Maximising the Volatility Return: A Risk-Based Strategy for Homogeneous Groups of Assets

June 2018
Table of Contents

Executive Summary ................................................................. 5
Diversification, Rebalancing and The Volatility Return ....................... 7
Properties of The Maximum Volatility Return Portfolio (MVR) ............. 11
Weighting Across Groups of Assets .............................................. 15
Empirical Implementation ........................................................ 19
Appendix ..................................................................................... 23
References ................................................................................... 31
About EDHEC-Risk Institute ......................................................... 35
Abstract

The long-term performance of any portfolio can be decomposed as the sum of the weighted average long-term return of its assets plus the volatility return of the portfolio. Hence, maximising the volatility return of portfolios of assets with similar characteristics, such as factor portfolios, yields an important increase in performance and risk-adjusted return relative to market-cap weighted factor portfolios. We derive the closed-form solution of the strategy as well as properties that relate it to other risk-based weighting rules, such as minimum variance and risk parity. In our empirical test, partitioning a universe of stocks into homogeneous groups and applying a block-wise maximum volatility return strategy (MVR) yields a more efficient portfolio than standard market-cap and equal-weighted indices.
About the Author

Daniel Mantilla-Garcia is Assistant Professor at Universidad de los Andes, School of Management and Research Associate at EDHEC-Risk Institute.
Executive Summary
It is well-known that the long-term return or growth rate of any portfolio can be decomposed as the sum of a pure portfolio return, called the volatility return (among many other names), plus the weighted average of its assets’ growth rates. Hence, maximising the volatility return of a portfolio also maximises its growth rate if all the assets in the portfolio have the same growth rate. The volatility return component is often more important than the asset return component in portfolios with homogeneous assets, such as factor portfolios.

We derive the weighting scheme that maximises the volatility return of any portfolio, and find that the formula is closely related to several well-known weighting schemes. For instance, for completely uncorrelated assets, the maximum volatility return rule (MVR) is equivalent to the equal-weighted (EW) rule, and when all correlations across assets are equal but different from zero, the MVR portfolio is a blend of the equal-weighted (EW) portfolio and the equal risk contribution (ERC) portfolio. Furthermore, when applied to groups of assets with exactly the same volatility, the MVR coincides with the Maximum Decorrelation (MDC), the Minimum Variance (MV), the Maximum Diversification (MD) portfolio of Choueifaty and Coignard (2008), and the equal-risk contribution portfolio (see Maillard, Roncalli, and Teïletche, 2010).

In our empirical illustration, we find that the risk-based strategies with the highest realised volatility return after the MVR portfolio are the MDC and the EW portfolios. The excess volatility return of the MDC and the EW portfolios represents circa 50% of their outperformance of the market-cap index, while the excess volatility return of the MV and MD strategies accounts for 18% and 25% of their long-term outperformance of market-cap portfolios.

The maximum volatility return portfolio (MVR) has some other interesting properties. It maximises the portfolio’s Sharpe ratio assuming that the expected returns of assets are proportional to their return variance, and it is a risk-based strategy that does not require estimating expected returns. Furthermore, maximising the volatility return also maximises portfolio diversification, as measured by the GLR diversification ratio (Goetzmann, Li, and Rouwenhorst, 2005). Maximising the volatility return is best suited to homogeneous groups of assets, such as factor portfolios. Hence, we propose partitioning assets into groups with approximately homogeneous growth rates, in order to maximise the volatility return in a block-wise fashion. We find that the block-wise MVR strategy applied on volatility deciles has superior average return, risk-adjusted performance and a lower level of risk than the market-cap and equal-weighted portfolios.
Diversification, Rebalancing and The Volatility Return
Diversification, Rebalancing and The Volatility Return

In this section, we discuss the concept of volatility return, and its relation with portfolio diversification and rebalancing benefits. Let \( \pi(t) \) denote the \( N \times 1 \) vector of risky assets returns at time \( t \), \( \mu(t) = E[\pi(t)] \) denote the vector of expected rates of return of the risky assets, and \( \Sigma(t) \) the \( N \times N \) covariance matrix where the element \( \Sigma_{i,j} = \sigma_{ij} \) denotes the covariance between assets \( i \) and \( j \). The growth rate of an asset \( i \) is \( \gamma_i(t) = \mu_i(t) - \frac{1}{2} \sigma_{ii}(t) \), where \( \sigma_{ii} \) denotes the variance of asset \( i \in \{1...N\} \). The growth rate is the continuously compounded rate of return, and is estimated as the geometric return average. The growth rate \( \gamma_i \) is a better proxy for the long-term value of an investment in asset \( i \) than its rate of return \( \mu_i \) (see appendix A.1 for an explanation).

Denote \( \pi(t) \) the \( N \times 1 \) weight vector of a portfolio composed by the \( N \) securities, such that \( \sum_{i=1}^{N} \pi_i(t) = 1 \). It can be shown that the expected return of the portfolio is \( \mu_\pi(t) = \sum_{i=1}^{N} \pi_i(t) \mu_i(t) \), the variance of the portfolio is \( \sigma_{\pi\pi}(t) = \pi'(t) \Sigma(t) \pi(t) \), and hence its growth rate is equal to

\[
\gamma_\pi(t) = \mu_\pi(t) - \frac{1}{2} \sigma_{\pi\pi}(t) \tag{1}
\]

as for any asset. By replacing \( \mu_i(t) = \gamma_i(t) + \frac{1}{2} \sigma_{ii}(t) \) in \( \mu_\pi(t) \) in definition (1) it follows that,

\[
\gamma_\pi(t) = \sum_{i=1}^{N} \pi_i(t) \gamma_i(t) + \gamma_\pi^*(t), \tag{2}
\]

\[
\gamma_\pi^*(t) = \frac{1}{2} \left( \sum_{i=1}^{N} \pi_i(t) \sigma_{ii}(t) - \sigma_{\pi\pi}(t) \right). \tag{3}
\]

Thus, the growth rate of a portfolio is equal to the weighted average of the assets growth rates plus the term \( \gamma_\pi^* \), which is always positive for long-only portfolios, i.e. \( \pi_i \geq 0 \) for all \( i \) (see Fernholz, 2002). The \( \gamma_\pi^* \) is a return in excess of the weighted average long-term return of the assets in the portfolio, and different authors have called it different names, including the “excess growth rate” (Fernholz and Shay, 1982), the “volatility return” (Willenbrock, 2011; Hallerbach, 2014), and the “diversification return” (Booth and Fama, 1992). In fact, notice that \( \gamma_\pi^* \) has a direct relationship with the following measure of portfolio diversification (see Goetzmann et al., 2005; Deguest, Martellini, and Meucci, 2013),

\[
\text{GLRD}(\pi) := (\text{GLR}(\pi))^{-1} = \sum_{i=1}^{N} \pi_i \sigma_{ii} \tag{4}
\]

Thus, a more diversified portfolio, as measured by (4), has a higher \( \gamma_\pi^* \) and viceversa. However, notice that if all correlations where equal to one (aka zero diversification potential), \( \gamma_\pi^* \) would still be positive and increasing with the average level of assets variances. On the other hand, if all volatilities were zero, \( \gamma_\pi^* \) would be zero as well, regardless of the correlation values, thus we follow Willenbrock (2011) and Hallerbach (2014) and refer to \( \gamma_\pi^* \) as the volatility return.

Notice that the growth rate decomposition (2) holds even for a buy-and-hold portfolio, for which the assets’ weights vary over time. Nonetheless, the volatility return is often associated with a return emanating from rebalancing, and some authors refer to it as a “rebalancing return” (e.g. Bouchey, Nemchinov, Paulsen, and Stein, 2012). The reason for this association is that any outperformance of a portfolio with fixed weights relative to a buy-and-hold portfolio with the same initial weights comes exclusively from its volatility return. To see this, consider the performance of any given portfolio \( \pi \) measured in shares of its buy-and-hold reference portfolio, instead of dollars. The relative performance of the rebalanced portfolio with fixed weights \( \pi \),
and value $Z_{FW}$ with respect to the buy-and-hold portfolio $Z_{BH}$ with the same initial weights $\pi$ is equal to

$$\log(Z_{FW}(t)/Z_{BH}(t)) = \sum_{s=0}^{t-1} \gamma^*_\pi(s) - D^\pi(A(t)), \quad \text{(5)}$$

where $A(t)$ denotes the vector of values of the $N$ assets at time $t$. From Jensen’s inequality it follows that $D^\pi(A(t))$ is always positive. It can be shown that for assets with homogeneous moments, $D^\pi(A(t)) \propto CSV^\pi(A)$, where CSV$^\pi(A)$ denotes the (weighted) cross-sectional variance of the assets values. Hence, $D^\pi(A(t))$ can be interpreted as a measure of dispersion of the asset values at each point in time. Thus, equation (5) shows that the relative performance of a constant-weighted portfolio and a buy-and-hold portfolio with the same initial weights can be decomposed as the sum of a strictly positive term, aka the cumulative volatility return and a strictly negative component or “discount” proportional to the dispersion of assets’ values. In a similar vein, Hallerbach (2014) provides an approximation of the difference in growth rates between portfolios $Z_{FM}$ and $Z_{BH}$ with the same initial weights, that he calls the rebalancing return, which is equal to the volatility return $\gamma^*_\pi$ minus a second strictly positive term he calls “dispersion discount”, which is proportional to the cross-sectional dispersion of asset’s growth rates. The difference between the volatility return and the dispersion discount can be positive or negative, depending on which term dominates. In his empirical illustration Hallerbach (2014) considers a portfolio composed of multiple asset classes with very different risk-return profiles (i.e. a stock market index, two long-short factor indices, a long-maturity bond index, a real estate index and a commodities index), and hence the dispersion discount tends to cancel the positive effect of $\gamma^*_\pi$. On the other hand, for portfolios of more homogeneous groups of assets such as value stocks in the same market, the volatility return is more likely to dominate. Recall that Hallerbach (2014)’s analysis and the decomposition in equation (5) above describe the relative performance of a portfolio with constant weights and a buy-and-hold portfolio with the same initial weights, and Hallerbach (2014) derivations assume constant growth rates over time. On the other hand, the decomposition in equation (2) of the growth rate of a portfolio also holds for time-varying growth rates and covariances and for portfolios with time-varying weights. Hence, a more general representation of the long-term outperformance of a portfolio with possibly varying weights and a potentially different initial allocation relative to a given benchmark, such as a cap-weighted portfolio (CW), can be described as the difference in the portfolios growth rates:

$$\gamma_\pi(t) - \gamma_{cw}(t) = \gamma^*_\pi(t) - \gamma^*_{cw}(t)$$

$$+ \sum_{i=1}^{N} \pi_i(t) \gamma_i(t) - \sum_{i=1}^{N} cw_i(t) \gamma_i(t) \quad \text{(6)}$$

where $cw_i$ is the market-cap weight of asset $i$. Thus, the relative performance of any two portfolios can be decomposed as the sum of the difference in the weighted average growth rates of their respective assets plus the excess volatility return $\gamma^*_\pi(t) - \gamma^*_{cw}(t)$. Notice that the difference of the last two terms in equation (6) can also be written as $\sum_{i=1}^{N} (\pi_i(t) - cw_i(t)) \gamma_i(t)$ and $\pi_i(t) - cw_i(t)$ are known as the “active weights” of a portfolio relative to the benchmark.
Diversification, Rebalancing and The Volatility Return
Properties of The Maximum Volatility Return Portfolio (MVR)
Maximising the growth rate of an investment is a good proxy for its long-term value (see appendix A.1 for an explanation on this point). For this reason, several authors consider that the maximisation of the portfolio's growth rate or Kelly criterion (Kelly, 1956) is a reasonable objective for long-term investors. Suppose for a moment that all the $N$ assets have the same long-term growth rate $\gamma$. In such case we can factorise in the definition (2) of the portfolio growth rate,

$$\gamma(\pi) = \frac{1}{N} \sum_{i=1}^{N} \pi_i(t) + \gamma^*_\pi(t).$$ (7)

Maximising the growth rate of the portfolio in general requires estimating the growth rates of individual assets, which can be very tricky. On the other hand, equation (7) shows that for groups of assets with homogeneous growth rates, maximising the total growth rate of the portfolio is equivalent to maximising the volatility return $\gamma^*_\pi$ (this is because the first term to the right of equation 7 is independent of the portfolio weights). Also, notice that maximising the excess volatility return $\gamma^*_\pi - \gamma^*_cw$ relative to any given benchmark portfolio $cw$ is equivalent to maximising the volatility return $\gamma^*_\pi$ because the volatility return of the benchmark $\gamma^*_cw$ is independent of the portfolio $\pi$. The maximisation of $\gamma^*_\pi$ does not require the estimation of the growth rates of assets, because $\gamma^*_\pi$ is solely a function of the elements of the covariance matrix and the portfolio weights.

Growth rates are time additive, which implies that maximising the time-average growth rate is equivalent to maximising the instantaneous growth rate at each point in time. Hence we can drop the time indices without loss of generality. Denoting the vector of variances as $\eta := \text{diag}(\Sigma)$, we can re-write the volatility return in vector form as $\gamma^*_\pi = \frac{1}{2} (\pi' \eta - \pi' \Sigma \pi)$, and the optimisation program that yields the maximum volatility return portfolio (MVR) is:

$$\pi_{mvr}^*: = \arg\max_{\pi} \pi' \eta - \pi' \Sigma \pi$$

$$s.t. \quad \pi \in \Omega$$

(8)

where the weight in the risk-free asset $r_f$ is determined implicitly by $1 - 1'\pi_{mvr}$, where $1$ denotes a $N \times 1$ vector of 1’s. As usual, the portfolio optimisation is often subject to a set of constraints $\pi \in \Omega$ that can be introduced to robustify the solution (e.g. regularisation constraints) or to comply with internal or external implementation rules, such as long-only constraints.

The MVR portfolio not only maximises the growth rate of the portfolio for groups of assets with homogenous growth rates, but it is also mean-variance optimal if expected returns are proportional to return variances. To see this, recall the optimisation program of a standard mean-variance investor:

$$\pi_{mvr}^*: = \arg\max_{\pi} \pi' \mu - \frac{\phi}{2} \pi' \Sigma \pi$$

$$s.t. \quad \pi \in \Omega$$

where $\phi$ is the investor’s coefficient of relative risk aversion. From the definition of the growth rate it follows that maximising the portfolio’s growth rate is equivalent to maximising the expected utility with a relative risk aversion parameter of $\phi = 1$. The latter value for the risk aversion parameter has been considered too low (e.g. Merton and Samuelson, 1974) resulting in a strategy deemed too aggressive even for most long-term investors. On the other
Properties of The Maximum Volatility Return Portfolio (MVR)

hand, from the optimisation programs above it is clear that the volatility return maximisation is equivalent to the mean-variance utility maximisation in the particular case of a risk aversion parameter $\phi = 2$ and assuming expected returns are proportional to their variances, i.e. $\mu \propto \eta$.

In appendix A.2 we derive the analytical closed-form solution of the MVR portfolio:

$$\pi_{mvr}^* = x \times \pi_u$$

where $x = \frac{1}{2}1^{\top}\Sigma^{-1}\eta$ is the allocation to the following portfolio,

$$\pi_u := \frac{\Sigma^{-1} \eta}{1^{\top}\Sigma^{-1}\eta}$$

and $1 - x$ is the allocation to the risk-free asset. Notice that the portfolio (9) is equal to the tangency portfolio $\pi_{tg} := \frac{\Sigma^{-1} \mu}{1^{\top}\Sigma^{-1}\mu}$ under the assumption that expected returns of stocks are proportional to their return variances, i.e. $\mu \propto \eta$.

The MVR portfolio is related to other risk-based strategies. In particular, in our empirical test we find that when applied to blocks of assets with similar volatilities, the MVR is very close to the maximum decorrelation portfolio (MDC)$^5$, which in the absence of constraints is equal to $\pi_{mde} = \frac{C^{-1}}{1^{\top}C^{-1}1}$. Indeed, for a set of assets with exactly equal variances, i.e. $\sigma_{ii} = \sigma_{jj} \forall i, j$, the minimum variance (MV) $\pi_{mv}^* = \frac{\Sigma^{-1} \mu}{1^{\top}\Sigma^{-1}\mu}$ and the max decorrelation portfolios are equivalent to the MVR portfolio.$^6$

In order to gain further intuition of the strategy, in appendix A.2 we show that in the particular case of a unique constant correlation across assets $\rho$ the MVR portfolio is a weighted average of an equal weighted portfolio, and an equal risk contribution portfolio (ERC)$^7$. Indeed, the weight of asset $i$ of the unconstrained MVR portfolio can be decomposed as follows:

$$[\pi_{mvr}]_i \propto \theta + \frac{\phi}{\sigma_i} \sum_{j=1}^N \sigma_j$$

where $\theta = \frac{1}{1 - \rho}$ is a constant proportional to the weight of the EW portfolio, and the weight of the ERC portfolio is proportional to $\varphi = N\frac{\rho}{N\rho + 1 - \rho}$. Notice that the weight of the EW portfolio is always positive, as the constant $\theta \geq \frac{1}{2}$ for any value of $\rho$, hence we can interpret the MVR portfolio as a tilted EW portfolio. However, the weight of the risk parity portfolio can be positive or negative depending on the sign of $\phi$, and the latter is negative for positive values of $\rho$ and positive for negative values of $\rho$, i.e. $\text{sign}(\rho) = -\text{sign}(\phi)$ (to see this, recall the lower bound of $\rho > -\frac{1}{N-1}$). Thus, loosely speaking, we can assert that whenever there are very good diversification opportunities (negative $\rho$) the MVR portfolio is tilted toward low volatility stocks. On the other hand, when correlations are positive, the MVR strategy is tilted toward riskier stocks. For a large number of assets, a tilt toward riskier assets, i.e. a negative weight to the ERC portfolio, is more likely to occur. The reason is that the lower bound for correlations $\rho > -\frac{1}{N-1}$ tends to zero from below as $N \rightarrow \infty$, thus the average correlation is likely to increase for larger $N$.

Notice that if all correlations among a set of assets were zero, i.e. $[C]_{i,j} = 0$ for all $i \neq j$ the MVR strategy is equivalent to the equal weighted portfolio (to see this notice $\phi = 0$ and $\theta = 1$ in equation 10).

The maximum diversification (MD) portfolio of Choueifaty and Coignard

---

5 - The max decorrelation portfolio is obtained by minimising portfolio variance assuming all assets’ variances are equal.

6 - To see this, notice that for $\eta = c\mathbf{1}$ the first term in the optimisation program (8) is a constant $c$ equal to the unique variance.

7 - It is well-known that for constant correlations the equal risk contribution portfolio is $\pi_{erc,i} \propto \sigma_i^{-1}$.
Properties of The Maximum Volatility Return Portfolio (MVR)

(2008) is mean-variance optimal under the assumption that the standard deviation (instead of the variance) is a good proxy for the expected returns of individual assets. Thus, the MVR portfolio is a more aggressive strategy that allocates more to stocks with higher variance. Nonetheless, the MD strategy is also equivalent to MV, MDC and MVR when applied to a set of assets with exactly the same variance. In our empirical tests, we find that the MVR is closer to the Maximum Decorrelation (MDC) than to the other risk-based strategies considered, i.e. Maximum Diversification (MD) and Minimum Variance (MV).

Short-selling and leverage constraints are very common in practice, and it is well known that unconstrained portfolio optimisation is very sensitive to parameter estimation error, resulting in poor out-of-sample performance. Furthermore, recall that the volatility return is known to be strictly positive (even ex-post) solely for long-only portfolios, i.e. $\pi_i > 0 \quad \forall i \in \{1...N\}$. Hence in our empirical illustrations we impose all weights to be positive for all the strategies considered (see appendix A.2 for details).
Weighting Across Groups of Assets
Maximising the Volatility Return: A Risk-Based Strategy for Homogeneous Groups of Assets — June 2018

Weighting Across Groups of Assets

The properties of the MVR portfolio implies that partitioning the universe of assets into \( K \) internally homogeneous groups in terms of long-term growth, constitute better candidates to maximise the volatility return. There are different alternatives to form homogenous groups of assets, using for instance economic sectors, or stock characteristics to form factor portfolios (e.g. value and growth).

In this article we choose to group stocks according to their current level of volatility.

Notice that, since \( \gamma_i = \mu_i - \frac{1}{2}\sigma_{ii} \), everything else equal, two stocks with similar volatilities are more likely to have similar growth rates than two stocks with very different volatilities. Furthermore, if expected returns have a cross-sectional relationship with return variance, grouping assets by their variance is likely to yield groups with roughly homogeneous growth rates, whether the relationship between risk and return is positive or negative. Thus we sort stocks according to their current level of volatility to form \( K \) volatility groups and denote \( G_k \) the group of stocks \( k \in \{1 \ldots K\} \).

Note that sorting stocks by volatility (or any other characteristic related to risk-factor exposures) uses the risk-return relation to form homogeneous groups of assets without having to estimate explicitly expected returns.

Assuming that the correlations between assets in different groups is zero, in appendix A.3 we show that the MVR optimisation across the full universe of assets is equivalent to \( K \) independent optimisations for the assets in each group yielding \( K \) MVR sub-portfolios \( \pi_k^* = \frac{\Sigma_k^{-1}\eta_k}{\sqrt{\Sigma_k^{-1}\eta_k\Sigma_k^{-1}\eta_k^T}} \), and that each of these sub-portfolios has a resulting weight in the overall portfolio equal to:

\[
\omega_k = \frac{1^T \Sigma_k^{-1} \eta_k}{1^T \Sigma^{-1} \eta} = \frac{GLRD(\pi_k^*)}{GLRD(\pi)} \quad (11)
\]

where \( \Sigma_k^{-1} \) is the covariance matrix of the \( k \)th group of stocks, \( \Sigma \) is the resulting block diagonal matrix with the \( K \) group-covariance matrices in the diagonal, and \( \eta_k \) is the vector of variances of the stocks in the \( k \)th group composed by \( n_k \) stocks. Hence the weight of each block in the MVR portfolio is proportional to its internal GLR diversification ratio, under the assumption of uncorrelated blocks.

It is common practice to determine the allocation of the overall portfolio to each of the different sub-portfolios in a two-step optimisation procedure; the first step consists in solving the \( K \) optimisations for each group of assets independently, and in the second step the investor is constrained to choose among the \( K \) assets created in the first step.

The second-step of the procedure is the optimal allocation across groups (AG):

\[
\omega^* := \arg \max_{\omega} f(\omega, \Sigma_{AG}) \quad \text{s.t.} \quad \sum_{i=1}^{K} \omega_k = 1
\]

where \( \Sigma_{AG} \) is the \( K \times K \) covariance matrix of the group portfolios and \( f \) is the objective function at this second step.

In our empirical implementation, we use for the two-step optimisation a minimum variance allocation criteria across the MVR block portfolios (i.e. \( \omega^* = \frac{\Sigma_{AG}^{-1} \eta}{\sqrt{\Sigma_{AG}^{-1} \eta \Sigma_{AG}^{-1} \eta^T}} \)), motivated by the findings in Ang, Hodrick, Xing, and Zhang (2006, 2009) that low volatility stocks outperform high volatility stocks.

Note that in general (without the uncorrelated blocks assumption), the
Weighting Across Groups of Assets

across groups covariance matrix can be calculated using the covariance matrix of the underlying assets as, \( \Sigma_{AG} = \mathcal{W}' \Sigma \mathcal{W} \), where, \( \mathcal{W} \) is the following \( N \times K \) sparse matrix with inputs obtained in the first optimisation step:

\[
\mathcal{W} = \begin{bmatrix}
\pi_{G_1}^* & 0 & \cdots & 0 \\
0 & \pi_{G_2}^* & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & \cdots & \cdots & \pi_{G_K}^*
\end{bmatrix}
\]
Weighting Across Groups of Assets
Empirical Implementation
In order to evaluate the behaviour of the block-wise MVR portfolio proposed, we run an out-of-sample Backtest using data from the Center for Research in Security Prices database (CRSP). The monthly data file used is constituted by market capitalisation and total returns of all stocks belonging to the S&P 500 index from January 1957 until December 2010. We construct portfolios each month using only the stocks that belong to the index at each point in time and that have no missing returns over the calibration period. Following Plyakha, Uppal, and Vilkov (2012) we use a calibration period of 10 years of monthly returns.

In order to estimate the covariance matrix we use a two step procedure. First we sort stocks into volatility deciles and estimate the covariance matrix of each decile independently, using the optimal statistical shrinkage methodology of Ledoit and Wolf (2003). Following Fernandes et al. (2012) we put together the block covariance matrix by setting all the correlations among stocks in different deciles equal to zero. Fernandes et al. (2012) find that, for min-variance portfolios, the loss of information incurred by ignoring the in-between group correlations is compensated by the gain in estimation precision due to the smaller number of parameters to estimate at once.

We compute the out-of-sample performance of the portfolios at month \( t \) with the weights calculated using data from month \( t - 121 \) up to month \( t - 1 \). Thus, our out-of-sample period spans from January 1967 to December 2010. All portfolios are rebalanced monthly to capture at most the volatility return but the optimal weights are recalculated every quarter to limit turnover (TO).

Each month we construct portfolios with all the stocks belonging to the index at each point in time and without missing returns over the past 10 years. Using this same universe of stocks for all portfolios we construct a market capitalisation weighted portfolio (CW), an equal weighted portfolio (EW), a minimum variance portfolio (MV), a Maximum Decorrelation portfolio (MDC) and a maximum volatility return portfolio (MVR). As discussed before, due to the block-wise structure used for the

<table>
<thead>
<tr>
<th>1966=12 – 2010=12</th>
<th>CW</th>
<th>EW</th>
<th>MV</th>
<th>MD</th>
<th>MDC</th>
<th>MV R</th>
<th>MV (MV)</th>
<th>MD(MV)</th>
<th>MDC(MV)</th>
<th>MV R(MV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gret</td>
<td>9.84</td>
<td>12.55</td>
<td>12.64</td>
<td>13.08</td>
<td>13.07</td>
<td>13.23</td>
<td>12.64</td>
<td>13</td>
<td>13.01</td>
<td>13.38</td>
</tr>
<tr>
<td>Vol</td>
<td>15.33</td>
<td>17.36</td>
<td>13.52</td>
<td>14.6</td>
<td>15.4</td>
<td>15.88</td>
<td>13.52</td>
<td>14</td>
<td>14.05</td>
<td>14.63</td>
</tr>
<tr>
<td>SR</td>
<td>0.44</td>
<td>0.55</td>
<td>0.71</td>
<td>0.69</td>
<td>0.65</td>
<td>0.64</td>
<td>0.71</td>
<td>0.71</td>
<td>0.7</td>
<td>0.7</td>
</tr>
<tr>
<td>eRet</td>
<td>0</td>
<td>2.71</td>
<td>2.8</td>
<td>3.24</td>
<td>3.23</td>
<td>3.39</td>
<td>2.8</td>
<td>3.15</td>
<td>3.17</td>
<td>3.54</td>
</tr>
<tr>
<td>TE</td>
<td>0</td>
<td>5.86</td>
<td>6.97</td>
<td>6.73</td>
<td>6.96</td>
<td>7.3</td>
<td>6.97</td>
<td>6.76</td>
<td>6.73</td>
<td>6.82</td>
</tr>
<tr>
<td>IR</td>
<td>Nan</td>
<td>0.46</td>
<td>0.48</td>
<td>0.46</td>
<td>0.46</td>
<td>0.4</td>
<td>0.46</td>
<td>0.47</td>
<td>0.47</td>
<td>0.51</td>
</tr>
<tr>
<td>ENC</td>
<td>92</td>
<td>440</td>
<td>61</td>
<td>78</td>
<td>81</td>
<td>67</td>
<td>61</td>
<td>69</td>
<td>69</td>
<td>58</td>
</tr>
<tr>
<td>( \gamma^*_{M} )</td>
<td>2.82</td>
<td>4.18</td>
<td>3.49</td>
<td>4.18</td>
<td>4.75</td>
<td>5.02</td>
<td>3.49</td>
<td>3.7</td>
<td>3.71</td>
<td>3.9</td>
</tr>
<tr>
<td>( \gamma^<em>_{M} - \gamma^</em>_{CW} )</td>
<td>0</td>
<td>1.35</td>
<td>0.66</td>
<td>1.35</td>
<td>1.92</td>
<td>2.19</td>
<td>0.66</td>
<td>0.87</td>
<td>0.88</td>
<td>1.07</td>
</tr>
<tr>
<td>PVR</td>
<td>Nan</td>
<td>49.97</td>
<td>23.78</td>
<td>41.74</td>
<td>59.64</td>
<td>64.82</td>
<td>23.78</td>
<td>27.76</td>
<td>27.93</td>
<td>30.4</td>
</tr>
</tbody>
</table>
covariance matrix, the MVR optimisation is equivalent to perform 10 independent optimisations and then weighting each of the 10 resulting portfolios proportionally to its GLR diversification ratio, as showed by equation (11). In the MV portfolio, the weight of each decile portfolio \( \omega_k \) is proportional to the inverse of their variance. For the MDC portfolio, the implied \( \omega_k \) are equal weights, 1/10. We also report another set of portfolios from the two-step optimisation procedure in which the deciles are weighted using the MV rule, instead of the implied \( \omega \) of the one-step optimisation. We denote those portfolios for instance as MVR(MV), for the portfolio allocating within groups using the MVR rule but weighting deciles with a MV rule. For the MV portfolio, the full MV optimisation and the MV(MV) are equivalent by construction.

Table 1 presents several performance statistics of the risk-based diversified portfolios mentioned. The MVR portfolio that maximises the volatility return ex-ante presents the largest average volatility return ex-post, which suggests that the estimation and optimisation procedure achieved their objective. This average result is confirmed by Figure 1 over time, which presents the cumulative volatility return of the diversified portfolios from the beginning of the out-of-sample period\(^{10}\).

While the block MV version of the MVR portfolio presents a lower volatility return, it is the highest among the three block MV portfolios. Furthermore, the highest Information ratio (IR), and Sharpe ratio (SR) over the sample are obtained by the block MV version of the MVR portfolio. The IR of the latter is 19% higher than the IR of the MV. The MDC(MV) is very close to the MVR(MV) as expected. The MD presents a similar level of average return than the MVR, but the percentage of it coming from the volatility return \( \gamma^*_\pi \) Section 0 is very different (35% vs. 58%).

Both the block MV and full versions of the MVR and MDC portfolios present an economically significant outperformance (eRet) of the cap-weighted portfolio of more than 3% per year, and their Sharpe ratio is between 36% to 50% higher than the Sharpe ratio of the CW portfolio. These portfolios also outperform the EW portfolio by around 30 basis points per annum, with an increase in Sharpe ratio of up to 20%. The MV portfolio presents, unsurprisingly the lowest annualised volatility (Vol) but its IR is the lowest after the one of the EW portfolio.

\[ r_i(t) = \log(1 + r_i(t)) - \sum_{t} n_t(t) \log(1 + r_p(t)) \]

\(^{10}\) Pal and Wong (2013) showed that the realised volatility return can be measured directly in discrete time, with no need of parameter estimation as.

**Figure 1:** Cumulative volatility return of diversified portfolios of all stocks in the S&P 500 index using monthly returns 1966-2010.
Empirical Implementation

The MVR portfolios present a higher turnover than the other portfolios, although maintaining the weights constant over a full quarter yields a reasonable figure in the range 81% — 84%. Hence, its outperformance relative to the CW benchmark after transaction costs is very likely to remain significant for this liquid universe of stocks. This behaviour is unsurprising, as the MVR is more sensible to changes in the estimates of the individual stock volatilities, as previously discussed. The last line of Table 1 (PEVR) shows the proportion of the excess volatility return $\gamma^*_\pi - \gamma^*_{cw}$ as a percentage of the total excess return (eRet) of each portfolio relative to the market cap-weighted portfolio (CW). We find that for the EW portfolio the excess volatility return is almost 50% which is similar in orders of magnitude of the augmented Fama-French alpha that Plyakha et al. (2012) report for the monthly rebalanced EW portfolio, i.e. 42% in their case. This figure is highest for the MVR portfolio, i.e. 58% over the sample period.

Finally, notice that despite its lower volatility return, the MVR(MV) presents a slightly higher return and a notably lower volatility than the full MVR portfolio. The higher return of the former is due to the fact that it allocates a higher proportion to the low volatility groups of stocks, which tend to outperform high volatility stocks.
A.1 Growth Rate as a Measure of Long Term Value

In the 1970s, the Geometric Brownian Motion (GBM) became in the seventies the preferred model for the evolution of stocks. The reason is that asset values are multiplicative processes

\[ S(t) = S(0) \prod_{i=0}^{t} (1 + R_i) = S(0) \prod_{i=0}^{t} \exp r_i = S(0) \exp \sum_{i=0}^{t} r_i \]

\[ 1 + R_t = \exp(r_t) \]

where \( r_i := \log S_{t+\Delta} - \log S_t \) and \( R_i := \frac{S_{t+\Delta} - S_t}{S_t} \). Assuming i.i.d. returns, for \( \Delta \to 0 \) \( S(t) \) converges to a GBM:

\[ S(t) = S(0)e^{\gamma t + \sigma W_t} = S(0)e^{(\mu - \frac{1}{2} \sigma^2) t + \sigma W_t} \]

\[ d \log S(t) = \gamma t + \sigma dW_t \]

\[ \frac{dS(t)}{S(t)} = \mu t + \sigma dW_t \]

where \( \mu \) is the instantaneous rate of return, \( \gamma \) is the growth rate of \( S \), \( \sigma \) is the volatility and \( W_t \) is a random Brownian Motion with difference distributed standard normal.

Notice that the growth rate and the rate of return, \( \gamma = \mu - \frac{1}{2} \sigma^2 \), have the same relation than the geometric and arithmetic return averages, i.e. \( \bar{G} \approx \bar{A} - \frac{1}{2} \bar{V} \), where \( \bar{A} \) is the arithmetic average, \( \bar{G} \) the geometric average and \( \bar{V} \) the return variance (see Becker, 2012, for instance).

Chambers and Zdanowicz (2014) point out that maximising the expected value of an investment, amounts to maximise the instantaneous expected rate of return \( \mu \), because the mean of the value \( S(t) \) is \( E[S(t)] = e^{\mu t} \). On the other hand, the median of the distribution of \( S(t) \) is \( e^{\mu t} \). While the intuition of an expected value as a proxy for the long-term value is appropriate for random variables with broadly symmetric distributions, such as returns (and other stationary series), in the case of the value of an investment this intuition is flawed. In fact, as Figure 3 illustrates, if \( \gamma > 0 \), as \( t \) increases the distribution of \( S(t) \) becomes more skewed to the right with an ever greater right tail. As a consequence, as time passes the mean of the distribution moves deeper into the right tail, exponentially diverging from the centre of the distribution, i.e. the median. To see this, notice that the ratio of the mean to the median is equal to \( e^{\frac{1}{2} \sigma \gamma t} \). Furthermore, notice that if \( \gamma > 0 \) then \( \lim_{T \to \infty} S(T) = \infty \), and if \( \gamma < 0 \) then \( \lim_{T \to \infty} S(T) = 0 \). On the other hand, \( \mu > 0 \) does not guarantee an increasing value of \( S \) in the long run, because if \( \sigma \) is large enough, \( \gamma \) could be negative even if \( \mu \) is positive. Hence, the growth rate \( \gamma \) is a better indicator of the long-term value of an investment than the rate of return \( \mu \).
A.2 Analytical Decomposition of the Maximum Volatility Return Portfolio (MVR)

In what follows, we derive the analytical formula of the MVR portfolio and then provide a decomposition under the assumption of constant correlations. Recall the portfolio vector $\pi$ represents the portfolio of risky assets. The augmented $(N + 1) \times 1$ portfolio vector $\tilde{\pi}$ includes the proportion of wealth invested in the risk-free asset $\pi_{rf}$. The corresponding augmented covariance matrix and vector of variances are:

$$
\tilde{\Sigma} = \begin{pmatrix} \Sigma & 0 \\ 0 & 0 \end{pmatrix}, \quad \tilde{\eta} = \text{diag}(\tilde{\Sigma}) = \begin{pmatrix} \eta \\ 0 \end{pmatrix}.
$$

The objective is to maximise the volatility return of the portfolio $\gamma_{\tilde{\pi}}^*$,

$$
\tilde{\pi}_{mer}^* := \arg\max_{\tilde{\pi}} \tilde{\pi}'\tilde{\eta} - \tilde{\pi}'\tilde{\Sigma}\tilde{\pi}
$$

subject to $1'\tilde{\pi} = 1$ (12)

Notice that the volatility return of $\tilde{\pi}$ is equal to the volatility return of $\pi$, i.e., $\gamma_{\tilde{\pi}}^* = \gamma_{\pi}^* = \pi'\eta - \pi'\Sigma\pi$, hence the optimisation program can be written as

$$
\max_{\pi, \pi_{rf}} \pi'\eta - \pi'\Sigma\pi
$$

subject to $1'\pi + \pi_{rf} = 1$.
for which the corresponding Lagrange function is:

$$\mathcal{L}(\pi, \pi_r, \lambda) = \pi'\eta - \pi'\Sigma\pi - \lambda(1'\pi + \pi_r - 1)$$

and the first order conditions are:

$$\partial_\pi \mathcal{L}(\pi, \pi_r, \lambda) = \eta - 2\Sigma\pi - \lambda = 0$$
$$\partial_{\pi_r} \mathcal{L}(\pi, \pi_r, \lambda) = \lambda = 0$$
$$\partial_\lambda \mathcal{L}(\pi, \pi_r, \lambda) = 1'\pi + \pi_r - 1 = 0$$

Thus, the solution of the optimisation program (12) is:

$$\pi^* = \frac{1}{2}\Sigma^{-1}\eta$$
$$\pi_r^* = 1 - \frac{1}{2}1'\Sigma^{-1}\eta$$

This solution can be re-written in terms of a portfolio \(\pi_u\) investing fully in the risky assets equal to,

$$\pi_u = \frac{\Sigma^{-1}\eta}{1'\Sigma^{-1}\eta}$$

(13)

and the proportion allocated to that portfolio is \(x := \frac{1}{2}1'\Sigma^{-1}\eta\). In other words, the MVR portfolio is

$$\tilde{\pi}_{mvr}^* = x \begin{pmatrix} \pi_u \\ 0 \end{pmatrix} + (1 - x) \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$  

(14)

In what follows, we decompose the structure of the MVR portfolio under the assumption of constant correlations across assets. The covariance matrix can be decomposed as:

$$\Sigma = S \circ C$$

(15)

where \(C\) is the correlation matrix and \(S = \sigma \sigma'\) and \([\sigma_i] = \sigma_i\) is the vector of volatilities, and the inverse covariance matrix \(\Sigma^{-1}\) can be decomposed as

$$\Sigma^{-1} = S^{o(-1)} \circ C^{-1}$$

(16)

where \(S^{o(-1)}\) is the Hadamard inverse of \(S\), i.e. \([S^{o(-1)}]_{i,j} := 1/[S]_{i,j} = \frac{1}{\sigma_i \sigma_j}\).
Appendix

In the particular case of a constant correlation across all assets $\rho$, $C$ can be written as

$$C = \rho \mathbf{1}\mathbf{1}' + (1 - \rho) I_N$$

(17)

where $I_N$ is the identity matrix of size $N$. Henderson and Searle (1981) showed that the inverse of a matrix such as (17) is

$$C^{-1} = \frac{1}{1 - \rho} \left( I_N - \frac{\rho}{1 + (N - 1)\rho} \mathbf{1}\mathbf{1}' \right).$$

(18)

Replacing equations (16) and (18) in (13), we find that the portfolio (13) can be written as a weighted average of an equal-weighted portfolio $\pi_{ew} = \left( \frac{1}{N} \right) \mathbf{1}$ and an equal risk contribution portfolio

$$\pi_{u} = x_{ew} \left( \frac{1}{N} \right) \mathbf{1} + x_{rp} \left( \sum_{i} \frac{1}{\sigma_i} \right)^{-1} \begin{pmatrix} \frac{1}{\sigma_1} \\ \vdots \\ \frac{1}{\sigma_N} \end{pmatrix}$$

where

$$x_{ew} = \frac{N \theta}{N \theta + \varphi \left( \sum_{i} \frac{N}{\sigma_i} \right) \left( \sum_{i} \frac{1}{\sigma_i} \right)}$$

and $x_{rp} = 1 - x_{ew}$ are the allocations to the equal-weighted and equal risk contribution portfolios which are proportional to the constants $\theta = \frac{1}{1 - \rho}$, and $\varphi = \frac{1 - \rho}{1 + (N - 1)\rho}$ respectively.

Sometimes the optimal allocation $x$ to $\pi_u$ is negative, and thus it is more intuitive to analyse the resulting optimal allocation to risky assets $\pi_{mvr}^* = x \times \pi_u = \frac{1}{2} \Sigma^{-1} \eta$.

$$\pi_{mvr}^* = \frac{1}{2} \begin{pmatrix} \frac{1}{\sigma_1} \\ \vdots \\ \frac{1}{\sigma_N} \end{pmatrix}$$

(19)

Notice that the constant $\theta \geq \frac{1}{2}$ is positive for any value of $\rho$, and hence the allocation to the EW portfolio, represented by the constant $\theta$ is always positive. The weight of the risk parity portfolio can be positive or negative depending on the sign of $\varphi$, and the latter is negative for positive values of $\rho$ and positive for negative values of $\rho$, i.e. $\text{sign}(\rho) = -\text{sign}(\varphi)$ (to see this, recall the lower bound of $\rho > \frac{1}{N - 1}$).

For ease of replicability of our empirical tests, we apply weight constraints on the analytical solutions of the optimal portfolios similar to the lambda constraints in Amenc, Goltz, Sivasubramanian, and Lodh (2014). Thus, in order to regularise the portfolios we use the normalised long leg of the unconstrained analytical solution. Thus, for the MVR

11 - It is well-known that for constant correlations the equal risk contribution portfolio is $[\pi_{erc}]_i \propto \sigma_i^{-1}$ (see for instance Roncalli, 2013).
strategy we use,

\[ \pi^*_{mv+} := \frac{\pi^+_u}{1' \pi^+_u} = \frac{\left(\Sigma^{-1} \eta\right)^+}{1' \left(\Sigma^{-1} \eta\right)^+} \]  

(20)

where \( \pi^+_u = \max(0, \pi^*_u) \) for all \( i \) and \( \pi^*_u = \frac{\Sigma^{-1} \eta}{1' \Sigma^{-1} \eta} \). Since the optimal allocation \( x \) to the portfolio \( \pi_u \) can be negative in some cases, we use the absolute value for the denominator of \( \pi_u \) to avoid flipping the signs of the weights when \( x < 0 \), as in DeMiguel, Garlappi, and Uppal (2009) (in their case for the tangency portfolio). Kirby and Ostdiek (2012) point out that the tangency portfolio yields very unstable and extreme weights when the denominator of \( \pi_t = \frac{\Sigma^{-1} \mu}{1' \Sigma^{-1} \mu} \) is close to zero, which also implies a high turnover merely induced by estimation risk. On the other hand, \( \pi^*_{mv+} \) is not subject to such problem, as the sum of the weights before normalisation \( 1' \pi^+_u \) is unlikely to be close to zero, since the negative weights are excluded in the sum of the denominator (this is also the case for the long-leg of the tangency portfolio).

For the MV, MD and MDC portfolios, we also use the corresponding normalised long leg of the analytical solution, thus the MV portfolio is implemented as

\[ \pi^*_{mv+,i} := c \times \max(0, \pi^*_{mv,i}) \text{ for all } i \in \{1...N\} \]  

(21)

where \( \pi^*_{mv} = \frac{\Sigma^{-1} \eta}{1' \Sigma^{-1} \eta} \) and \( c \) is the constant that ensures the positive weights sum to one. For the MDC portfolio we use the same formula (21), except that all volatility terms in \( \Sigma \) are set equal to a constant, thus \( \pi^*_{mdc} = \frac{C^{-1} \eta}{1' C^{-1} \eta} \), and for MD we start with \( \pi^*_{md} = \frac{\Sigma^{-1} \sigma}{1' \Sigma^{-1} \sigma} \) where \( \sigma \) is the vector of volatilities, and use also the resulting normalised long leg of the portfolio.

### A.3 Derivation of the Block-Wise MVR Portfolio

Equation (11) in the text states that the weight of each block in the overall MVR portfolio is proportional to its internal GLR diversification ratio, under the assumption that assets in different blocks are uncorrelated. In order to see this, we use the fact that the inverse of the now diagonal blockwise covariance matrix is equal to a diagonal blockwise matrix composed by the inverse matrices of the blocks. Hence, assuming that stocks are sorted by their current level of volatility the unconstrained MVR portfolio can be rewritten as,

\[
\pi_u(\Sigma) = [\Sigma_{G_1}^{-1}, \ldots, 0, \ldots] \cdot [\eta_{G_1}, \ldots, 0, \ldots] = \frac{1}{1' \Sigma^{-1} \eta} \begin{bmatrix}
\Sigma_{G_1}^{-1} & 0 & \cdots & 0 \\
0 & \Sigma_{G_2}^{-1} & \cdots & \\
\vdots & \vdots & \ddots & \\
0 & \cdots & \cdots & \Sigma_{G_K}^{-1}
\end{bmatrix}
\begin{bmatrix}
\eta_{G_1} \\
\vdots \\
\eta_{G_K}
\end{bmatrix} = \begin{bmatrix}
\omega_1 \pi^*_{G_1} \\
\vdots \\
\omega_K \pi^*_{G_K}
\end{bmatrix}
\]

where \( \pi^*_{G_k} = \frac{\Sigma_{G_k}^{-1} \eta_{G_k}}{1' \Sigma_{G_k}^{-1} \eta_{G_k}} \) and \( \omega_k = \frac{1'}{1' \Sigma^{-1} \eta} \) for \( k \in \{1...K\} \).
Furthermore, notice that for the MVR portfolio $\pi_u = \frac{\Sigma^{-1} \eta}{1^\top \Sigma^{-1} \eta}$, and the definition of the GLRD it follows that $GLRD_{\pi} = 1^\top \Sigma^{-1} \eta$ and for a block covariance matrix we have,

$$1^\top \Sigma^{-1} \eta = \sum_{k=1}^{K} 1^\top_k \Sigma_k^{-1} \eta_\theta_k$$

$$GLRD_{\pi} = \sum_{k=1}^{K} GLRD_{\pi_k}$$

Thus, the sum of the GLR diversification ratios of the uncorrelated block MVR portfolios is equal to the GLR diversification ratio of the overall MVR portfolio (for a block-wise covariance matrix $\Sigma$). In the case where all block sizes are equal to 1 security in every block, the GLR diversification ratio is by definition equal to 1 for every security and the optimal allocation to each “block” is $\omega_k = 1/N$ for all $k$, recovering the result that the MVR portfolio is equal to the EW portfolio when all correlations are nil.
Appendix
References
References

References

About EDHEC-Risk Institute
About EDHEC-Risk Institute

The Choice of Asset Allocation and Risk Management and the Need for Investment Solutions
EDHEC-Risk has structured all of its research work around asset allocation and risk management. This strategic choice is applied to all of the Institute’s research programmes, whether they involve proposing new methods of strategic allocation, which integrate the alternative class; taking extreme risks into account in portfolio construction; studying the usefulness of derivatives in implementing asset-liability management approaches; or orienting the concept of dynamic “core-satellite” investment management in the framework of absolute return or target-date funds. EDHEC-Risk Institute has also developed an ambitious portfolio of research and educational initiatives in the domain of investment solutions for institutional and individual investors.

Seven research programmes have been conducted by the centre to date:
- Investment Solutions in Institutional and Individual Money Management
- Equity Risk Premia in Investment Solutions
- Fixed-Income Risk Premia in Investment Solutions
- Alternative Risk Premia in Investment Solutions
- Multi-Asset Multi-Factor Investment Solutions
- Reporting and Regulation for Investment Solutions
- Technology, Big Data and Artificial Intelligence for Investment Solutions

These programmes receive the support of a large number of financial companies. The results of the research programmes are disseminated through the EDHEC-Risk locations in the City of London in the United Kingdom; Nice and Paris in France.

Academic Excellence and Industry Relevance
In an attempt to ensure that the research it carries out is truly applicable, EDHEC has implemented a dual validation system for the work of EDHEC-Risk. All research work must be part of a research programme, the relevance and goals of which have been validated from both an academic and a business viewpoint by the Institute’s advisory board. This board is made up of internationally recognised researchers, the Institute’s business partners, and representatives of major international institutional investors. Management of the research programmes respects a rigorous validation process, which guarantees the scientific quality and the operational usefulness of the programmes.

EDHEC-Risk has developed a close partnership with a small number of sponsors within the framework of research chairs or major research projects:
- ETF, Indexing and Smart Beta Investment Strategies, in partnership with Amundi
- Regulation and Institutional Investment, in partnership with AXA Investment Managers
- Asset-Liability Management and Institutional Investment Management, in partnership with BNP Paribas Investment Partners
- New Frontiers in Risk Assessment and Performance Reporting, in partnership with CACEIS
- Exploring the Commodity Futures

Founded in 1906, EDHEC is one of the foremost international business schools. Accredited by the three main international academic organisations, EQUIS, AACSB, and Association of MBAs, EDHEC has for a number of years been pursuing a strategy of international excellence that led it to set up EDHEC-Risk Institute in 2001. This institute now boasts a team of close to 50 permanent professors, engineers and support staff, as well as 39 research associates from the financial industry and affiliate professors.
About EDHEC-Risk Institute

Risk Premium: Implications for Asset Allocation and Regulation, in partnership with CME Group
• Asset-Liability Management Techniques for Sovereign Wealth Fund Management, in partnership with Deutsche Bank
• The Benefits of Volatility Derivatives in Equity Portfolio Management, in partnership with Eurex
• Innovations and Regulations in Investment Banking, sponsored by the French Banking Federation (FBF)
• Maximising and Harvesting the Rebalancing Premium in Equity Markets, in partnership with the French Central Bank (BDF Gestion)
• Risk Allocation Solutions, in partnership with Lyxor Asset Management
• Infrastructure Equity Investment Management and Benchmarking, in partnership with Meridiam and Campbell Lutyens
• Risk Allocation Framework for Goal-Driven Investing Strategies, in partnership with Merrill Lynch Wealth Management
• Investment and Governance Characteristics of Infrastructure Debt Investments, in partnership with Natixis
• Advanced Modelling for Alternative Investments, in partnership with Société Générale Prime Services (Newedge)
• Advanced Investment Solutions for Liability Hedging for Inflation Risk, in partnership with Ontario Teachers’ Pension Plan
• Cross-Sectional and Time-Series Estimates of Risk Premia in Bond Markets”, in partnership with PIMCO
• Active Allocation to Smart Factor Indices, in partnership with Rothschild & Cie
• Solvency II, in partnership with Russell Investments
• Structured Equity Investment Strategies for Long-Term Asian Investors, in partnership with Société Générale Corporate & Investment Banking

The philosophy of the Institute is to validate its work by publication in international academic journals, as well as to make it available to the sector through its position papers, published studies, and global conferences.

To ensure the distribution of its research to the industry, EDHEC-Risk also provides professionals with access to its website, https://risk.edhec.edu, which is entirely devoted to international risk and asset management research. The website is aimed at professionals who wish to benefit from EDHEC-Risk’s analysis and expertise in the area of applied portfolio management research. Its quarterly newsletter is distributed to more than 150,000 readers.


<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of permanent staff</td>
<td>48</td>
</tr>
<tr>
<td>Number of research associates &amp; affiliate professors</td>
<td>36</td>
</tr>
<tr>
<td>Overall budget</td>
<td>€6,500,000</td>
</tr>
<tr>
<td>External financing</td>
<td>€7,025,695</td>
</tr>
<tr>
<td>Nbr of conference delegates</td>
<td>1,087</td>
</tr>
</tbody>
</table>
| Nbr of participants at research seminars and executive education seminars | 1,465
About EDHEC-Risk Institute

Research for Business
The Institute’s activities have also given rise to executive education and research service offshoots. EDHEC-Risk’s executive education programmes help investment professionals to upgrade their skills with advanced risk and asset management training across traditional and alternative classes. In partnership with CFA Institute, it has developed advanced seminars based on its research which are available to CFA charterholders and have been taking place since 2008 in New York, Singapore and London.

In 2012, EDHEC-Risk Institute signed two strategic partnership agreements with the Operations Research and Financial Engineering department of Princeton University to set up a joint research programme in the area of asset-liability management for institutions and individuals, and with Yale School of Management to set up joint certified executive training courses in North America and Europe in the area of risk and investment management.

As part of its policy of transferring know-how to the industry, in 2013 EDHEC-Risk Institute also set up ERI Scientific Beta. ERI Scientific Beta is an original initiative which aims to favour the adoption of the latest advances in smart beta design and implementation by the whole investment industry. Its academic origin provides the foundation for its strategy: offer, in the best economic conditions possible, the smart beta solutions that are most proven scientifically with full transparency in both the methods and the associated risks.
EDHEC-Risk Institute
Publications and Position Papers
(2015–2018)

2018
• Mantilla-Garcia, D. Maximising the Volatility Return: A Risk-Based Strategy for Homogeneous Groups of Assets (June).
• Martellini, L. and V. Milhau. Smart Beta and Beyond: Maximising the Benefits of Factor Investing (February).

2017
• Amenc, N., F. Goltz, V. Le Sourd. EDHEC Survey on Equity Factor Investing (November).
• Maeso, J.M., Martellini, L. Maximising an Equity Portfolio Excess Growth Rate: A New Form of Smart Beta Strategy?
• Amenc, N., F. Goltz, V. Le Sourd. The EDHEC European ETF and Smart Beta Survey 2016 (May).
• Esakia, M., F. Goltz, S. Sivasubramanian and J. Ulahel. Smart Beta Replication Costs (February).

2016
• Amenc, N., F. Goltz, V. Le Sourd. Investor Perceptions about Smart Beta ETFs (August).
• Giron, K., L. Martellini and V. Milhau Multi-Dimensional Risk and Performance Analysis for Equity Portfolios (July).
• Maeso, J.M., L. Martellini. Factor Investing and Risk Allocation. From Traditional to Alternative Risk Premia Harvesting (June).
• Martellini, L. Mass Customisation versus Mass Production in Investment Management (January).

2015
• Amenc, N., G. Coqueret, and L. Martellini. Active Allocation to Smart Factor Indices (July).
• Goltz, F., and V. Le Sourd. Investor Interest in and Requirements for Smart Beta ETFs (April).

• Amenc, N., F. Ducoulombier, F. Goltz, V. Le Sourd, A. Lodh and E. Shirbini. The EDHEC European Survey 2014 (March).
• Blanc-Brude, F., and M. Hasan. The Valuation of Privately-Held Infrastructure Equity Investments (January).

2016 Position Paper
• Amenc, N., F. Ducoulombier, F. Goltz and J. Ulahel. Ten Misconceptions about Smart Beta (June).
• O’Kane, D. Initial Margin for Non-Centrally Cleared OTC Derivatives (June).