measures computed at date $t$ are likely to bear little, if any, relation to pension fund performance at date $t + n$ months. Depending on the policies adopted by a fund, its actual performance at date $t + n$ months may therefore have nothing to do with its diversification measure at date $t$.

In our test, we compute the fund returns over two different periods: over the year directly following the date of computation of the diversification measures (from 28/09/2007 to 26/09/2008), and over the worst period of the subprime crisis for the financial sector (from 05/09/2008 to 27/02/2009). For each diversification measure, we first plot the relationship between the US pension funds’ annualised performances at date $t + n$ months according to their level of diversification measure at date $t$ (end of September 2007). Then, we statistically test the degree of significance
4. Empirical Analysis for Pension Funds

of our results. We replicate this test for each diversification measure and for the two periods of time considered. We display our results in Figure 12, and panels (a) and (b) of Table 11. In Figure 12, it is first striking to see that the relationship between US pension fund performances and their level of ENC is negative, while this relationship is positive for the ENB. This is true for the two periods of performance computation. We test the degree of significance of these results for the two periods and note that the coefficients between performances and diversification measures are indeed positive for the ENB and negative for the ENC whatever the period of performance chosen, and that t-stats and p-values are always significant. These results mean that, at the end of September 2007, a pension fund that had a higher ENB was more likely to reach higher performances during 28/09/2007-26/09/2008 and during 05/09/2008-27/02/2009 (which translates into lower losses because we are looking at performances during the subprime crisis) than a pension fund that had a lower ENB, assuming the policy portfolio weights remaining constant. However, higher levels of ENC for a pension fund at the end of September are likely to have no impact if not negative effects on its performances during 28/09/2007-26/09/2008 and during 05/09/2008-27/02/2009 compared to another pension fund with lower levels of ENC, all weights remaining equal.

Nevertheless, in Figure 12, we notice that the pension funds that performed best during the two periods considered seem to behave differently to the others. For these pension funds, there seems to be a negative, as opposed to positive, relationship between their performances and their level of ENB at the end of September 2007. For the annualised performances observed during 28/09/2007-26/09/2008, this observation applies to the pension funds that performed over -5%; for the annualised performances observed during 05/09/2008-27/02/2009, it applies to the pension funds that performed over -20%. Therefore, for each period, we split up our population and test the link between performances and diversification measures on the US pension funds that performed over -5% (respectively over -20%) on the one hand, and the ones that performed under -5% (respectively under -20%) on the other hand during 28/09/2007-26/09/2008 (respectively during 05/09/2008-27/02/2009). For each diversification measure and for each period, we scatter plot these relationships and test their levels of significance. We display our results in Figure 13, and panels (c) and (e) of Table 11 for performances computed during 28/09/2007-26/09/2008; and in Figure 14, and panels (d) and (f) of Table 11 for performances computed during 05/09/2008-27/02/2009. We always display annualised performances expressed in percentages.

For each period and for the US pension funds that performed under the threshold of -5% (or -20%), the results of our linear regressions are very close to the ones obtained on the whole sample and their scatter plots look similar to the scatter plot obtained on the whole sample. However, results are very different for the US pension funds that performed over the threshold of -5% (or -20%). Indeed, we see that for the two periods considered, the relationship between pension funds performances and ENB becomes negative. This relationship is also negative for the ENC. This means
that, all weights remaining equal in 2007, the pension funds that performed over -5% (respectively over -20%) between 28/09/2007 and 26/09/2008 (respectively between 05/09/2008 and 27/02/2009) were more likely to perform better than other pension funds from the same sample if they were less diversified in terms of uncorrelated risk factors than these other pension funds. Indeed, we can clearly see in Figure 13 and in Figure 14 that there is a negative relationship between the performances that are over the thresholds and their ENB. We also see that, among the pension funds that performed above the threshold (-5% or -20%), the slopes of the linear relationship between the ENC measure and the funds’ performances are much steeper than the slopes obtained with ENB.

One may wonder which asset classes these pension funds were invested in during the subprime crisis in order to be able to limit the losses even though the funds showed lower levels of diversification. In Figure 15, we display the charts representing the average asset allocation of the pension funds that perform under and over -5% during 28/09/2007-26/09/2008 and under and over -20% during 05/09/2008-27/02/2009. We clearly see that the pension funds that perform over the thresholds are the ones that allocate 80% to 85% on average to US bonds and almost nothing in the other asset classes. This explains the negative relationship between ENC/ENB and annualised performances. These pension funds performed best during the two periods considered because they were fully invested in safe asset classes that resisted well during the crisis. Indeed, we see in Table 12 that fixed income asset classes are among the classes of assets that suffered the least during the two periods considered. They made positive returns during 28/09/2007-26/09/2008 and above -0.5% during 05/09/2008-27/02/2009. These pension funds were very poorly diversified in terms of asset classes since their aim is not to diversify their portfolio so as to maximise their risk reward but it is to invest into a safe, low risk-rewarding asset class. In other words, they seem to maintain a focus on liability hedging, as opposed to performance generation. As a result, their strategy is to have a highly concentrated portfolio in the asset class that represents the best hedge for the liabilities. The opportunity cost of this exceedingly cautious strategy is of course prohibitive in terms of renouncement to the access of the risk premia on risky asset classes that is allowed by a well-diversified portfolio.

4.3 Diversification Measures for the World’s 10 Largest Pension Funds

In this section we analyse the policy portfolios held by the world’s 10 largest pension funds (except for the Central Provident Fund (Singapore) and the Employees Provident Fund (Malaysia), which are replaced by PFZW (Netherlands) and California State Teachers (U.S.A.), as mentioned previously). These funds are very different from each other in terms of geographic location, portfolio investments, management strategies, structure and purposes, and we provide below a brief description of their mode of operations.

Government Pension Investment Fund (GPIF) is a Japanese pension fund that managed US$ 1,394,873 million at year-end 2011. It is established as an independent...
4. Empirical Analysis for Pension Funds

Government Pension Fund (GPF) is a Norwegian pension fund meant to invest the surplus generated by the Norwegian petroleum sector. This fund has been established in order to smooth out the effects of oil price wavering and to better manage an expected future decline of income in this sector. The Norwegian Central bank manages this fund through Norges Bank Investment Management (NBIM). The size of its assets under management reached US$ 575,527 million at year-end 2011. NBIM orientates its portfolio management decisions towards equity and fixed income asset classes. Since 2010, it invests in real estate asset class as well but this type of investment still represents a minor part of its portfolio’s allocation.

Korea National Pension Service (KNPS) is a South Korean public pension fund. It is the main and biggest national pension fund in Korea and managed at the end of 2011 US$ 313,981.00 million amount of assets. This pension plan is a compulsory coverage for Korean employees and employers in any workplace with one or more employees. Any foreign worker settled in South Korea, regardless of its age, workplace and number of employees, is also included in this mandatory pension plan. National Pension Service invests in domestic and international fixed income (65% in total, 60% in domestic bonds in 2012), in domestic and international equities (around 30% of their 2012 total portfolio) and in dedicated alternative portfolios (around 10% of its total portfolio in 2012).

Federal Retirement Thrift Investment Board (FRTIB) is an independent agency of the US government. It has been established by the Federal Employees Retirement System Act in order to administer the Thrift Savings Plan (TSP). TSP is a defined contribution plan and is a pay-as-you-go pension system. Its role is mostly to manage and invest the Reserve Funds of two government pension plans entrusted by the Minister of Health, Labour and Welfare: the Employee’s Pension Insurance and the National Pension. Since it works as a pay-as-you-go pension system, GPIF has a social responsibility toward Japanese citizens and residents. Its portfolio investment decisions are closely related to this task and this is why this pension fund is mostly invested in domestic fixed income (almost 70% in 2012 and up to 75% if we include international bonds). The rest of the portfolio is invested in domestic and international equity. This type of investment decision is meant to insure constant future cash flows (bond investments) while trying to get access to higher gains (equity investments) with moderate risk (small portion allocated to this asset class).

Stichting Pensioenfonds ABP means "National Civil Pension Fund”. It is a Dutch pension fund for employees in the government, public and education sectors in the Netherlands. At the end of 2011, its assets under management reached the amount of US$ 320,356 million. Unlike the two first pension funds, ABP portfolio seems to be invested in a lot of asset classes. In 2012, this fund invested in domestic and international fixed income (about 30% of its total portfolio), high-yield bonds, inflation-linked bonds, domestic and international equities (about 30% of its total portfolio as well), private equity, commodity, real estate and other alternative investments.
plan (the largest in the world) for Federal employees and members of the uniformed services. It offers five types of investments in the following different funds: G, F, C, S, I and L funds. The G Fund is a government securities investment fund that invests in short-term US Treasury securities and delivers rates of interest that are close to those of long-term government securities. The F Fund is a fixed income index investment fund that replicates US government, mortgage-backed, corporate and foreign bond securities issued in the US market. The C Fund is a common stock index investment fund that replicates the S&P500 and tracks the 500 large to medium-sized US companies. The S Fund is a small-capitalisation stock index fund that follows small and medium-sized companies that are not included in the S&P500 index. The I Fund is an international stock index investment fund that tracks international equities from large companies based in Europe, Australasia and the Far East. Lastly, the L Fund gathers life cycle funds that are invested in the five former funds and follow liability-driven strategies e.g. pre-determined asset allocations. According to Towers Watson’s ranking, FRTIB managed US$ 281,359 million at year-end 2011.

California Public Employees (CalPERS) is a Californian public administration that manages institutional Californian funds for public schools, local agencies and state employers and provides them health and retirement benefits. At the end of 2011, it managed US$ 220,638 million. Its main activity is to administer retirement benefits through four different programs: CalPERS Retirement Benefits (which accounts for more than 90% of the total assets under management), Judges’ Retirement System Retirement Benefits (I and II) and Legislators’ Retirement System Retirement Benefits. CalPERS’ portfolio is widely diversified in terms of asset classes (fixed income securities, equities, private equity, commodities, real estates and other alternative derivatives) and allocates more than 50% of its total investments to domestic and international equities in 2012.

Chikyoren (Pension Fund for Local Government Officials) is a Japanese pension plan that managed US$ 199,549 million at year-end 2011. Like GPIF, Chikyoren has a social responsibility toward Japanese citizens and residents and has to dedicate a great portion of its investment portfolio to government debt issued by the Fiscal Investment and Loan Program. And indeed, in 2012, it invests more than 60% of its portfolio in domestic fixed income, 15% in Japanese equities and only 10% in foreign bonds and 10% in foreign stocks. However, Local Government Officials only has a Japanese official website and it is very hard to find detailed information on its portfolio allocation in English or to reach an English-speaking contact. Therefore, we only managed to collect partial information on its investment allocation.

Canada Pension Plan (CPP) is a pension fund that is administered by the Canada Pension Plan Investment Board (CPPIB), a government-independent organisation. CPPIB managed US$ 158,669 million at the end of 2011. CPPIB invests in a wide range of asset classes and follows a global portfolio strategy. In 2012, Canada Pension Plan invested indeed in domestic and international bonds, in mortgages, domestic and international equities, private
equities, real estate and other alternative investments. It follows two types of investment strategy: the CPP Reference Portfolio (CRP) and active investment management with value-added strategies. The CRP is a simple portfolio that invests in assets traded on public markets and is meant to insure the long-term 4.0% real rate of return while the value-added active investment portfolios use the CRP as a benchmark to access upward potential gains.

Stichting Pensioenfonds Zorg en Welzijn (PFZW) which means “Pension Fund for Care and Wellbeing”, is a Dutch pension fund. It manages US$ 150,479 million at year-end 2011. PFZW is a non-profit company administered by PGGM, a cooperative of trade unions and employers working in healthcare and social sectors. PGGM’s portfolio is well balanced in terms of asset class diversification, investing around 20% of its portfolio in both domestic and international bonds, 20% in domestic and international equity and the rest in high-yield bonds, inflation-linked bonds, private equity, commodities, real estate, hedge funds and other alternative investments.

California State Teachers (CalSTRS) is a pension plan that is part of the State of California’s Government Operations Agency and is meant to secure and manage the funds of educators. It is the largest educator-only pension fund in the world and managed US$ 139,534 million amount of assets at the end of 2011. It holds defined benefit and cash balance benefit programs. Like CalPERS, it is widely diversified in terms of asset classes. Even if it allocates more than 50% of its portfolio to international and national equities, it also invests in fixed income securities (domestic only), high-yield, inflation-linked bonds, private equities, commodities and other alternative investments.

We hand collect in Table 13 the allocations of these 10 funds using public information gathered from their official website, from their financial statements and from their comprehensive annual reports and compute, in Table 14, the ENC (top panel) measures and the ENB (bottom panel) measures at two different dates: 2007 and 2012. For the computation of the ENB, we use the MLT approach where the covariance matrix of the 10 risky benchmarks representing the asset classes is estimated using five years of historical daily returns before the date at which we perform the computation. We note that the two Dutch pension funds are by far the most diversified in terms of the ENC and ENB measures in 2007 and 2012 with ENC (ENB) values equal to 6.99 (5.05) for ABP and equal to 7.53 (4.95) for PFZW in 2012. Given that the maximum ENC is equal to the number of risky asset classes, that is 10 in this analysis, this shows that their asset allocation portfolio is closest, amongst all pension funds in the sample, to an equally-weighted scheme. On the other hand, the least diversified pension funds, both in terms of ENC and ENB, are KNPS (South Korea) and FRTIB (USA). They exhibit ENC (ENB) values below equal to 2.63 (3.05) for KNPS and 2.56 (3.56) for FRTIB in 2012, which shows that their allocation is very concentrated since they are exposed to only 2 to 3 asset classes (3 or 4 risk factors). It is worth noting that the ENC and ENB measures are consistent to determine the two most and least diversified pension funds among the
4. Empirical Analysis for Pension Funds

world 10 biggest ones. Also, we find the ENB measure to be higher than the ENC measure in some cases, suggesting that a seemingly poorly diversified diversification in terms of asset classes can hide a more diversified allocation in terms of underlying risk factors.

Now, if we look at the third column of both panels in Table 14, we can assess the evolution of the allocation strategy of the pension funds after the big international turmoil caused by the subprime financial crisis. Almost all diversification measures increased except for the ENC of the Norwegian pension fund GPF, which decreased by 17.06%. However, it is interesting to notice that the ENB measure of the same fund slightly increased by 5.68%, which suggests that if GPF allocation is more concentrated in terms of asset classes in 2012 than it used to be in 2007, it is not necessarily less diversified in terms of underlying factor exposures. On the other hand, the Canadian pension fund CPC has significantly reduced its concentration (since its ENC has increased by 20.39%), but its diversification in terms of independent sources of risk has not substantially improved (only by 2.55%). Finally, we can see that even though both Dutch pension funds already had the highest diversification measures in 2007, they kept improving the degree of diversification in their portfolios from 2007 to 2012.
5. Conclusion
This paper analyses various measures of portfolio diversification, and explores the implication in terms of advanced risk reporting techniques. We use the minimal linear torsion approach (Meucci et al. (2013)) to turn correlated constituents into uncorrelated factors, and focus on the **effective number of (uncorrelated) bets** (ENB), the entropy of the distribution of risk factor contribution to portfolio risk, as a meaningful measure of the degree of diversification in a portfolio. In an attempt to assess whether a relationship exists between the degree of diversification of a portfolio and its performance in various market conditions, we empirically analyse the diversification of various equity indices and pension fund policy portfolios. We find strong evidence of a significantly positive time-series and cross-sectional relationship between the ENB risk diversification measure and performance in bear markets. This relationship, however, is highly linear, and the top performing portfolios in severe bear markets are typically portfolios concentrated in safe assets, as opposed to well-diversified portfolios. We also find statistical and economic evidence that this diversification measure has predictive power for equity market returns, a predictive power which becomes substantial over long holding period. Overall our results suggest that the ENB measure could be a useful addition to the list of risk indicators reported for equity and policy portfolios.

Our work can be extended in several dimensions. In particular, the analysis developed in this paper could be used not only to measure, but also to manage, diversification within an equity or policy portfolio. We encourage interested readers to look at Meucci et al. (2013) and Deguest et al. (2013) for a thorough empirical and theoretical analysis of the properties of portfolios designed to maximise the effective number of bets. Regarding the empirical analysis of diversification for equity portfolios, another natural extension of our work would consist in comparing the degree of diversification for various weighting schemes based on the same investment universe. While we have looked at cap-weighted and equally-weighted equity index portfolios only, it would be useful to also assess the degree of diversification achieved by minimum variance portfolios or risk parity portfolios, amongst other examples of so-called smart beta indices. Yet another useful extension of the paper would consist in repeating the analysis we have conducted for pension funds for endowments. We leave this extension for further research.
Appendices
Appendices

A. Robustness Checks

In this paper, we chose as a base case to compute the different diversification measures for the period starting in 1959 and ending in 2012 using a 1-year rolling window without overlap and using the entropy metric, see Equations (2.3) and (2.6). However, we could have used a rolling window with overlap, as well as other rolling window periods, and other kinds of metrics to compute the dispersion of the weights in the ENC, or of the factor contributions in the ENB. In order to test the robustness of our analysis, we present in this section some results using different time-periods and another type of dispersion measure than the one used in the base case to compute the diversification measures. We first present the ENC and the ENB computed with a PCA, an MLT and the Fama-French factors using the inverse Herfindhal metric. In a second step, we present the diversification measures we obtain using a 1-year rolling window with a 1-week lag overlap, and we present 2-year and 5-year rolling windows with a 1-week lag overlap as well.

As a base case, we use the entropy metric to compute the ENC and the ENB. We propose now to look at the inverse Herfindhal metric (also called the "$L_2$-norm") calculated as follows:

\[
ENC_2(w) = \frac{1}{\sum_{k=1}^{N} (w_k^k)^2}
\]

\[
ENB_2(w, A) = \frac{1}{\sum_{k=1}^{N} (p_k^k)^2},
\]

where each \( p_k \) is a function of \( w_1, ..., w_N \).

The ENC and ENB measures computed with the inverse Herfindhal metric can be seen in Figure 16. On this figure we notice that the diversification measures obtained with the entropy metric (Figure 1) have similar fluctuations as the ones obtained using the inverse Herfindhal metric. We also use the inverse Herfindhal metric with a 1-year window without overlap in order to compute the ENC and ENB (using the MLT approach) of the equally-weighted version of the S&lP500 (see Figure 17). As for the cap-weighted S&lP500, we notice that the diversification measures computed with both metrics have similar fluctuations. However, we notice that the values obtained with the inverse Herfindhal metric are in general lower than the ones obtained with the entropy metric.

Finally, we present the results of diversification measures computed with a 1-year rolling window on a weekly basis (with overlap) and the results obtained using a 2-year and a 5-year rolling window with overlap (respectively in Figures 18, 19 and 20). We notice that the 2-year and 5-year rolling window periods with overlap lead to similar shapes for the diversification measures than the 1-year rolling window period with overlap. However, the longer the length of the rolling-window, the smoother the curves of the three diversification measures.

Therefore, from these robustness checks we can conclude that the findings we obtained with our diversification measures do not depend on our choice of the metric, nor on the length of the time period used to estimate the covariance matrix.
B. Technical Details

B.1 Proof of Lemma 1
Since the weights of a portfolio sum up to 1, then the mean of the weights is simply equal to 1/N, and the expression of the sample variance applied to the weights takes to the following form:

\[ N \text{Var}(w) = N \left( \frac{1}{N} \sum_{k=1}^{N} \left( w_k - \frac{1}{N} \right)^2 \right) \]

\[ = \sum_{k=1}^{N} \left( w_k^2 - 2w_k \frac{1}{N} + \frac{1}{N^2} \right) \]

\[ = \sum_{k=1}^{N} w_k^2 - 2 \frac{1}{N} \sum_{k=1}^{N} w_k + \frac{1}{N} = \sum_{k=1}^{N} w_k^2 - \frac{1}{N} \]

Then, we conclude the proof by recalling the expression of the ENC_2 measure:

\[ \text{ENC}_2(w) = \frac{1}{\sum_{k=1}^{N} w_k^2} = \frac{1}{N \text{Var}(w) + \frac{1}{N}}. \]

B.2 Derivation of the Minimal Linear Transformation
We look for matrix \( A \) such that the returns on the factors \( r_F \), defined as:

\[ r_F = A'r, \]  

are uncorrelated with a variance matrix \( A^*\Sigma A \) equal to a diagonal matrix \( D^2 \), and also such that the basis of the factors is the smallest linear torsion of the basis of the original assets in the sense that the sum of squared tracking errors between the new uncorrelated factors’ returns \( r_F \) and the original correlated assets’ returns \( r \) is minimal:

\[ A^* = \arg\min_A \left( \text{TE}\{r_{F1}, r_1\}^2 + \cdots + \text{TE}\{r_{FN}, r_N\}^2 \right) \]  

Matrix \( D^2 \) can be anything, e.g. one may want to impose that the factors have the same variance as the initial asset, and pick \( D^2 = \text{diag}(\Sigma) \), or one may want to impose that the factors have the same unit variance, in which case we pick \( D^2 = I_N \).

If we compute the sum of squared tracking errors, we have:

\[ \sum_{k=1}^{N} \text{TE}\{r_{Fk}, r_k\}^2 = \sum_{k=1}^{N} \text{Var}(r_{Fk} - r_k) \]

\[ = \sum_{k=1}^{N} \text{Var}(a_k'r - e_k'r) \]

\[ = \sum_{k=1}^{N} \text{Var}(a_k - e_k')r \]

\[ = \sum_{k=1}^{N} [a_k - e_k']\Sigma[a_k - e_k] \]

\[ = \text{tr}([A - I_N]'\Sigma[A - I_N]) \]

\[ = \text{tr}(A^*\Sigma A - A^*\Sigma - \Sigma A + \Sigma) \]

\[ = \text{tr}(D^2) + \text{tr}(\Sigma) - 2\text{tr}(A^*\Sigma) \]

where \( a_k \) is the kth column of matrix \( A \), and \( e_k \) the kth elementary vector.

Since \( \Sigma \) and \( D \) are given, so are their traces, then minimising the sum of the squared tracking errors is equivalent to maximising \( \text{tr}(A^*\Sigma) \). Now, if we use the PCA decomposition of \( \Sigma = PA^2P' \), we can rewrite \( \text{tr}(A^*\Sigma) \) as:

\[ \text{tr}(A^*\Sigma) = \text{tr}(D D^{-1}A'PA\Lambda P') \]

\[ = \text{tr}(Q'\Lambda P'D), \]

where \( Q \) satisfies the following property:

\[ Q'Q = I_N \].

Therefore we are now solving the following problem:

\[ Q^* = \arg\max_{Q \text{ s.t. } Q'Q = I_N} \text{tr}(Q'\Lambda P'D) \]  

8 - Note that we could also use the Cholesky decomposition as well, leading to: \( \Sigma = LL' \)
Appendices

In order to do so, we compute the singular value decomposition of $\Lambda P'D$, leading to:

$$\Lambda P'D = USV^\prime,$$  \hspace{1cm} (B.4)

where $U$ and $V$ are two orthogonal matrices, and $S$ is a diagonal matrix containing the singular values of $\Lambda P'D$. This leads to:

$$\text{tr}(Q'\Lambda P'D) = \text{tr}(Q'USV') = \text{tr}(V'Q'US)$$

where $Z = V'Q'U$ satisfies $ZZ' = I_N$. Hence, we obtain:

$$\text{tr}(V'Q'US) = \text{tr}(ZS) = \sum_{k=1}^{N} z_{kk}s_{kk} \leq \sum_{k=1}^{N} s_{kk}. \hspace{1cm} (B.5)$$

From Equation (B.5), it is straightforward to see that the maximum is attained for $z_{kk} = 1$ for all $k = 1, .., N$, leading to $Z = I_N$. Finally, we obtain $Q' = US$, which gives us:

$$A^* = P\Lambda^{-1}UV'D. \hspace{1cm} (B.6)$$

**Remark 1** In the simple case where we impose $D = I_N$, i.e. the factors have a unit variance, then the solution $A^*$ is simpler since we are left with the singular value decomposition of $\Lambda P'$. Noticing that $\Lambda$ is a diagonal matrix and $P$ an orthogonal matrix simply leads to: $U = I_N$, $S = \Lambda$ and $V = P$. Therefore, $Q^*$ is simply given by $UV' = P'$, and matrix $A^*$ takes the following expression:

$$A^* = P\Lambda^{-1}P',$$  \hspace{1cm} (B.7)

which coincides with the Riccati decomposition of Meucci (2009b).

**Remark 2** The problem of maximising $\text{tr}(Q'M)$ over orthogonal matrices $Q$ is also known as a particular case of the orthogonal Procrustes problem solved in Schönemann (1966). Note that in our case, $Q$ is not orthogonal so we cannot apply directly the results obtained by P. Schönemann.
C. Tables

Table 1: Correlations in % Among the Diversification Measures

<table>
<thead>
<tr>
<th></th>
<th>ENC</th>
<th>ENB-MLT</th>
<th>ENB-PCA</th>
<th>ENB-FFF</th>
</tr>
</thead>
<tbody>
<tr>
<td>ENC</td>
<td>100.00</td>
<td>11.82</td>
<td>-11.18</td>
<td>1.64</td>
</tr>
<tr>
<td>ENB-MLT</td>
<td>11.82</td>
<td>100.00</td>
<td>1.58</td>
<td>-29.10</td>
</tr>
<tr>
<td>ENB-PCA</td>
<td>-11.18</td>
<td>1.58</td>
<td>100.00</td>
<td>10.43</td>
</tr>
<tr>
<td>ENB-FFF</td>
<td>1.64</td>
<td>-29.10</td>
<td>10.43</td>
<td>100.00</td>
</tr>
</tbody>
</table>

This table displays the correlations (in %) of increment of the Effective Number of Constituents (ENC) and increments of the various versions of the Effective Number of Uncorrelated Bets (ENB) for the S&P500. The ENB has been computed with a PCA approach, an MLT approach, and uncorrelated Fama-French factors. The PCA and MLT methods are based on a robustified version of the sample covariance matrix estimated with rolling-windows of 52 weeks of constituents’ returns between December 1959 until the end of December 2012. Then the correlations that are reported in the table are computed with diversification measures obtained at a weekly frequency from December 1959 to December 2012.

Table 2: Cross-Sectional Analysis of the Relationship Between the Number of Constituents of 14 Equity Indices and their Diversification Measures

<table>
<thead>
<tr>
<th></th>
<th>ENC</th>
<th>ENB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficients</td>
<td>0.42</td>
<td>0.49</td>
</tr>
<tr>
<td>R-Squared (%)</td>
<td>96.06%</td>
<td>99.25%</td>
</tr>
<tr>
<td>t-stat</td>
<td>16.39</td>
<td>38.26</td>
</tr>
<tr>
<td>p-value (%)</td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
</tbody>
</table>

This table displays the diagnostics of the linear regression between the amount of constituents composing the 14 equity indices and the ENC and ENB of each index. The amount of constituents are the indices’ facial number of constituents. The diversification measures are the averages of diversification measures computed over the largest historical data-set available for each equity index.

Table 3: Correlations in % Between Diversification Measures and Economic Indicators

<table>
<thead>
<tr>
<th>Correlation</th>
<th>S&amp;P500 Vol.</th>
<th>GARCH Vol.</th>
<th>Credit Spread</th>
<th>Int. Rate</th>
<th>Term Spread</th>
<th>FFF1</th>
<th>FFF2</th>
<th>FFF3</th>
<th>FFF4</th>
</tr>
</thead>
<tbody>
<tr>
<td>ENC</td>
<td>1.90</td>
<td>-10.97</td>
<td>-7.95</td>
<td>-0.69</td>
<td>3.21</td>
<td>2.39</td>
<td>9.94</td>
<td>5.61</td>
<td>-4.42</td>
</tr>
<tr>
<td>ENB</td>
<td>-2.64</td>
<td>-0.72</td>
<td>-3.97</td>
<td>2.56</td>
<td>-3.52</td>
<td>-3.42</td>
<td>-6.38</td>
<td>-2.4</td>
<td>2.05</td>
</tr>
</tbody>
</table>

This table displays the correlations (in %) of three diversification measures of the S&P500 and several economic indicators. The diversification measures are the Effective Number of Constituents (ENC) and the Effective Number of Uncorrelated Bets (ENB) computed using an MLT approach. The economic indicators are the cumulated returns of the S&P500, the GARCH Volatility, the Term Spread, the Credit Spread, the Interest Rate and the four Fama-French factors (FFF). These correlations have been computed on the period ranging from January 1959 to December 2012.
Appendices

Table 4: Cross-Sectional Analysis of the Relationship Between 14 Equity Indices Performances and their Diversification Measures during the Subprime Crisis

<table>
<thead>
<tr>
<th></th>
<th>(a) ENC</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Δτ</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>End of August 2008</td>
<td>Previous Month</td>
<td>Previous Quarter</td>
<td>Previous Semester</td>
<td>Previous Year</td>
<td>Previous 2-Y</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coefficients</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R-Squared</td>
<td>17.12%</td>
<td>16.74%</td>
<td>16.56%</td>
<td>16.73%</td>
<td>17.08%</td>
<td>13.31%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>t-stat</td>
<td>1.51</td>
<td>1.49</td>
<td>1.48</td>
<td>1.48</td>
<td>1.51</td>
<td>1.24</td>
<td></td>
<td></td>
</tr>
<tr>
<td>p-value</td>
<td>15.99%</td>
<td>16.51%</td>
<td>16.76%</td>
<td>16.52%</td>
<td>16.04%</td>
<td>24.36%</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(b) ENB</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Δτ</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>End of August 2008</td>
<td>Previous Month</td>
<td>Previous Quarter</td>
<td>Previous Semester</td>
<td>Previous Year</td>
<td>Previous 2-Y</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coefficients</td>
<td>0.09</td>
<td>0.08</td>
<td>0.09</td>
<td>0.1</td>
<td>0.11</td>
<td>0.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R-Squared</td>
<td>24.71%</td>
<td>21.55%</td>
<td>25.99%</td>
<td>29.42%</td>
<td>35.34%</td>
<td>25.99%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>t-stat</td>
<td>1.9</td>
<td>1.74</td>
<td>1.97</td>
<td>2.14</td>
<td>2.45</td>
<td>1.87</td>
<td></td>
<td></td>
</tr>
<tr>
<td>p-value</td>
<td>8.39%</td>
<td>11.00%</td>
<td>7.51%</td>
<td>5.55%</td>
<td>3.21%</td>
<td>9.04%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

These two tables display the diagnostics of a linear regression between the annualised index performances of the 14 equity indices between September 2008 and February 2009 and their respective average ENC and ENB computed over different periods. Each average diversification measure is computed over six periods immediately preceding the period of computation of the annualised performance: on the previous week (at the end of August 2008), previous month, previous quarter, previous semester, previous year and previous 2-years. The statistical analysis displayed here excludes an outlier: the FTSE 100.

Table 5: Cross-Sectional Analysis of the Relationship Between 14 Equity Indices Performances and their Diversification Measures during the Recovery of the Subprime Crisis

<table>
<thead>
<tr>
<th></th>
<th>(a) ENC</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Δτ</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>End of August 2009</td>
<td>Previous Month</td>
<td>Previous Quarter</td>
<td>Previous Semester</td>
<td>Previous Year</td>
<td>Previous 2-Y</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coefficients</td>
<td>-0.43</td>
<td>-0.43</td>
<td>-0.43</td>
<td>-0.43</td>
<td>-0.41</td>
<td>-0.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R-Squared</td>
<td>27.12%</td>
<td>28.43%</td>
<td>28.76%</td>
<td>29.36%</td>
<td>29.34%</td>
<td>28.94%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>p-value</td>
<td>6.80%</td>
<td>6.06%</td>
<td>5.89%</td>
<td>5.58%</td>
<td>5.59%</td>
<td>5.79%</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(b) ENB</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Δτ</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>End of August 2009</td>
<td>Previous Month</td>
<td>Previous Quarter</td>
<td>Previous Semester</td>
<td>Previous Year</td>
<td>Previous 2-Y</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coefficients</td>
<td>-0.48</td>
<td>-0.45</td>
<td>-0.45</td>
<td>-0.45</td>
<td>-0.4</td>
<td>-0.61</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R-Squared</td>
<td>30.74%</td>
<td>27.97%</td>
<td>29.56%</td>
<td>20.49%</td>
<td>16.45%</td>
<td>27.68%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>t-stat</td>
<td>-2.21</td>
<td>-2.07</td>
<td>-2.15</td>
<td>-1.68</td>
<td>-1.47</td>
<td>-2.05</td>
<td></td>
<td></td>
</tr>
<tr>
<td>p-value</td>
<td>4.93%</td>
<td>6.31%</td>
<td>5.48%</td>
<td>12.04%</td>
<td>16.92%</td>
<td>6.49%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

These two tables display the diagnostics of a linear regression between the annualised index performances of the 14 equity indices between March 2009 and February 2010 and their respective average ENC and ENB computed over different periods. Each average diversification measure is computed over six periods immediately preceding the period of computation of the annualised performance: on the previous week (at the end of February 2009), previous month, previous quarter, previous semester, previous year and previous 2-years.
Table 6: Time-Series Analysis of the Relationship between the Performances of the S&P500 and its Diversification Measures

<table>
<thead>
<tr>
<th>Delta T</th>
<th>ENC Coefficients</th>
<th>ENC R-Squared</th>
<th>ENC t-stat</th>
<th>ENC p-value</th>
<th>ENB Coefficients</th>
<th>ENB R-Squared</th>
<th>ENB t-stat</th>
<th>ENB p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Following Quarter</td>
<td>Following Semester</td>
<td>Following Year</td>
<td>Following 2-Y</td>
<td>Following 5-Y</td>
<td>Following 10-Y</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.22</td>
<td>0.22</td>
<td>0.28</td>
<td>0.33</td>
<td>0.28</td>
<td>0.27</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.14%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Following Quarter</td>
<td>Following Semester</td>
<td>Following Year</td>
<td>Following 2-Y</td>
<td>Following 5-Y</td>
<td>Following 10-Y</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ENB</td>
<td>0.39</td>
<td>0.30</td>
<td>0.21</td>
<td>0.25</td>
<td>0.33</td>
<td>0.42</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.86%</td>
<td>0.96%</td>
<td>0.99%</td>
<td>2.86%</td>
<td>11.02%</td>
<td>32.00%</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4.98</td>
<td>5.25</td>
<td>5.31</td>
<td>9.02</td>
<td>17.98</td>
<td>33.25</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

These two tables display the diagnostics of the linear regression between the annualised performance of the S&P500 computed on different periods and its ENC and ENB. Each diversification measure is computed weekly over the whole historical data period of the S&P500. Annualised performances of the S&P500 are calculated at a weekly frequency on each quarter, each semester, each year, each 2-year period, each 5-year period and each 10-year period immediately following the dates of computation of each diversification measure.

Table 7: Descriptive Statistics of Asset Classes

(a) Risk and Performance Statistics

<table>
<thead>
<tr>
<th>Asset Class</th>
<th>Av. Ret. (%)</th>
<th>Volatility (%)</th>
<th>Sharpe Ratio</th>
<th>VaR5% (%)</th>
<th>VaR1% (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>US Bonds</td>
<td>6.29</td>
<td>4.45</td>
<td>0.83</td>
<td>0.98</td>
<td>1.58</td>
</tr>
<tr>
<td>Global Bonds</td>
<td>6.35</td>
<td>7.11</td>
<td>0.53</td>
<td>1.47</td>
<td>2.16</td>
</tr>
<tr>
<td>US HY Bonds</td>
<td>7.00</td>
<td>7.60</td>
<td>0.58</td>
<td>1.31</td>
<td>2.94</td>
</tr>
<tr>
<td>US IL Bonds</td>
<td>7.38</td>
<td>5.66</td>
<td>0.84</td>
<td>1.13</td>
<td>2.11</td>
</tr>
<tr>
<td>US MBS</td>
<td>6.11</td>
<td>3.12</td>
<td>1.12</td>
<td>0.65</td>
<td>1.25</td>
</tr>
<tr>
<td>US Eq.</td>
<td>5.02</td>
<td>19.23</td>
<td>1.03</td>
<td>4.27</td>
<td>6.91</td>
</tr>
<tr>
<td>Ex-US Eq.</td>
<td>5.18</td>
<td>19.96</td>
<td>0.13</td>
<td>4.21</td>
<td>8.05</td>
</tr>
<tr>
<td>Global Eq.</td>
<td>5.25</td>
<td>18.58</td>
<td>0.14</td>
<td>3.84</td>
<td>7.70</td>
</tr>
<tr>
<td>Private Eq.</td>
<td>7.72</td>
<td>23.06</td>
<td>0.14</td>
<td>5.30</td>
<td>9.29</td>
</tr>
<tr>
<td>Real Est.</td>
<td>9.21</td>
<td>25.73</td>
<td>0.22</td>
<td>4.95</td>
<td>11.75</td>
</tr>
<tr>
<td>Commo.</td>
<td>2.55</td>
<td>23.95</td>
<td>0.00</td>
<td>5.42</td>
<td>9.74</td>
</tr>
</tbody>
</table>

(b) Correlation in %

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>US Bonds</td>
<td>100.00</td>
<td>56.76</td>
<td>11.31</td>
<td>71.39</td>
<td>79.29</td>
<td>-19.44</td>
<td>-15.81</td>
<td>-17.96</td>
<td>-22.87</td>
<td>-12.54</td>
<td>-7.68</td>
</tr>
<tr>
<td>Global Bonds</td>
<td>56.76</td>
<td>100.00</td>
<td>1.83</td>
<td>52.16</td>
<td>48.40</td>
<td>-12.57</td>
<td>13.61</td>
<td>1.28</td>
<td>-11.81</td>
<td>-4.03</td>
<td>12.11</td>
</tr>
<tr>
<td>US HY Bonds</td>
<td>11.31</td>
<td>1.83</td>
<td>100.00</td>
<td>14.82</td>
<td>7.73</td>
<td>46.49</td>
<td>52.59</td>
<td>53.17</td>
<td>47.67</td>
<td>43.41</td>
<td>24.04</td>
</tr>
<tr>
<td>US IL Bonds</td>
<td>71.39</td>
<td>52.16</td>
<td>14.82</td>
<td>100.00</td>
<td>56.86</td>
<td>-12.24</td>
<td>-2.48</td>
<td>-6.94</td>
<td>-13.56</td>
<td>-4.42</td>
<td>8.52</td>
</tr>
<tr>
<td>US MBS</td>
<td>79.29</td>
<td>48.40</td>
<td>7.73</td>
<td>56.86</td>
<td>100.00</td>
<td>-9.21</td>
<td>-5.40</td>
<td>-7.22</td>
<td>-15.23</td>
<td>-8.55</td>
<td>-6.33</td>
</tr>
<tr>
<td>US Eq.</td>
<td>-19.44</td>
<td>-12.57</td>
<td>-12.24</td>
<td>-9.21</td>
<td>100.00</td>
<td>78.48</td>
<td>93.70</td>
<td>87.97</td>
<td>63.63</td>
<td>23.68</td>
<td>-7.68</td>
</tr>
<tr>
<td>Ex-US Eq.</td>
<td>-15.81</td>
<td>13.61</td>
<td>52.59</td>
<td>-2.48</td>
<td>78.48</td>
<td>100.00</td>
<td>94.99</td>
<td>74.04</td>
<td>55.53</td>
<td>37.50</td>
<td>-6.33</td>
</tr>
<tr>
<td>Global Eq.</td>
<td>-17.96</td>
<td>1.28</td>
<td>53.17</td>
<td>-6.94</td>
<td>93.70</td>
<td>94.99</td>
<td>100.00</td>
<td>85.34</td>
<td>62.42</td>
<td>27.87</td>
<td>33.15</td>
</tr>
<tr>
<td>Private Eq.</td>
<td>-22.87</td>
<td>-15.23</td>
<td>-15.36</td>
<td>-8.57</td>
<td>87.97</td>
<td>74.04</td>
<td>85.34</td>
<td>100.00</td>
<td>71.52</td>
<td>19.23</td>
<td>100.00</td>
</tr>
<tr>
<td>Real Est.</td>
<td>-12.54</td>
<td>-4.42</td>
<td>-4.42</td>
<td>-8.57</td>
<td>63.63</td>
<td>55.53</td>
<td>62.42</td>
<td>71.52</td>
<td>100.00</td>
<td>19.23</td>
<td>100.00</td>
</tr>
<tr>
<td>Commo.</td>
<td>-7.68</td>
<td>24.04</td>
<td>8.52</td>
<td>-6.33</td>
<td>23.68</td>
<td>37.50</td>
<td>33.15</td>
<td>27.87</td>
<td>19.23</td>
<td>100.00</td>
<td>100.00</td>
</tr>
</tbody>
</table>

The top table displays various statistics on the asset classes that are used in the empirical analysis. The average return and volatility are annualised, while both VaR computations are done with weekly returns. The risk-free return for the Sharpe ratio computation is the US T-bill with a 3-month maturity. The bottom table displays correlations between each asset class. All these statistics are computed over a data set that uses 787 weekly returns from September 1997 to the end of September 2012.
### Table 8: Factors' Exposures in % with the PCA Approach

<table>
<thead>
<tr>
<th>Asset Class</th>
<th>F1</th>
<th>F2</th>
<th>F3</th>
<th>F4</th>
<th>F5</th>
<th>F6</th>
<th>F7</th>
<th>F8</th>
<th>F9</th>
<th>F10</th>
<th>F11</th>
</tr>
</thead>
<tbody>
<tr>
<td>US Bonds</td>
<td>-2.00</td>
<td>-0.41</td>
<td>1.26</td>
<td>15.20</td>
<td>40.20</td>
<td>16.84</td>
<td>3.28</td>
<td>-14.70</td>
<td>-58.06</td>
<td>65.25</td>
<td>-2.76</td>
</tr>
<tr>
<td>Global Bonds</td>
<td>-0.39</td>
<td>-6.00</td>
<td>-0.87</td>
<td>38.02</td>
<td>51.41</td>
<td>-5.48</td>
<td>-38.02</td>
<td>48.48</td>
<td>17.64</td>
<td>-65.58</td>
<td>-9.97</td>
</tr>
<tr>
<td>US HY Bonds</td>
<td>9.58</td>
<td>-0.98</td>
<td>-0.76</td>
<td>13.06</td>
<td>17.99</td>
<td>1.20</td>
<td>91.78</td>
<td>29.94</td>
<td>8.18</td>
<td>-4.77</td>
<td>-1.15</td>
</tr>
<tr>
<td>US IL Bonds</td>
<td>-0.96</td>
<td>-3.63</td>
<td>2.27</td>
<td>20.52</td>
<td>48.48</td>
<td>17.64</td>
<td>3.85</td>
<td>-65.58</td>
<td>49.82</td>
<td>-9.97</td>
<td>-0.78</td>
</tr>
<tr>
<td>US MBS</td>
<td>-0.78</td>
<td>-0.07</td>
<td>-0.44</td>
<td>9.97</td>
<td>23.00</td>
<td>11.68</td>
<td>-1.12</td>
<td>-10.61</td>
<td>-59.45</td>
<td>-74.74</td>
<td>0.51</td>
</tr>
<tr>
<td>US Eq.</td>
<td>39.95</td>
<td>11.39</td>
<td>-27.21</td>
<td>-24.96</td>
<td>0.08</td>
<td>70.82</td>
<td>-7.99</td>
<td>15.46</td>
<td>8.07</td>
<td>-1.27</td>
<td>-39.07</td>
</tr>
<tr>
<td>Ex-US Eq.</td>
<td>39.81</td>
<td>-7.50</td>
<td>-36.57</td>
<td>55.78</td>
<td>-24.09</td>
<td>-34.97</td>
<td>-1.93</td>
<td>-16.02</td>
<td>-6.56</td>
<td>1.60</td>
<td>-42.44</td>
</tr>
<tr>
<td>Global Eq.</td>
<td>39.97</td>
<td>1.20</td>
<td>-32.51</td>
<td>16.67</td>
<td>-11.08</td>
<td>16.11</td>
<td>-2.72</td>
<td>-2.67</td>
<td>-0.91</td>
<td>2.81</td>
<td>81.61</td>
</tr>
<tr>
<td>Private Eq.</td>
<td>48.71</td>
<td>12.58</td>
<td>-8.18</td>
<td>-55.04</td>
<td>41.07</td>
<td>-51.49</td>
<td>-2.94</td>
<td>-4.39</td>
<td>-2.51</td>
<td>-0.56</td>
<td>-0.56</td>
</tr>
<tr>
<td>Real Est.</td>
<td>47.27</td>
<td>27.95</td>
<td>79.48</td>
<td>21.26</td>
<td>-12.02</td>
<td>6.96</td>
<td>-4.40</td>
<td>-0.76</td>
<td>-0.83</td>
<td>0.07</td>
<td>0.57</td>
</tr>
<tr>
<td>Commo.</td>
<td>22.74</td>
<td>-93.93</td>
<td>21.74</td>
<td>-11.74</td>
<td>-1.60</td>
<td>6.33</td>
<td>-1.25</td>
<td>0.59</td>
<td>-2.12</td>
<td>0.20</td>
<td>-0.25</td>
</tr>
<tr>
<td>Total Position (%)</td>
<td>243.90</td>
<td>-59.40</td>
<td>-1.80</td>
<td>98.76</td>
<td>173.23</td>
<td>55.64</td>
<td>38.54</td>
<td>-5.89</td>
<td>-43.77</td>
<td>-26.06</td>
<td>-2.91</td>
</tr>
<tr>
<td>Variance (%)</td>
<td>0.37</td>
<td>0.10</td>
<td>0.06</td>
<td>0.03</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Percent Explained (%)</td>
<td>63.35</td>
<td>17.01</td>
<td>9.46</td>
<td>4.26</td>
<td>2.15</td>
<td>1.59</td>
<td>1.33</td>
<td>0.51</td>
<td>0.25</td>
<td>0.08</td>
<td>0.02</td>
</tr>
<tr>
<td>Cumulative (%)</td>
<td>63.35</td>
<td>80.36</td>
<td>89.82</td>
<td>94.08</td>
<td>96.23</td>
<td>99.15</td>
<td>99.91</td>
<td>99.98</td>
<td>100.00</td>
<td>100.00</td>
<td>100.00</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
<td>0.20</td>
<td>0.10</td>
<td>0.14</td>
<td>0.33</td>
<td>0.95</td>
<td>0.11</td>
<td>0.33</td>
<td>-0.19</td>
<td>-0.32</td>
<td>-0.67</td>
<td>0.02</td>
</tr>
</tbody>
</table>

This table displays the exposures (in %) with respect to asset classes of each factor obtained with the PCA approach applied to the sample covariance matrix estimated with 787 weekly returns from September 1997 to the end of September 2012.

### Table 9: Factors' Exposures in % with the MLT Approach

<table>
<thead>
<tr>
<th>Asset Class</th>
<th>F1</th>
<th>F2</th>
<th>F3</th>
<th>F4</th>
<th>F5</th>
<th>F6</th>
<th>F7</th>
<th>F8</th>
<th>F9</th>
<th>F10</th>
<th>F11</th>
</tr>
</thead>
<tbody>
<tr>
<td>US Bonds</td>
<td>190.48</td>
<td>-32.15</td>
<td>-16.78</td>
<td>-43.95</td>
<td>-61.83</td>
<td>5.67</td>
<td>23.64</td>
<td>-24.20</td>
<td>3.50</td>
<td>0.21</td>
<td>2.27</td>
</tr>
<tr>
<td>US HY Bonds</td>
<td>-16.78</td>
<td>103.7</td>
<td>125.05</td>
<td>-9.50</td>
<td>1.02</td>
<td>14.37</td>
<td>-1.60</td>
<td>-28.78</td>
<td>-3.80</td>
<td>-6.03</td>
<td>-2.10</td>
</tr>
<tr>
<td>US MBS</td>
<td>-61.83</td>
<td>-7.13</td>
<td>1.02</td>
<td>6.80</td>
<td>161.21</td>
<td>-2.60</td>
<td>0.97</td>
<td>-3.21</td>
<td>2.91</td>
<td>0.29</td>
<td>0.93</td>
</tr>
<tr>
<td>Ex-US Eq.</td>
<td>23.64</td>
<td>-31.59</td>
<td>-1.60</td>
<td>9.37</td>
<td>0.97</td>
<td>370.93</td>
<td>636.22</td>
<td>-925.62</td>
<td>-65.7</td>
<td>-14.19</td>
<td>-14.46</td>
</tr>
<tr>
<td>Private Eq.</td>
<td>3.50</td>
<td>4.78</td>
<td>-3.80</td>
<td>4.02</td>
<td>2.91</td>
<td>-69.90</td>
<td>-6.57</td>
<td>-51.04</td>
<td>208.60</td>
<td>-65.50</td>
<td>-8.37</td>
</tr>
<tr>
<td>Real Est.</td>
<td>0.21</td>
<td>-1.46</td>
<td>-6.03</td>
<td>-1.40</td>
<td>0.29</td>
<td>-15.51</td>
<td>-14.19</td>
<td>4.13</td>
<td>-45.50</td>
<td>132.92</td>
<td>-0.93</td>
</tr>
<tr>
<td>Commo.</td>
<td>2.27</td>
<td>-2.14</td>
<td>-2.10</td>
<td>-2.88</td>
<td>0.93</td>
<td>8.55</td>
<td>-14.46</td>
<td>-10.42</td>
<td>-8.37</td>
<td>-0.93</td>
<td>106.27</td>
</tr>
<tr>
<td>Total Position (%)</td>
<td>46.86</td>
<td>73.37</td>
<td>82.22</td>
<td>54.52</td>
<td>85.75</td>
<td>96.74</td>
<td>47.10</td>
<td>-11.31</td>
<td>38.64</td>
<td>52.52</td>
<td>76.74</td>
</tr>
<tr>
<td>Variance (%)</td>
<td>0.00</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.00</td>
<td>0.07</td>
<td>0.08</td>
<td>0.07</td>
<td>0.10</td>
<td>0.13</td>
<td>0.11</td>
</tr>
<tr>
<td>Percent Explained (%)</td>
<td>66.66</td>
<td>1.66</td>
<td>1.89</td>
<td>1.05</td>
<td>0.32</td>
<td>12.12</td>
<td>13.06</td>
<td>11.32</td>
<td>17.44</td>
<td>21.70</td>
<td>18.80</td>
</tr>
<tr>
<td>Cumulative (%)</td>
<td>66.66</td>
<td>2.30</td>
<td>4.20</td>
<td>5.25</td>
<td>5.57</td>
<td>17.69</td>
<td>30.75</td>
<td>42.07</td>
<td>59.51</td>
<td>81.20</td>
<td>100.00</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
<td>0.25</td>
<td>0.40</td>
<td>0.50</td>
<td>0.62</td>
<td>0.92</td>
<td>-0.06</td>
<td>-0.02</td>
<td>0.03</td>
<td>0.21</td>
<td>0.21</td>
<td>-0.05</td>
</tr>
</tbody>
</table>

This table displays the exposures (in %) with respect to asset classes of each factor obtained with the MLT approach applied to the sample covariance matrix estimated with 787 weekly returns from September 1997 to the end of September 2012.
Appendices

Table 10: Analysis of Diversification Measures Based on Pension Funds’ Characteristics

<table>
<thead>
<tr>
<th></th>
<th>(a) Analysis Based on AUM</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2002</td>
<td>2007</td>
<td>2012</td>
</tr>
<tr>
<td>ENC</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>t-stat</td>
<td>14.23</td>
<td>16.70</td>
<td>10.85</td>
</tr>
<tr>
<td>p-value</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
<tr>
<td>ENB-MLT</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>t-stat</td>
<td>10.12</td>
<td>3.55</td>
<td>-3.70**</td>
</tr>
<tr>
<td>p-value</td>
<td>0.00%</td>
<td>0.04%</td>
<td>0.03%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>(b) Analysis Based on Public/Corporate Sector</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2002</td>
<td>2007</td>
<td>2012</td>
</tr>
<tr>
<td>ENC</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>t-stat</td>
<td>2.08</td>
<td>9.13</td>
<td>6.37</td>
</tr>
<tr>
<td>p-value</td>
<td>3.87%</td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
<tr>
<td>ENB-MLT</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>t-stat</td>
<td>1.88</td>
<td>0.60</td>
<td>-7.08**</td>
</tr>
<tr>
<td>p-value</td>
<td>6.14%*</td>
<td>55.13%*</td>
<td>0.00%</td>
</tr>
</tbody>
</table>

In panel (a), we perform a t-test of the null hypothesis that diversification measures of funds with low AUM and funds with high AUM are independent random samples from normal distributions with equal means, against the alternative that the means are not equal. In panel (b), we perform a t-test of the null hypothesis that diversification measures of public funds and corporate funds are independent random samples from normal distributions with equal means, against the alternative that the means are not equal. (***) shows that the null hypothesis cannot be rejected with a 95% confidence level. (*) shows that the null hypothesis is rejected, but the negative t-stat indicates a switch of the sector with the highest mean compared to positive t-stat.
## Table 11: Cross-Sectional Analysis of the Relationship Between US Pension Funds’ Performances and their Diversification Measures

### (a) All Performances from 28/09/2007 to 26/09/2008

<table>
<thead>
<tr>
<th>ENC</th>
<th>ENB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficients</td>
<td>46.06</td>
</tr>
<tr>
<td>R-Squared (%)</td>
<td>23.33</td>
</tr>
<tr>
<td>t-stat</td>
<td>-4.31</td>
</tr>
<tr>
<td>p-value (%)</td>
<td>0.00</td>
</tr>
</tbody>
</table>

### (b) All Performances from 05/09/2008 to 27/02/2009

<table>
<thead>
<tr>
<th>ENC</th>
<th>ENB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficients</td>
<td>-2.49</td>
</tr>
<tr>
<td>R-Squared (%)</td>
<td>6.91</td>
</tr>
<tr>
<td>t-stat</td>
<td>-7.60</td>
</tr>
<tr>
<td>p-value (%)</td>
<td>0.00</td>
</tr>
</tbody>
</table>

### (c) Performances from 28/09/2007 to 26/09/2008 under -5%

<table>
<thead>
<tr>
<th>ENC</th>
<th>ENB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficients</td>
<td>0.08</td>
</tr>
<tr>
<td>R-Squared (%)</td>
<td>0.12</td>
</tr>
<tr>
<td>t-stat</td>
<td>0.96</td>
</tr>
<tr>
<td>p-value (%)</td>
<td>33.57</td>
</tr>
</tbody>
</table>

### (d) Performances from 05/09/2008 to 27/02/2009 under -20%

<table>
<thead>
<tr>
<th>ENC</th>
<th>ENB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficients</td>
<td>-1.03</td>
</tr>
<tr>
<td>R-Squared (%)</td>
<td>1.88</td>
</tr>
<tr>
<td>t-stat</td>
<td>-3.82</td>
</tr>
<tr>
<td>p-value (%)</td>
<td>0.01</td>
</tr>
</tbody>
</table>

### (e) Performances from 28/09/2007 to 26/09/2008 over -5%

<table>
<thead>
<tr>
<th>ENC</th>
<th>ENB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficients</td>
<td>-3.62</td>
</tr>
<tr>
<td>R-Squared (%)</td>
<td>75.03</td>
</tr>
<tr>
<td>t-stat</td>
<td>-8.31</td>
</tr>
<tr>
<td>p-value (%)</td>
<td>0.00</td>
</tr>
</tbody>
</table>

### (f) Performances from 05/09/2008 to 27/02/2009 over -20%

<table>
<thead>
<tr>
<th>ENC</th>
<th>ENB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficients</td>
<td>15.04</td>
</tr>
<tr>
<td>R-Squared (%)</td>
<td>87.81</td>
</tr>
<tr>
<td>t-stat</td>
<td>11.39</td>
</tr>
<tr>
<td>p-value (%)</td>
<td>0.00</td>
</tr>
</tbody>
</table>

These tables display the diagnostics of the linear regressions between the annualised index performances of the US pension funds in the P&I database and their respective ENC and ENB at the end of September 2007. The performances are computed over two different periods: on the year immediately following the date of computation of the diversification measures (from 28/09/2007 to 26/09/2008) and during the worst of the subprime crisis (from 05/09/2008 to 27/02/2009). From top to bottom, we display the results made for all pension funds performances, for the pension funds that performed under -5% (from 28/09/2007 to 26/09/2008) and under -20% (from 05/09/2008 to 27/02/2009), and for the pension funds that performed over -5% (from 28/09/2007 to 26/09/2008) and over -20% (from 05/09/2008 to 27/02/2009). We consider that pension funds’ asset allocations has not not changed since the end of September 2007, therefore, the performances displayed here are only estimates.

## Table 12: Annualised Performance of Asset Classes’ Benchmarks (in %)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>05/09/2008 - 27/02/2009</td>
<td>-0.24</td>
<td>-0.40</td>
<td>-41.01</td>
<td>-14.03</td>
<td>11.06</td>
<td>-63.96</td>
<td>-64.49</td>
<td>-64.28</td>
<td>-69.74</td>
<td>-84.31</td>
<td>-81.76</td>
</tr>
</tbody>
</table>

This table displays the annualised performance of the indices used as asset class benchmarks for the US pension funds. The performances are computed over two different periods: on the year immediately following the date of computation of the diversification measures (from 28/09/2007 to 26/09/2008) and during the worst of the subprime crisis (from 05/09/2008 to 27/02/2009).
## Table 13: Allocation of the World’s 10 Biggest Pension Funds

### (a) Year 2007

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>GPIF</td>
<td>Japan</td>
<td>67.60%</td>
<td>8.10%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>13.70%</td>
<td>10.60%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
<tr>
<td>GPF</td>
<td>Norway</td>
<td>26.24%</td>
<td>22.74%</td>
<td>0.51%</td>
<td>4.75%</td>
<td>12.48%</td>
<td>33.27%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
<tr>
<td>ABP</td>
<td>Netherlands</td>
<td>10.30%</td>
<td>27.86%</td>
<td>0.00%</td>
<td>7.21%</td>
<td>0.00%</td>
<td>36.22%</td>
<td>5.15%</td>
<td>10.17%</td>
<td>3.09%</td>
<td>0.00%</td>
</tr>
<tr>
<td>KNPS</td>
<td>Korea</td>
<td>73.82%</td>
<td>8.14%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>15.51%</td>
<td>2.53%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
<tr>
<td>FRTIB</td>
<td>U.S.A</td>
<td>7.81%</td>
<td>26.36%</td>
<td>0.28%</td>
<td>1.10%</td>
<td>13.67%</td>
<td>47.72%</td>
<td>0.00%</td>
<td>0.33%</td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
<tr>
<td>CalPERS</td>
<td>U.S.A</td>
<td>22.78%</td>
<td>2.82%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>32.16%</td>
<td>26.71%</td>
<td>7.56%</td>
<td>7.16%</td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
<tr>
<td>Chikyoren</td>
<td>Japan</td>
<td>61.05%</td>
<td>7.57%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>17.64%</td>
<td>13.74%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
<tr>
<td>CPP</td>
<td>Canada</td>
<td>28.41%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>3.88%</td>
<td>15.09%</td>
<td>37.38%</td>
<td>8.26%</td>
<td>6.98%</td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
<tr>
<td>PFZW</td>
<td>Netherlands</td>
<td>13.95%</td>
<td>10.19%</td>
<td>1.41%</td>
<td>9.07%</td>
<td>13.10%</td>
<td>26.90%</td>
<td>5.74%</td>
<td>14.35%</td>
<td>5.30%</td>
<td>0.00%</td>
</tr>
<tr>
<td>CalSTRS</td>
<td>U.S.A</td>
<td>20.04%</td>
<td>0.00%</td>
<td>1.40%</td>
<td>0.00%</td>
<td>39.28%</td>
<td>21.74%</td>
<td>7.82%</td>
<td>9.72%</td>
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</table>

### (a) Year 2012

<table>
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<tr>
<td>GPIF</td>
<td>Japan</td>
<td>62.84%</td>
<td>10.23%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>13.47%</td>
<td>13.47%</td>
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<td>0.00%</td>
<td>0.00%</td>
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<tr>
<td>GPF</td>
<td>Norway</td>
<td>10.55%</td>
<td>26.36%</td>
<td>0.28%</td>
<td>1.10%</td>
<td>13.67%</td>
<td>47.72%</td>
<td>0.00%</td>
<td>0.33%</td>
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<td>ABP</td>
<td>Netherlands</td>
<td>21.27%</td>
<td>12.36%</td>
<td>2.73%</td>
<td>7.95%</td>
<td>5.49%</td>
<td>29.74%</td>
<td>5.68%</td>
<td>10.23%</td>
<td>4.55%</td>
<td>0.00%</td>
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<tr>
<td>KNPS</td>
<td>Korea</td>
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<td>5.06%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>20.53%</td>
<td>8.77%</td>
<td>0.00%</td>
<td>0.00%</td>
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<tr>
<td>FRTIB</td>
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<td>0.00%</td>
<td>0.00%</td>
<td>5.33%</td>
<td>68.96%</td>
<td>14.36%</td>
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<tr>
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<td>0.31%</td>
<td>1.78%</td>
<td>24.66%</td>
<td>28.44%</td>
<td>13.96%</td>
<td>8.71%</td>
<td>1.47%</td>
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<tr>
<td>Chikyoren</td>
<td>Japan</td>
<td>61.80%</td>
<td>10.29%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>15.61%</td>
<td>12.30%</td>
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<td>CPP</td>
<td>Canada</td>
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<td>1.30%</td>
<td>0.76%</td>
<td>5.33%</td>
<td>26.99%</td>
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<td>PFZW</td>
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<td>5.43%</td>
<td>8.21%</td>
<td>23.36%</td>
<td>7.17%</td>
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<td>0.20%</td>
<td>36.47%</td>
<td>15.73%</td>
<td>14.40%</td>
<td>14.40%</td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
</tbody>
</table>

This table displays the allocations (in risky asset classes) of the world’s 10 biggest pension funds in year 2007, and 2012, hand collected using public information gathered from their official website, from their financial statements and from their comprehensive annual reports. Alternative investments have been removed from these allocations.
### Table 14: Diversification Measures of the World's 10 Biggest Pension Funds Using a 5-Year Window for the Covariance in Year 2007 and 2012

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<th>2007</th>
<th>2012</th>
<th>Evolution</th>
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<tr>
<td><strong>(a) Effective Number of Constituents</strong></td>
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<tr>
<td>Government Pension Investment Fund (Japan)</td>
<td>2.66</td>
<td>2.90</td>
<td>9.05%</td>
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<td>Government Pension Fund (Norway)</td>
<td>4.42</td>
<td>3.66</td>
<td>-17.06%</td>
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<td>Stichting Pensioenfonds ABP (Netherlands)</td>
<td>5.16</td>
<td>6.99</td>
<td>35.48%</td>
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<tr>
<td>Korea National Pension Service (South Korea)</td>
<td>2.25</td>
<td>2.63</td>
<td>16.83%</td>
</tr>
<tr>
<td>Federal Retirement Thrift Investment Board (USA)</td>
<td>2.14</td>
<td>2.56</td>
<td>19.48%</td>
</tr>
<tr>
<td>California Public Employees (USA)</td>
<td>4.87</td>
<td>5.61</td>
<td>15.16%</td>
</tr>
<tr>
<td>Chikyoren (Japan)</td>
<td>2.93</td>
<td>2.94</td>
<td>0.37%</td>
</tr>
<tr>
<td>Canada Pension Plan (Canada)</td>
<td>4.61</td>
<td>5.55</td>
<td>20.39%</td>
</tr>
<tr>
<td>Stichting Pensioenfonds Zorg en Welzijn (Netherlands)</td>
<td>7.41</td>
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<td>1.62%</td>
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<tr>
<td>California State Teachers (USA)</td>
<td>4.51</td>
<td>4.90</td>
<td>8.60%</td>
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<table>
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<th>2007</th>
<th>2012</th>
<th>Evolution</th>
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</thead>
<tbody>
<tr>
<td><strong>(b) Effective Number of Bets</strong></td>
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<td></td>
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<tr>
<td>Government Pension Investment Fund (Japan)</td>
<td>4.03</td>
<td>4.33</td>
<td>7.38%</td>
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<tr>
<td>Government Pension Fund (Norway)</td>
<td>4.00</td>
<td>4.22</td>
<td>5.68%</td>
</tr>
<tr>
<td>Stichting Pensioenfonds ABP (Netherlands)</td>
<td>4.63</td>
<td>5.05</td>
<td>8.96%</td>
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<tr>
<td>Korea National Pension Service (South Korea)</td>
<td>3.00</td>
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<td>1.50%</td>
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<tr>
<td>Federal Retirement Thrift Investment Board (USA)</td>
<td>2.85</td>
<td>3.56</td>
<td>24.87%</td>
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<tr>
<td>California Public Employees (USA)</td>
<td>3.45</td>
<td>4.16</td>
<td>20.65%</td>
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<tr>
<td>Chikyoren (Japan)</td>
<td>3.52</td>
<td>3.76</td>
<td>6.63%</td>
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<tr>
<td>Canada Pension Plan (Canada)</td>
<td>3.80</td>
<td>3.90</td>
<td>2.55%</td>
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<tr>
<td>Stichting Pensioenfonds Zorg en Welzijn (Netherlands)</td>
<td>4.00</td>
<td>4.95</td>
<td>23.81%</td>
</tr>
<tr>
<td>California State Teachers (USA)</td>
<td>3.35</td>
<td>3.90</td>
<td>16.36%</td>
</tr>
</tbody>
</table>

In panel (a), we display the evolution of the effective number of constituents (ENC) from 2007 to 2012, and in panel (b) we do the same with the effective number of bets (ENB) computed with an MLT approach for the world's 10 biggest pension funds. The ENC measure is based on the weight values of the pensions' positions, while the ENB is based on uncorrelated factors. The MLT method uses the sample covariance matrix of 10 risky benchmarks representing the fund's asset allocation, and estimated with 5 years of historical returns.
Appendices

D. Figures

Figure 1: Diversification Measures for the S&P500 at a Yearly Frequency and Using Rolling-Windows of 1 Year for the Covariance (Entropy Metric)

These figures display the effective number of constituents (ENC) and the effective number of uncorrelated bets (ENB) for the S&P500 computed with a PCA approach, an MLT approach, and four uncorrelated Fama-French factors. The PCA and MLT methods are based on a robustified version of the sample covariance matrix estimated with rolling-windows of 52 weeks of constituents’ returns between January 1959 until the end of December 2012.
Appendices

Figure 2: Diversification Measures for the Equally-Weighted S&P500 at a Yearly Frequency and Using Rolling-Window of 1 Year for the Covariance (Entropy Metric)

These figures display the effective number of constituents (ENC) and the effective number of uncorrelated bets (ENB) computed with a PCA approach and an MLT approach for the equally-weighted scheme of the S&P500 constituents. The PCA and MLT methods are based on a robustified version of the sample covariance matrix estimated with rolling-windows of 52 weeks of constituents’ returns between January 1959 until the end of December 2012.

Figure 3: Nominal Number of Constituents in the S&P500 Index

This figure displays the nominal number of constituents in the S&P500 (white+black), together with the number of constituents available in the database CRSP that have a historical time series of one year at each point in time (white). The study covers the
time period between January 1958 and the end of December 2012.

Figure 4: Average Diversification Measures of 14 Equity Indices with respect to their Number of Constituents

These figures display the average effective number of constituents (ENC) and the average effective number of uncorrelated bets (ENB) of 14 equity indices computed over their respective whole historical data available with respect to the facial number of constituents of each equity index. For clarity, the FTSE All World is not displayed in this figure.
Appendices

Figure 5: Diversification Measures for the CAC40 at a Yearly Frequency and Using Rolling-Windows of 1 Year for the Covariance (Entropy Metric)

These figures display the effective number of constituents (ENC) and the effective number of uncorrelated bets (ENB) computed with an MLT approach for the CAC40. The MLT method is based on a robustified version of the sample covariance matrix estimated with rolling-windows of 52 weeks of constituents’ returns between July 2000 until the end of May 2013.
Appendices

Figure 6: Diversification Measures for the Stoxx Europe 600 at a Yearly Frequency and Using Rolling-Windows of 1 Year for the Covariance (Entropy Metric)

These figures display the effective number of constituents (ENC) and the effective number of uncorrelated bets (ENB) computed with an MLT approach for the Stoxx Europe 600. The MLT method is based on a robustified version of the sample covariance matrix estimated with rolling-windows of 52 weeks of constituents’ returns between August 1999 until the end of May 2013.
Appendices

Figure 7: Performances of 14 Equity Indices with respect to their Diversification Measures during the Subprime Crisis

These figures display the annualised performances of 14 equity indices during the worst of the subprime crisis (between the beginning of September 2008 and the end of February 2009) with respect to their respective effective number of constituents (ENC) and their effective number of uncorrelated bets (ENB) computed at the end of August 2008. These figures display the outlier (the FTSE 100) but the slope of the linear regression is computed without this outlier.
Figure 8: Performances of 14 Equity Indices with respect to their Diversification Measures during the Recovery of the Subprime Crisis

These figures display the annualised performances of 14 equity indices during the recovery of the subprime crisis (between the beginning of March 2009 and the end of February 2010) with respect to their respective effective number of constituents (ENC) and their effective number of uncorrelated bets (ENB) computed at the end of February 2009.
Appendices

Figure 9: Distribution of Diversification Measures of US Pension Funds Using a 5-Year Window for the Covariance in Year 2002, 2007 and 2012

These figures display the distribution of the effective number of constituents (ENC), and the effective number of bets (ENB) computed with a PCA approach, and an MLT approach for the US pension funds of the P&I database in year 2002, 2007 and 2012. The PCA and MLT methods are based on the sample covariance matrix of 11 risky benchmarks representing the fund’s asset allocation, and estimated with 5 years of historical returns.
Appendices

Figure 10: Distribution of Diversification Measures of US Pension Funds Using a 5-Year Window for the Covariance in Year 2002, 2007 and 2012 according to their amount of Assets Under Management (AUM)

These figures display the distribution of the effective number of constituents (ENC) and the effective number of bets (ENB) computed with a PCA approach, and an MLT approach for the US pension funds of the P&I database in year 2002, 2007 and 2012. The figures distinguish the distribution of each diversification measure according to the amount of assets under management in the pension funds. The distribution in red concerns the pension funds having the lowest 30% of assets under management, while the distribution in blue concerns the pension funds having the highest 30% of assets under management.
These figures display the distribution of the effective number of constituents (ENC) and the effective number of bets (ENB) computed with a PCA approach, and an MLT approach for defined benefit US pension funds of the P&I database in year 2002, 2007 and 2012. The figures distinguish the distribution of the diversification measures of public and corporate pension funds. The distribution in red concerns the corporate pension funds, while the distribution in blue concerns the public pension funds.
Figure 12: Performances of US Pension Funds with respect to their Diversification Measures at the End of September 2007

These figures display the annualised performances of the US pension funds of the P&I database computed on two different periods with respect to their diversification measures at the end of September 2007. The annualised performances are calculated on the year immediately following the date of computation of the diversification measures (from 28/09/2007 to 26/09/2008) and during the worst of the subprime crisis (from 05/09/2008 to 27/02/2009). We consider that pension funds’ asset allocations has not changed since the end of September 2007, therefore, the performances displayed here are only estimates.
Appendices

Figure 13: Performances of US Pension Funds from 28/09/2007 to 26/09/2008 with respect to their Diversification Measures at the end of September 2007

These figures display the annualised performances of US pension funds computed on the period immediately following the date of computation of the diversification measures (from 28/09/2007 to 26/09/2008) with respect to their diversification measures computed at the end of September 2007. For each diversification measure, we make the distinction between the US pension funds that performed under -5% and over -5% from 28/09/2007 to 26/09/2008. We consider that pension funds’ asset allocations has not changed since the end of September 2007, therefore, the performances displayed here are only estimates.
Figure 14: Performances of US Pension Funds from 05/09/2008 to 27/02/2009 with respect to their Diversification Measures at the end of September 2007

These figures display the annualised performances of US pension funds computed during the worst of the subprime crisis (from 05/09/2008 to 27/02/2009) with respect to their diversification measures computed at the end of September 2007. For each diversification measure, we make the distinction between the US pension funds that performed under -20% and over -20% from 05/09/2008 to 27/02/2009. We consider that pension funds’ asset allocations has not changed since the end of September 2007, therefore, the performances displayed here are only estimates.
Figure 15: Average Asset Allocations of US Pension Funds

These figures display the average asset allocation of US pension funds with respect to their return computed on two different periods. The two top figures concern the period from 28/09/2007 to 26/09/2008, while the two figures of the bottom concern the period from 05/09/2007 to 27/02/2009. For the period 28/09/2007-26/09/2008 (respectively 05/09/2008-27/02/2009), we display the average asset allocation of US pension funds that performed under a threshold of -5% (respectively -20%) on a first place and the average asset allocation of US pension funds that performed over this threshold on a second place.
Appendices

Figure 16: Diversification Measures for the S&P500 at a Yearly Frequency and Using Rolling-Windows of 1 Year for the Covariance (Inverse Herfindahl Metric)

These figures display the effective number of constituents (ENC) and the effective number of uncorrelated bets (ENB) of the S&P500 computed with a PCA approach, an MLT approach, and uncorrelated Fama-French factors. The PCA and MLT methods are based on a robustified version of the sample covariance matrix estimated with rolling-windows of 52 weeks of constituents’ returns between January 1959 until the end of December 2012.
Appendices

Figure 17: Diversification Measures for the Equally-Weighted S&P500 at a Yearly Frequency and Using Rolling-Windows of 1 Year for the Covariance (Inverse HerfindahlMetric)

These figures display the effective number of constituents (ENC) and the effective number of uncorrelated bets (ENB) computed with a PCA approach and an MLT approach of the equally-weighted scheme of the S&P500 constituents. The PCA and MLT methods are based on a robustified version of the sample covariance matrix estimated with rolling-windows of 52 weeks of constituents’ returns between January 1959 until the end of December 2012.
Appendices

Figure 18: Diversification Measures for the S&P500 at a Weekly Frequency and Using Rolling-Windowsof 1 Year for the Covariance (Entropy Metric)

These figures display the effective number of constituents (ENC) and the effective number of uncorrelated bets (ENB) of the S&P500 computed with a PCA approach, an MLT approach, and uncorrelated Fama-French factors. The PCA and MLT methods are based on a robustified version of the sample covariance matrix estimated with rolling-windows of 52 weeks of constituents’ returns between January 1959 until the end of December 2012.
Appendices

Figure 19: Diversification Measures for the S&P500 at a Weekly Frequency and Using Rolling-Windows of 2 Years for the Covariance (Entropy Metric)

These figures display the effective number of constituents (ENC) and the effective number of uncorrelated bets (ENB) of the S&P500 computed with a PCA approach, an MLT approach, and uncorrelated Fama-French factors. The PCA and MLT methods are based on a robustified version of the sample covariance matrix estimated with rolling-windows of 104 weeks of constituents’ returns between January 1969 until the end of December 2012.
Appendices

Figure 20: Diversification Measures for the S&P500 at a Weekly Frequency and Using Rolling-Windows of 5 Years for the Covariance (Entropy Metric)

These figures display the effective number of constituents (ENC) and the effective number of uncorrelated bets (ENB) computed with a PCA approach and an MLT approach of the S&P500. The PCA and MLT methods are based on a robustified version of the sample covariance matrix estimated with rolling-windows of 260 weeks of constituents’ returns between January 1959 until the end of December 2012.
Appendices
References

References


About CACEIS
About CACEIS

CACEIS is one of the world’s market leaders in asset servicing and the largest depositary bank and premier fund administrator in Europe, holding assets under custody of €2.5 trillion and assets under administration of €1.3 trillion for institutional and corporate clients. Through an extensive network of offices spanning Europe, North America and Asia, CACEIS offers a broad range of services covering depositary and custody, fund administration, middle office outsourcing, clearing, fund distribution support and issuer services. With strong yet steady expansion in terms of assets, clients and geographical coverage, and the solid support of our principal shareholder Crédit Agricole, CACEIS has proven itself a reliable and capable asset servicing partner.

The CACEIS group is focused on providing innovative servicing solutions that enable clients to achieve their growth and expansion objectives. The group has the business agility to give clients a first-starter advantage in today’s changing regulatory environment, and by handling clients’ non-core functions in an effective and efficient manner, CACEIS helps them focus on generating value for investors.
About EDHEC-Risk Institute
About EDHEC-Risk Institute

The Choice of Asset Allocation and Risk Management
EDHEC-Risk structures all of its research work around asset allocation and risk management. This strategic choice is applied to all of the Institute’s research programmes, whether they involve proposing new methods of strategic allocation, which integrate the alternative class; taking extreme risks into account in portfolio construction; studying the usefulness of derivatives in implementing asset-liability management approaches; or orienting the concept of dynamic “core-satellite” investment management in the framework of absolute return or target-date funds.

Academic Excellence and Industry Relevance
In an attempt to ensure that the research it carries out is truly applicable, EDHEC has implemented a dual validation system for the work of EDHEC-Risk. All research work must be part of a research programme, the relevance and goals of which have been validated from both an academic and a business viewpoint by the Institute’s advisory board. This board is made up of internationally recognised researchers, the Institute’s business partners, and representatives of major international institutional investors. Management of the research programmes respects a rigorous validation process, which guarantees the scientific quality and the operational usefulness of the programmes.

Six research programmes have been conducted by the centre to date:
• Asset allocation and alternative diversification
• Style and performance analysis
• Indices and benchmarking
• Operational risks and performance
• Asset allocation and derivative instruments
• ALM and asset management

These programmes receive the support of a large number of financial companies. The results of the research programmes are disseminated through the EDHEC-Risk locations in Singapore, which was established at the invitation of the Monetary Authority of Singapore (MAS); the City of London in the United Kingdom; Nice and Paris in France; and New York in the United States.

EDHEC-Risk has developed a close partnership with a small number of sponsors within the framework of research chairs or major research projects:
• Core-Satellite and ETF Investment, in partnership with Amundi ETF
• Regulation and Institutional Investment, in partnership with AXA Investment Managers
• Asset-Liability Management and Institutional Investment Management, in partnership with BNP Paribas Investment Partners
• Risk and Regulation in the European Fund Management Industry, in partnership with CACEIS
• Exploring the Commodity Futures Risk Premium: Implications for Asset Allocation and Regulation, in partnership with CME Group

Founded in 1906, EDHEC is one of the foremost international business schools. Accredited by the three main international academic organisations, EQUIS, AACSB, and Association of MBAs, EDHEC has for a number of years been pursuing a strategy of international excellence that led it to set up EDHEC-Risk Institute in 2001. This institute now boasts a team of 90 permanent professors, engineers and support staff, as well as 48 research associates from the financial industry and affiliate professors.

Founded in 1906, EDHEC is one of the foremost international business schools. Accredited by the three main international academic organisations, EQUIS, AACSB, and Association of MBAs, EDHEC has for a number of years been pursuing a strategy of international excellence that led it to set up EDHEC-Risk Institute in 2001. This institute now boasts a team of 90 permanent professors, engineers and support staff, as well as 48 research associates from the financial industry and affiliate professors.
Improved Risk Reporting with Factor-Based Diversification Measures — February 2014

About EDHEC-Risk Institute

- Asset-Liability Management in Private Wealth Management, in partnership with Coutts & Co.
- Asset-Liability Management Techniques for Sovereign Wealth Fund Management, in partnership with Deutsche Bank
- The Benefits of Volatility Derivatives in Equity Portfolio Management, in partnership with Eurex
- Structured Products and Derivative Instruments, sponsored by the French Banking Federation (FBF)
- Optimising Bond Portfolios, in partnership with the French Central Bank (BDF Gestion)
- Asset Allocation Solutions, in partnership with Lyxor Asset Management
- Infrastructure Equity Investment Management and Benchmarking, in partnership with Meridiam and Campbell Lutyens
- Investment and Governance Characteristics of Infrastructure Debt Investments, in partnership with Natixis
- Advanced Modelling for Alternative Investments, in partnership with Newedge Prime Brokerage
- Advanced Investment Solutions for Liability Hedging for Inflation Risk, in partnership with Ontario Teachers’ Pension Plan
- The Case for Inflation-Linked Corporate Bonds: Issuers’ and Investors’ Perspectives, in partnership with Rothschild & Cie
- Solvency II, in partnership with Russell Investments
- Structured Equity Investment Strategies for Long-Term Asian Investors, in partnership with Société Générale Corporate & Investment Banking

The philosophy of the Institute is to validate its work by publication in international academic journals, as well as to make it available to the sector through its position papers, published studies, and conferences.

Each year, EDHEC-Risk organises three conferences for professionals in order to present the results of its research, one in London (EDHEC-Risk Days Europe), one in Singapore (EDHEC-Risk Days Asia), and one in New York (EDHEC-Risk Days North America) attracting more than 2,500 professional delegates.

EDHEC also provides professionals with access to its website, www.edhec-risk.com, which is entirely devoted to international asset management research. The website, which has more than 58,000 regular visitors, is aimed at professionals who wish to benefit from EDHEC’s analysis and expertise in the area of applied portfolio management research. Its monthly newsletter is distributed to more than 1.5 million readers.

### EDHEC-Risk Institute: Key Figures, 2011-2012

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About EDHEC-Risk Institute

The EDHEC-Risk Institute PhD in Finance

The EDHEC-Risk Institute PhD in Finance is designed for professionals who aspire to higher intellectual levels and aim to redefine the investment banking and asset management industries. It is offered in two tracks: a residential track for high-potential graduate students, who hold part-time positions at EDHEC, and an executive track for practitioners who keep their full-time jobs. Drawing its faculty from the world’s best universities, such as Princeton, Wharton, Oxford, Chicago and CalTech, and enjoying the support of the research centre with the greatest impact on the financial industry, the EDHEC-Risk Institute PhD in Finance creates an extraordinary platform for professional development and industry innovation.

Research for Business

The Institute’s activities have also given rise to executive education and research service offshoots. EDHEC-Risk’s executive education programmes help investment professionals to upgrade their skills with advanced risk and asset management training across traditional and alternative classes. In partnership with CFA Institute, it has developed advanced seminars based on its research which are available to CFA charterholders and have been taking place since 2008 in New York, Singapore and London.

In 2012, EDHEC-Risk Institute signed two strategic partnership agreements with the Operations Research and Financial Engineering department of Princeton University to set up a joint research programme in the area of risk and investment management, and with Yale School of Management to set up joint certified executive training courses in North America and Europe in the area of investment management.

As part of its policy of transferring know-how to the industry, EDHEC-Risk Institute has also set up ERI Scientific Beta. ERI Scientific Beta is an original initiative which aims to favour the adoption of the latest advances in smart beta design and implementation by the whole investment industry. Its academic origin provides the foundation for its strategy: offer, in the best economic conditions possible, the smart beta solutions that are most proven scientifically with full transparency in both the methods and the associated risks.
Improved Risk Reporting with Factor-Based Diversification Measures — February 2014


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• Badaoui, S., Deguest, R., L. Martellini and V. Milhau. Dynamic Liability-Driven Investing Strategies: The Emergence of a New Investment Paradigm for Pension Funds? (February).

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• Blanc-Brude, F. and O.R.H. Ismail. Who is afraid of construction risk? (March).
• Lixia, L., L. Martellini, and S. Stoyanov. The relevance of country- and sector-specific model-free volatility indicators (March).
• Deguest, R., L. Martellini, and V. Milhau. The benefits of sovereign, municipal and corporate inflation-linked bonds in long-term investment decisions (February).
• Deguest, R., L. Martellini, and V. Milhau. Hedging versus insurance: Long-horizon investing with short-term constraints (February).
• Padmanaban, N., M. Mukai, L. Tang, and V. Le Sourd. Assessing the quality of asian stock market indices (February).
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• Almeida, C., and R. Garcia. Robust assessment of hedge fund performance through nonparametric discounting (June).
• Amenc, N., F. Goltz, V. Milhau, and M. Mukai. Reactions to the EDHEC study “Optimal design of corporate market debt programmes in the presence of interest-rate and inflation risks” (May).
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• Le Sourd, V. Performance of socially responsible investment funds against an efficient SRI Index: The impact of benchmark choice when evaluating active managers – an update (March).
• Martellini, L., V. Milhau, and A. Tarelli. Dynamic investment strategies for corporate pension funds in the presence of sponsor risk (March).

- Sender, S. Shifting towards hybrid pension systems: A European perspective (March).
- Blanc-Brude, F. Pension fund investment in social infrastructure (February).
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- Scherer, B. An integrated approach to sovereign wealth risk management (June).

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