Construction Rules of Retirement Goal-Based Investing Indices

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EDHEC-RISK Institute

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Contents

Meaningful investment solutions should start with an understanding of clients' goals. In retirement planning, the main problem faced by individuals is to finance a sufficient and stable stream of replacement income in retirement. EDHEC-Risk Institute is grateful to Merrill Lynch for having supported the research project that has provided the conceptual foundations for the design of the EDHEC-Princeton Retirement Goal-based Investing Index series. The Retirement Goal-Based Investing Indices, developed with Princeton University Operations Research and Financial Engineering Department in the context of our joint research program on Investment Solutions for Institutions and Individuals, are an example of implementation of these concepts.

This document presents the second family of EDHEC-Princeton Retirement Goal-Based Investing Indices. The Retirement Goal-Based Investing Indices represent the performance of dynamic strategies that aim to deliver high probabilities of reaching high levels of wealth upon retirement, or high levels of replacement income for a period equal to life expectancy at retirement. Moreover, these strategies control short-term risk by preserving 80% of the purchasing power of accumulated savings in terms of replacement income or retirement wealth on an annual basis. At each point in time, the maximum retirement capital or replacement income that can be financed with savings is obtained by dividing savings by the price of one dollar of wealth or income in the future: the other family of EDHEC-Princeton Retirement Goal-Based Investing Indices, which consists of Retirement Goal Price Indices, precisely aims to measure this price.

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1. Building Blocks of Goal-Based Investing Strategy

A Retirement Goal-Based Investing Index represents the performance of a dynamic goal-based investing (in short, GBI) strategy. This section describes the two building blocks of a GBI strategy: the goal-hedging portfolio and the performance-seeking portfolio. The next section will present the allocation strategy.

Goal-Based Investing Strategies at a Glance

GBI strategies are studied in depth in Deguest et al. (2015) and Martellini and Milhau (2016), and they pursue a twofold objective of performance and safety, not in absolute terms but relative to an objective that is relevant to individual investors. The goal of interest in retirement planning is either to generate a target replacement income for a fixed period in retirement (income goal) or to reach a target wealth level by retirement (wealth goal). Therefore, a GBI strategy must secure a minimum income or wealth level by giving the opportunity to reach a higher level.

To accomplish this goal, the strategy makes use of two building blocks: a goal-hedging portfolio and a performance-seeking one. The former must guarantee a fixed income for a specified period that corresponds to the life expectancy of a recently retired individual, or deliver a fixed wealth on retirement leave. The latter is intended to generate performance beyond the goal-hedging portfolio in order to reach attractive amounts of income or wealth. In addition to the use of appropriate building blocks, the GBI approach also requires a sound investment process that reliably secures the minimum level while giving access to the upside potential of the performance-seeking component.

Goal-Hedging Portfolio

The goal-hedging portfolio (in short, GHP) must deliver a fixed replacement income for a predefined period, or a fixed capital at the retirement date. Without loss of generality, the income level can be normalized to \$1 per year and the wealth level to \$1: to secure an arbitrary goal, it then suffices to purchase as many units of the GHP as desired dollars of wealth or annual income. The price of \$1 of retirement income or wealth is precisely the value of an EDHEC-Princeton Goal Price Index. A Goal Price Index is the present value of future income or wealth cash flows. For the income goal, it is the price of a hypothetical "retirement bond" that pays \$1 every year in retirement and makes no final redemption payment. For the wealth goal, a Goal Price Index is the price of a zero-coupon bond that pays \$1 at the retirement date. The Construction Rules of Goal Price Indices (EDHEC-Risk Institute 2018) describe the cash-flow schedule of retirement bonds in detail.

A Goal Price Index associated with an income goal is determined by five characteristics: the geographic zone, the duration of the decumulation period, the provision for cost-of-living adjustment, the inception date and the retirement date. This information defines an investor's profile. To write the mathematical expression for the index, denote the inception date as date 0 and the retirement date as date T, and let τ be the decumulation period and π be the annual growth rate in cash flows. Let also $y(t, s-t)$ denote the nominal zero-coupon rate to be applied at date t to a cash flow occurring

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at date s. Then, the price of the retirement bond is

$$P_t = \sum_{s=T}^{T+\tau-1} e^{-[s-t]y(t,s-t)}[1+\pi]^s. \qquad (1)$$

For a wealth goal, the retirement bond price is still given by this formula, where the decumulation period is set to one year: a one-year decumulation means that the investor targets a single cash flow.

For a non-zero π, the bond price is distinct from the Goal Price Index, defined by

$$\beta_t = \frac{P_t}{[1+\pi]^t}. \qquad (2)$$

The difference can be put as follows: P_t is the price of \$1 of replacement income expressed in dollars of date 0, while β_t is the price of \$1 of income in dollars of date t. The GHP must replicate the bond, so that its returns must match those of the bond. In practice, it can be synthesized by constructing a cash-flow or a duration-matching matching portfolio portfolio, or by equating durations and convexities for a better replication.

Performance-Seeking Portfolio

The performance-seeking portfolio (PSP) is intended to outperform the GHP in the long run, but unlike the GHP, it is not tailored to a particular investor's profile, except for the geographic zone, which is taken to be identical to that of the GHP in order to avoid currency hedging issues.

Since Goal-Based Investing Indices have United States as their geographic zone, the PSP is taken to be a broad US equity index, which is weighted by capitalization because cap-weighted indices serve as a standard reference for the evaluation of the performance of equity strategies. Specifically, the PSP is the ERI Scientific Beta US cap-weighted index (see Section 6 for data sources).

The choice of a widespread reference is intended to be relatively neutral, as opposed to being optimal. It has been widely documented by research on equity investing that cap-weighted indices are dominated by a number of alternative strategies taking advantage from the links between certain characteristics and expected returns, and/or exploiting the properties of certain weighting schemes. For instance, "smart factor indices" introduced by Amenc, Goltz, and Lodh (2012); Amenc et al. (2014) combine a stock selection step to filter in stocks with a characteristic known to positively impact expected returns with a weighting step that aims to optimize the reward per unit of risk taken: the cited work shows that these indices post better long-term average returns and Sharpe ratios than their broad cap-weighted counterparts.

It is also possible in principle to construct a multi-class PSP in order to capture the

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risk premia found in non-equity asset classes. Martellini and Milhau (2015) present a framework for the construction of well-diversified portfolios containing equities, sovereign bonds and commodities.

Index Characteristics

A Goal-Based Investing Index inherits the five characteristics of the GHP. The geographic zone is the United States. For an income goal, the decumulation period is set to 20 years, which roughly corresponds to the life expectancy of a US individual at 65.[1] For a wealth goal, the decumulation period is 1 year. As far as the cost-of-living adjustment is concerned, two versions of each index are available: the basic one assumes no adjustment – so that cash flows are equal to $1 in constant dollars – and the adjusted one includes an annual growth rate of 2% in order to make up for the expected effects of inflation on the prices of consumption goods and services.

1 – The exact life expectancy is 19.4 years (National Vital Statistics Reports 2017).

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The allocation strategy to the building blocks (goal-hedging portfolio and performance-seeking portfolio, abbreviated as GHP and PSP) aims to secure a minimum level of replacement income or retirement wealth while giving the opportunity to eventually reach a higher level. The general investment principle is similar to dynamic core-satellite management (see Amenc, Malaise, and Martellini (2004) for a presentation), and it consists in a dynamic management of the tracking error with respect to the GHP. Moreover, the strategy respects the general allocation pattern of deterministic target date funds – while deviating from it to secure the minimum – by letting the allocation to the PSP follow a decreasing trend over time. This choice ensures that the strategy is anchored with respect to standard forms of target date funds.

Essential Goal and Floor

The goal-based investing (GBI) strategy aims to secure a minimum level of replacement income or retirement wealth known as an *essential level*. A key risk management indicator is the "present value" of the minimum amount to protect, in other words the capital required to secure the desired stream of income or amount of wealth. Indeed, in the absence of arbitrage opportunities, an essential goal cannot be secured with a 100% confidence level if the portfolio value falls short the minimum capital required at some point.

The essential goal that the GBI strategy aims to protect can be put as follows: secure at least 80% of the purchasing power of retirement savings in terms of replacement income or retirement wealth on an annual basis. This means that the purchasing power should never decrease by more than 20% over a year, from one beginning of January to the next. Thus, the portfolio value should be greater than or equal to the minimum wealth needed to have 80% of purchasing power at the beginning of the next year. This condition is of course verified when a new year begins, but it is precisely the objective of the investment policy to make sure that it holds at any point within the year.

Mathematically, let date 0 be the inception date and take the year as the time unit, so that integer dates $n = 1, 2, 3, \ldots$ represent beginnings of years. Retirement takes place at date T, after an integer number of years. At date t, let P_t be the retirement bond price implied by the investor's profile and given by Equation (1), and W_t be the value of retirement savings. The purchasing power of savings measured in terms of the investor's goal (income or wealth) is

$$\frac{W_t}{P_t}. \tag{3}$$

For a goal with no cost-of-living adjustment, the bond price equals the index value, β_t. For a cost-of-living-adjusted goal, the bond price is greater than the index value (see Equation (2)), and Equation (3) measures the purchasing power in terms of dollars of the inception date.

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The essential goal is to have, for any integer n equal to 0, 1, ..., $T-1$,

$$W_{n+1} \geq \delta_{ess} \frac{W_n}{P_n} P_{n+1}, \tag{4}$$

where $\delta_{ess} = 80\%$. Within year $n+1$ (i.e., the year beginning at date n), the ratio W_n/P_n is a constant, so the minimum capital needed to secure the right-hand side is

$$F_t = \delta_{ess} \frac{W_n}{P_n} P_t. \tag{5}$$

This equation defines the *floor* that the portfolio value should respect at all times. The index t lies in the half-open interval $[n, n+1)$. (Date n is included, but $n+1$ is excluded.)

Floor Resets

As implied by Equation (5), the floor is reset every year to δ_{ess} times the current portfolio value. As a result, it is discontinuous at year changes. Indeed, the floor just before date $n+1$ is

$$F_{[n+1]-} = \delta_{ess} \frac{W_n}{P_n} P_{n+1}, \tag{6}$$

while the floor immediately after is

$$F_{[n+1]+} = F_{n+1} = \delta_{ess} W_{n+1}. \tag{7}$$

F_{n+1} is in general different from $F_{[n+1]-}$, and the sign and the magnitude of the jump depends on the return spread between savings and the retirement bond. In details, we have

$$F_{[n+1]+} - F_{n+1} = \delta_{ess} W_n \left[\frac{W_{n+1}}{W_n} - \frac{P_{n+1}}{P_n} \right], \tag{8}$$

so the floor jumps up if the portfolio outperforms the bond, jumps down if it underperforms and does not jump if the portfolio happens to exactly replicate the bond return this year.

If the GBI strategy were applied to a real-world investor, savings would most likely include periodic contributions, because most individuals build up their nest egg by periodically putting money in a savings account, as opposed to investing a lump sum. In the above equations, wealth at dates n and $n+1$ would include the contributions made on these dates, if any, so the change in savings as measured by the ratio W_{n+1}/W_n does not reflect solely the return of the underlying strategy. In Goal-Based Investing Indices, however, no such contributions take place, so that the returns of the index coincide with those of the GBI strategy.

Another way to look at the floor is to measure it relative to the retirement bond price: this represents the replacement income or retirement wealth that must be secured. By Equation (5), the "relative floor" is piecewise constant over time, being equal to δ_{ess} times the income or wealth level that was affordable at the beginning of the year.

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Dynamic Management of Risk Budget

To say that the strategy respects the floor is mathematically equivalent to saying that the risk budget, defined as the distance between the current value of savings and the floor, is nonnegative at all times:

$$RB_t = W_t - F_t. \tag{9}$$

In order to maintain a nonnegative risk budget, the GBI strategy lets the allocation to the PSP shrink to zero as savings value approaches the floor. The percentage of savings invested in the PSP at a rebalancing date t is

$$w_{S,t} = \min\left[1, \max\left[0, m_t \frac{RB_t}{W_t}\right]\right], \tag{10}$$

where m_t is a multiplier independent from W_t but depending on time (more on this below). This percentage is essentially equivalent to RB_t/W_t with a floor at 0 and a cap at 100% in order to avoid short sales and leverage. The remainder of savings, that is the fraction $1 - w_{S,t}$, is invested in the GHP, the performance of which replicates floor returns within the year.

This investment rule is similar to that implemented in the constant proportion portfolio insurance (CPPI) strategies described by Black and Perold (1992), but it extends them to a "relative" context, in which the objective is to avoid downside risk with respect to a benchmark (here, the retirement bond) as opposed to avoiding capital losses. Similar extensions of CPPI rules were brought to the literature by Amenc, Malaise, and Martellini (2004) with the introduction of "dynamic core-satellite strategies" and by Martellini and Milhau (2012) with "liability-driven investing strategies" aiming to protect a minimum funding ratio for defined-benefit pension plans. The general motivation here is the same, namely to avoid downside risk while taking advantage of the long-term outperformance of the PSP with respect to the GHP to eventually outperform the benchmark. Here, downside risk is a short-term risk because it is defined as the risk to lose more than 20% of purchasing power over one year. Outperformance, if it materializes, means that the purchasing power of any dollar invested in the portfolio at inception increases.

Gap Risk and Rebalancing Frequency

Leverage can arise if

$$m_t \frac{RB_t}{W_t} > 1, \tag{11}$$

which never happens if m_t is 1 because the risk budget is a fraction less than 100% of savings, but cannot be ruled out if m_t is greater than 1. Hence, the cap at 100% in Equation (10) matters.

Short sales can occur only if the risk budget at the rebalancing date turns out to be negative. This situation is a manifestation of the *gap risk* that arises when a port-

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folio insurance strategy is rebalanced in discrete time. With continuous rebalancing, the portfolio value remains above the floor at all times because the allocation can be continuously adjusted to the relative movements of the PSP with respect to those of the GHP, no matter how large the underperformance is (see e.g. Proposition 16 in Deguest et al. (2015) for a mathematical statement and a proof). With discrete rebalancing, however, the portfolio value can fall below the floor if the PSP underperforms the GHP by sufficiently large an amount between two consecutive rebalancing dates. In details, it can be shown that if t and $t+1$ are two rebalancing dates, the floor is respected at date t and the PSP weight at date t is less than 100%, then the floor is also respected at date $t+1$ if, and only if, the gross returns of the PSP and the GHP between dates t and $t+1$, denoted respectively as $R_{t,t+1}(S)$ and $R_{t,t+1}(G)$, satisfy

$$R_{t,t+1}(S) \geq \left[1 - \frac{1}{m_t}\right] R_{t,t+1}(G). \tag{12}$$

In words, the gross performance of the PSP must be at least $[1 - 1/m_t]$ that of the GHP. When m_t is 1, this condition is always verified and there is no gap risk. The higher the multiplier, the more likely it gets to encounter at least a few periods in which the PSP does not sufficiently outperform the GHP, and gap risk becomes more likely too.

The probability of a floor violation can be reduced in two main ways: decrease the multiplier or increase the rebalancing frequency. In the construction of Goal-Based Investing Indices, the multiplier is exogenously fixed as explained below, so the first option is not available. But it has been found in Monte-Carlo simulations and historical backtests (see Giron et al. (2018)) that gap risk is negligible in frequency and magnitude if the portfolio is rebalanced every month. It is this frequency that is retained for the Indices.[2]

Multiplier and Reference Target Date Fund

In a deterministic target date fund, the allocation to equities is a predetermined function of the horizon, an example of which is shown as the blue line in Figure 1. This glidepath is the one taken as a benchmark for Goal-Based Investing Indices. It starts at 100% forty years before retirement and decreases by one percentage point every year until year 20 before retirement, when it reaches 80%. The decrease then continues at a faster pace of two percentage points per year until retirement: at this point, the allocation falls to 40%. This allocation pattern is coherent with those implemented in commercial target date funds, but providers offer a number of variants, in particular to accommodate different levels of risk aversion (see examples of glidepaths in Morningstar (2017)).

In order to reproduce the general pattern of a decreasing allocation to performance-

2 – This does not imply that an asset manager replicating a Goal-Based Investing Index would need to rebalance every month. In practice, other techniques can be employed, like move-based rebalancing or increasing the rebalancing frequency in bear markets, in order to keep turnover at reasonable levels without compromising the essential goal.

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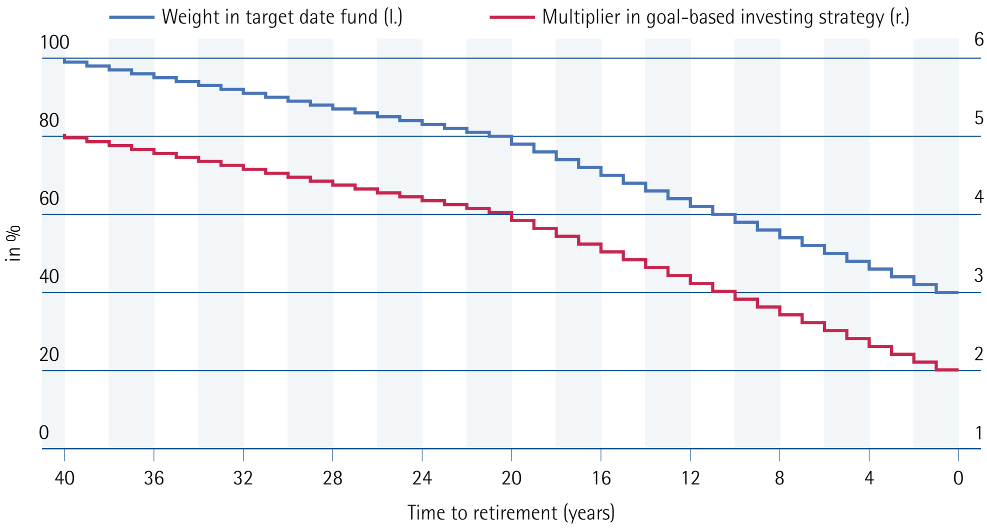
Figure 1: Glidepath of target date fund and goal-based investing strategy.



In the target date fund, the weight allocated to the performance-seeking component is a deterministic function of the time to retirement that starts at 100% forty years ahead of retirement and ends at 20% just before retirement. In the goal-based investing strategy, it is the multiplier that is a deterministic function of the horizon. To translate the weights into multiplier values, it is assumed in this figure that the fraction of the purchasing power to be secured on an annual basis is 80%.

seeking assets, the GBI strategy on which Goal-Based Investing Indices are based has a time-varying multiplier, which is revised every year so as to match the allocation of the target date fund. By Equations (5) and (10), the fraction of savings invested in the PSP at date n (the beginning of a year) is

$$w_{S,n} = \max\left[1, \min\left[m_n\left[1 - \delta_{ess}\right]\right]\right]. \tag{13}$$

Let TDF_n be the percentage allocation to equities at date n in the target date fund, as given by Figure 1. To equate $w_{S,n}$ and TDF_n, it suffices to take

$$m_n = \frac{TDF_n}{1 - \delta_{ess}}. \tag{14}$$

The multiplier is then held constant for the period $[n, n+1)$, which corresponds to year $n+1$.

With $\delta_{ess} = 80\%$, the multiplier for the period $[T-40, T-39)$, which corresponds to the thirty-ninth year before retirement is

$$m_{T-40} = \frac{100}{100-80} = 5. \tag{15}$$

For the period $[T-1, T)$, which is the last year before retirement, the multiplier is

$$m_{T-1} = \frac{40}{100-80} = 2. \tag{16}$$

The glidepath for the multiplier is plotted in red in Figure 1.

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3. Index Valuation

Each index is valued as a portfolio invested in two building blocks: the performance-seeking portfolio (PSP) and the goal-hedging portfolio (GHP).

Budget Equation

The budget equation expresses the return on the portfolio as a function of the composition and the returns on the building blocks. Consider a valuation date t posterior to inception, and let s denote the date of the latest rebalancing until date t exclusive. (If t is itself a rebalancing date, then s is the date of the previous rebalancing operation.) Portfolio composition was last set at time s, with the weights $w_{S,s}$ and $w_{G,s}$ respectively in the PSP and the GHP. Then, from date s to date t, the portfolio was left buy-and-hold.

Let $R_{s,t}(\alpha)$ denote the portfolio return over the period that goes from time s to time t, and let similarly $R_{s,t}(S)$ and $R_{s,t}(G)$ denote the returns of the constituents. Here, returns are understood to be gross returns: the gross return on a portfolio of value V between dates s and t is V_t/V_s. Then, the budget equation reads

$$R_{s,t}(\alpha) = w_{S,s}R_{s,t}(S) + w_{G,s}R_{s,t}(G). \tag{17}$$

It is important to note that this equation holds when s is a rebalancing time and no rebalancing takes place after date s until date t, though t itself may be a rebalancing time.

Initial Conditions

Equation (17) defines index *returns*. In order to have index *values*, we must specify some initial value. By convention, the initial date for an index (date 0) is taken to be the inception date, which is the date from which backtested index performance is calculated and reported on the website. The index value is set to 1 on this date, so that the index value at each point in time, α_t, represents the current value of one dollar that was invested at date 0 in the dynamic strategy. Similarly, we define S_t and G_t to be "the PSP and the GHP values", that is the values at date t of one dollar invested at date 0 respectively in the PSP and the GHP.

With this notation, Equation (17) can be rewritten as

$$\frac{\alpha_t}{\alpha_s} = w_{S,s}R_{s,t}(S) + w_{G,s}R_{s,t}(G). \tag{18}$$

Here, t denotes any valuation date and s is the date of the latest rebalancing before or at date t.

Index Values from Building Block Returns

In fact, index values are calculated by stepping from one valuation date to the next, as opposed to starting from a rebalancing date and going forward. This is done by rewriting the budget equation between two consecutive valuation dates, t and $t+1$. Suppose first that t is not a rebalancing date, so that the latest rebalancing time is the

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same for dates t and $t+1$, say s. We have, by Equation (18),

$$\frac{\alpha_t}{\alpha_s} = w_{S,s} R_{s,t}(S) + w_{G,s} R_{s,t}(G), \tag{19}$$

$$\frac{\alpha_{+1}}{\alpha_s} = w_{S,s} R_{s,t+1}(S) + w_{G,s} R_{s,t+1}(G). \tag{20}$$

Define the *pre-rebalancing weights* of the PSP and the GHP in the portfolio at date t as

$$v_{S,t} = w_{S,s} \frac{\alpha_s}{\alpha_t} R_{s,t}(S), \tag{21}$$

$$v_{G,t} = w_{G,s} \frac{\alpha_s}{\alpha_t} R_{s,t}(G). \tag{22}$$

These are the weights of the constituents in the portfolio before any rebalancing operation. On a date with no rebalancing, they coincide with the *post-rebalancing* weights, but on rebalancing times, the pre- and the post-rebalancing allocations may differ: the turnover is a formal measure of these changes in composition.

Dividing Equation (20) by Equation (19) and using the expressions for the pre-rebalancing weights, we obtain

$$\frac{\alpha_{t+1}}{\alpha_t} = v_{S,t} R_{t,t+1}(S) + v_{G,t} R_{t,t+1}(G), \tag{23}$$

where $R_{t,t+1}$ denotes a return over the period $[t, t+1]$, that is $R_{t,t+1} = R_{s,t+1}/R_{s,t}$. The pre-rebalancing weights at date $t+1$ are given by the same formulas as at date t:

$$v_{S,t+1} = w_{S,s} \frac{\alpha_s}{\alpha_{t+1}} R_{s,t+1}(S), \tag{24}$$

$$v_{G,t+1} = w_{G,s} \frac{\alpha_s}{\alpha_{t+1}} R_{s,t+1}(G). \tag{25}$$

This implies

$$v_{S,t+1} = v_{S,t} R_{t,t+1}(S) \frac{\alpha_t}{\alpha_{t+1}}, \tag{26}$$

$$v_{G,t+1} = v_{G,t} R_{t,t+1}(G) \frac{\alpha_t}{\alpha_{t+1}}. \tag{27}$$

With Equations (23), (26) and (27), index values can be calculated in an iterative way starting from inception knowing the returns of the PSP and the GHP.

Figure 2 illustrates this process with fictitious numbers. Assume that weights at inception are 20% in the GHP and 80% in the PSP, and that the returns on the two building blocks from date 0 to date 1 are respectively 2% and −6%. Then, by Equation (23), the

3. Index Valuation

Figure 2: Valuation of a Goal-Based Investing Index.

Date 0 is the inception date, at which the first rebalancing takes place. No other rebalancing takes place until date 4. The third axis displays the returns of the building blocks (PSP and GHP) between two consecutive valuation dates. Pre-rebalancing and post-rebalancing allocations are identical by construction on dates at which no rebalancing occurs. The index value at inception is normalized to 1. Weights and index values are rounded.

index return between dates 0 and 1 is

$$\frac{\alpha_1}{\alpha_0} = 0.20 \times [1 + 0.02] + 0.80 \times [1 - 0.06], \qquad (28)$$

3. Index Valuation

which is 0.956. Because the index is 1 at time 0, it follows that the index value at time 1 is

$$\alpha_1 = 0.956. \tag{29}$$

The pre-rebalancing weights at date 1 are given by Equations (26) and (27):

$$\begin{aligned} v_{S,1} &= 0.20 \times [1 + 0.02] \times \frac{1}{0.956} \\ &= 0.213, \end{aligned} \tag{30}$$

$$\begin{aligned} v_{S,2} &= 0.80 \times [1 - 0.06] \times \frac{1}{0.956} \\ &= 0.787. \end{aligned} \tag{31}$$

At date 1, no rebalancing takes place, so the post-rebalancing weights coincide with the pre-rebalancing ones. Calculation starts over with these initial conditions and cycles through the same sequence, to generate the displayed path for the index.

Valuation Frequency

Index values are calculated for every weekday from Monday to Friday inclusive, regardless of business day issues. Of course, the input data (the zero-coupon rates) may not be available on some valuation days, due to market closures or bank holidays: see Section 6 for the approach to missing data issues.

4. Content of Website Pages

The EDHEC-Princeton Retirement Goal-Based Investing Indices are published on EDHEC-Risk Institute website at

`https://risk.edhec.edu/indices-investment-solutions`

in the "Retirement Goal Indices" tab. The main index page displays the latest available monthly returns of indices, for the versions with and without a cost-of-living adjustment. Pages dedicated to individual indices can be accessed by clicking each row.

Data Visualization

Tab "Historical Values" shows the monthly values of the index and its two building blocks since inception. All values are set to 1 at inception. A table displays the monthly returns on the index and its two building blocks over the past months. Tab "Rebalancing Weights" shows the index composition at the last rebalancing time in terms of its two building blocks. Also displayed is the index composition since inception.

The third tab, "Success Probabilities", displays the estimated probabilities for the GBI strategy and for its reference target date fund to achieve an essential goal and three aspirational goals. The essential goal is to keep the annual loss in the level of affordable income or wealth at 20% at most for all remaining years until retirement. The change in the affordable level is calculated from the beginning of January to the beginning of the next year. For probabilities reported within a year, the current (incomplete) year is included in the calculation. Except in the event of gap risk, the probability for the index to respect this condition is 100%, and should gap risk occur, the probability is expected to fall by no more than a few percentage points.

An aspirational goal is defined as a multiple of the initially affordable income or wealth level to be reached at least once before retirement. Probabilities are reported for three values: 130%, 150% and 200%. Each of them is defined as the probability of hitting the aspirational level at least once before retirement. They are calculated under the assumption that the strategy (GBI or deterministic target date fund) continues according to its rules, so the displayed probability can fall under 100% even if the strategy has reached the aspirational goal at some point of its history. This approach is taken because indices and target date funds are intended to be independent from any aspirational goal, but it is distinct from what would be implemented for an individual: he/she would be recommended to do a "stop-gain" and to transfer retirement savings to the GHP as soon as the aspirational goal is reached. This would imply that the affordable income or wealth level remains constant as of this point until retirement, so that the probability would be stuck at 100%.

All goals can be stated as funding ratio objectives, where the funding ratio is defined as the ratio between the initially affordable income or wealth level and the currently affordable level. A funding ratio of 100% means that the strategy has had the exact same cumulative performance as the corresponding Goal Price Index, so that the purchasing power is exactly the same as at inception. A ratio of 150% means that the

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strategy has outperformed the Goal Price Index by 50%, so the purchasing power is higher than at inception.

All probabilities are based on a stochastic model and on hypothetical values for parameters like risk premia, volatilities and correlations. Section 5 below describes the model in detail. As a result, they are speculative in nature, and even a 100% probability should not be regarded as a formal guarantee that the strategy will reach a particular goal. That said, some metrics are more sensitive to parameter values than others. There is a firm reason for the GBI strategy to respect the annual loss condition, so the probability should be close to 100% for any set of reasonable parameter values, but probabilities of reaching aspirational goals are much more sensitive to the assumed equity risk premium (see Giron et al. (2018)).

Downloads

The monthly values of the index and its two building blocks are available for download in a comma-separated value file. Index rebalancing weights since inception are provided in the same format.

Calculators

In the "Success Probabilities" tab, a calculator allows funding ratios to be translated into levels of retirement wealth or replacement income. This information is provided because individuals will find dollar amounts to be more concrete than funding levels. The input expected from the user is the current amount of retirement savings, W. The calculator uses the latest available value of the Goal Price Index with the same investor's profile and the same goal (income or wealth) as the Goal-Based Investing Index. If β denotes the Goal Price Index, the maximum amount of wealth or annual income that can be secured with savings is

$$\frac{W}{\beta}, \tag{32}$$

and an aspirational goal corresponds to a multiple (greater than 1) of this quantity.

For indices with a cost-of-living adjustment, β is the price of \$1 of future income or wealth, with this dollar being a dollar of the current date. Thus, Equation (32) gives the affordable income or wealth level expressed in dollars of the current date.

To fix the ideas, assume that savings are \$100,000, that the goal of interest is to produce replacement income and that the current price index is $\beta = 13.56$. This means that with the \$100,000, the investor can secure a replacement income of

$$\frac{100,000}{13.56} = \$7,375 \quad \text{per year.} \tag{33}$$

If he/she reaches the 130% goal by retirement, then he/she will be able to secure a higher income, of

$$1.3 \times \frac{100,000}{13.56} = \$9,587 \quad \text{per year.} \tag{34}$$

4. Content of Website Pages

If he/she reaches the 200% aspirational goal, then he/she will double his/her income with respect to the current date, with

$$2 \times \frac{100,000}{13.56} = \$14,749 \quad \text{per year.} \tag{35}$$

It should be noted that these levels of income are based solely on the amount of savings and the current Goal Price Index value, which is itself independent from any model or parameter value. What is model- and parameter-dependent is the probability of reaching a level.

4. Content of Website Pages

5. Simulation Model for the Estimation of Probabilities

This section describes the stochastic model and the parameter values used in stochastic simulations of target date funds and goal-based investing (GBI) strategies. The risk factors involved in this process are:

- the nominal term structure, which impacts the returns of the goal-hedging portfolio (GHP);
- the value of the stock index used as the performance-seeking portfolio (PSP) in target date funds and GBI strategies;
- the value of the bond index that serves as a safe building block in deterministic target date funds.

It is important to note that the model described in this section and the associated parameter values are not used at any point in the construction methodology or in the valuation of Goal-Based Investing Indices. They are involved only at the reporting stage, to estimate the probabilities of reaching essential or aspirational goals. These probabilities are reported in the tab "Success Probabilities" of each Goal-Based Investing Index' page.

Nominal Term Structure

For simplicity, we adopt a one-factor model for the term structure, namely the model of Vasicek (1977). The risk factor is the short-term rate, which follows a mean-reverting process:

$$\mathrm{d}r_t = a[b - r_t]\,\mathrm{d}t + \sigma_r\,\mathrm{d}z_{rt}. \tag{36}$$

Here, z_r is a standard Brownian motion. The price of interest rate risk is a constant λ_r, which must be negative for long-term bonds to have a positive expected excess returns over short-term bills.

The dataset used to estimate nominal rate parameters consists of the nominal zero-coupon rates calculated by the method of Gürkaynak, Sack, and Wright (2007) and currently available on the website of the Federal Reserve at

`https://www.federalreserve.gov/pubs/feds/2006/200628/200628abs.html`

which we complete with the secondary market rate on three-month Treasury bills, also available from the Fed at

`https://www.federalreserve.gov/releases/h15/`

All series are sampled at the monthly frequency.

There are five parameters to estimate: the speed of mean reversion a, the long-term mean b, the short-term volatility σ_r, the price of risk λ_r and the initial value r_0. Estimates are revised at each date a simulation is run in order to achieve consistency between the model-implied term structure and the observed one at the start date. To

5. Simulation Model for the Estimation of Probabilities

ensure that estimates do not vary too fast over time, we use a mixture of historical estimation over a moving window, which by nature implies stable estimates, and calibration, which implies a good fit between the model-implied and the observed yield curves.

Specifically, b is estimated as the mean of the nominal three-month rate, which is is the shortest maturity available in the sample. a and σ_r are chosen in such a way that the model-implied long-term volatility and one-year autocorrelation match the sample moments of the short-term rate estimated over the same time frame. Formally, the long-term moments are defined as the limits of the volatility and the one-year autocorrelation as time goes to infinity (and therefore memory of initial conditions is lost):

$$\sigma_\infty = \lim_{t\uparrow\infty} \sqrt{\mathbb{V}[r_t]}, \quad \rho_\infty = \lim_{t\uparrow\infty} \mathbb{C}\mathrm{orr}[r_{t-12}, r_t]. \tag{37}$$

It can be shown that the long-term moments implied by the Vasicek model are

$$\mu_\infty = b, \quad \sigma_\infty = \frac{\sigma_r}{\sqrt{2a}}, \quad \rho_\infty = e^{-a}, \tag{38}$$

so that the parameters can be recovered from the long-term moments as

$$a = -\log \rho_\infty, \quad b = \mu_\infty, \quad \sigma_r = \sigma_\infty \sqrt{2a}. \tag{39}$$

Quantities ρ_∞, μ_∞ and σ_∞ are estimated as the sample moments over the estimation period.

Table 1: Examples of short-term interest rate parameter values.

Reference date	a	b	σ_r	λ_r	r_0
1 Jan. 2018	0.2313	0.0192	0.0137	−0.2199	0.0145
1 Jan. 2017	0.2063	0.0213	0.0136	−0.2941	0.0053
1 Jan. 2016	0.1950	0.0235	0.0135	−0.2499	0.0030
1 Jan. 2015	0.1908	0.0262	0.0136	−0.1755	0.0007
1 Jan. 2014	0.2200	0.0283	0.0142	−0.3868	−0.0024

These parameter values for the Vasicek model have been estimated by the historical-calibration procedure described in the text.

Parameters λ_r and r_0 are estimated by minimizing the sum of squared differences between the model-implied zero-coupon yields and the "observed" ones.[3] In the Vasicek model, the zero-coupon rate of maturity u is given by

$$y(0, u) = \frac{D(u)}{u} r_0 - \frac{E(u)}{u}, \tag{40}$$

3 – Zero-coupon rates are not directly observed in practice, and they must be inferred from the market prices of Treasuries. See Gürkaynak, Sack, and Wright (2007).

5. Simulation Model for the Estimation of Probabilities

with

$$D(u) = \frac{1 - e^{-au}}{a}, \tag{41}$$

$$E(u) = \left[b - \frac{\sigma_r \lambda_r}{a}\right][D(u) - u] + \frac{\sigma_r^2}{2a^2}\left[u - 2D(u) + \frac{1 - e^{-2au}}{2a}\right]. \tag{42}$$

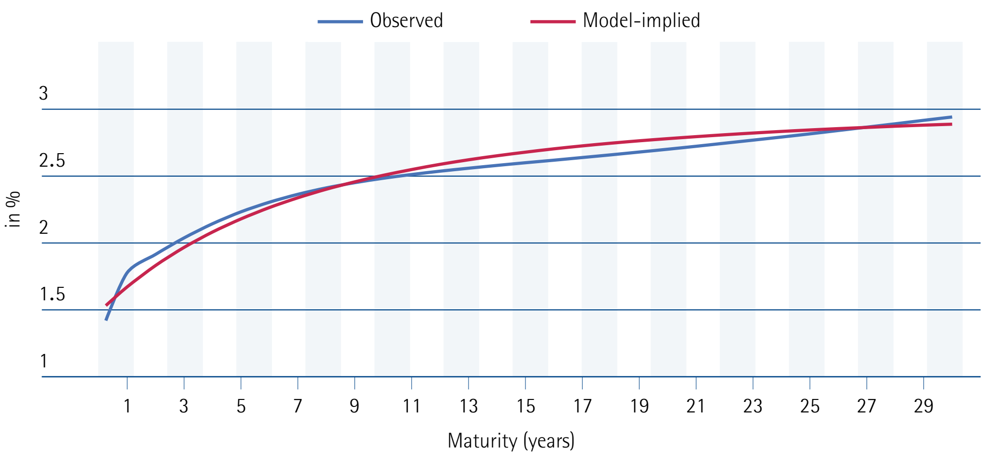


Figure 3: Comparison of observed and model-implied zero-coupon yield curves on 2 January 2018.

"Observed" zero-coupon rates on 2 January 2018 are inferred from the market prices of Treasuries. Model-implied rates are the zero-coupon rates implied by the Vasicek model with parameters estimated by the mixed historical-calibration procedure described in the text over the period from January 1998 through January 2018.

Table 1 shows the parameter estimates obtained through this procedure for the first day of January of the past five years. The most stable estimate is that of σ_r, which is a volatility parameter, and is thus more robust to the choice of the sample than a first-order moment. Long-term mean estimates decrease over time because interest rates have been following a decreasing trend since the early 1980s. Because long-term mean is estimated over the 20 years preceding the calibration date, the decreasing pattern in rates implies that the initial short-term rate is always less than the long-term mean. Due to the mean reversion in the short rate process, this initial condition implies that a rise in interest rates is simulated.

Figure 3 displays the observed and model-implied yield curves on 2 January 2018. The fit is good, and it could be further improved by minimizing the sum of squared errors over the five parameters, not just r_0 and λ_r. However, not any arbitrary shape of the yield curve can be accurately matched because the Vasicek model has only one factor. Moreover, focusing on pricing error minimization on a given date would lead to more instability over time in the estimates of long-term parameters.

Stock and Bond Indices

The stock and the bond indices are modeled as diffusion processes with a constant volatility and a constant expected excess return over the nominal short-term rate. For

5. Simulation Model for the Estimation of Probabilities

instance, the stock index evolves as

$$\frac{\mathrm{d}S_t}{S_t} = [r_t + \sigma_S \lambda_S]\,\mathrm{d}t + \sigma_S\,\mathrm{d}z_{St}, \tag{43}$$

where z_S is a Brownian motion correlated to the Brownian motions of the nominal and the real short-term rates.

Sharpe ratios and volatilities are borrowed from Merrill Lynch's capital market assumptions for 2017. They are summarized in Tables 2 and 3.

Correlations

Instantaneous correlations are the correlations between the innovations to the various processes. They are set to the neutral value of zero, with two exceptions. The first is the correlation between the bond index and the nominal short-term rate, estimated as the empirical correlation between changes in the three-month Treasury bill rate and the returns on the BofA Merrill Lynch AAA US Treasury/Agency Master index between April 1978 and May 2017: the value is −0.588. The second non-zero correlation is the correlation between the stock and the bond indices, which is borrowed from Merrill Lynch's capital market assumptions for 2017 and is −14.75%.

Table 2: Constant parameter values: Sharpe ratios and volatilities.

	Sharpe ratio	Volatility (%)
Stock index	0.395	16.20
Bond	0.234	6.40

"Constant parameter values" are held constant from one simulation to the other, regardless of the date at which interest rate parameters are estimated.

Table 3: Constant parameter values: Correlations.

	Correlations (%)	
	Stock index	Bond index
Nominal rate	0	−58.8
Stock index		−14.8

6. Data Sources

Valuation of Goal-Based Investing Indices requires two types of data: nominal zero-coupon rates to calculate the returns of the goal-hedging portfolio, and the returns of a performance-seeking portfolio.

Interest Rates

The interest rate data is the same as for the valuation of Goal Price Indices: it consists of US nominal zero-coupon rates calculated by the methodology of Gürkaynak, Sack, and Wright (2007) and obtained from the Quandl online database at

`https://www.quandl.com/data/FED/SVENY-US-Treasury-Zero-Coupon-Yield-Curve`

The original data is posted on the Federal Reserve website at

`https://www.federalreserve.gov/pubs/feds/2006/200628/200628abs.html`

Publication lag – as measured by the time interval between current date and the date of the latest observation – is approximately from 3 to 7 weekdays, excluding Saturday and Sunday.

Performance-Seeking Portfolio

The performance-seeking portfolio is represented by the ERI Scientific Beta US cap-weighted index, which includes the 500 largest US stocks. Index values are delivered by FTP, and publication lag is exactly one weekday, excluding Saturday and Sunday, hence much shorter than for interest rate data.

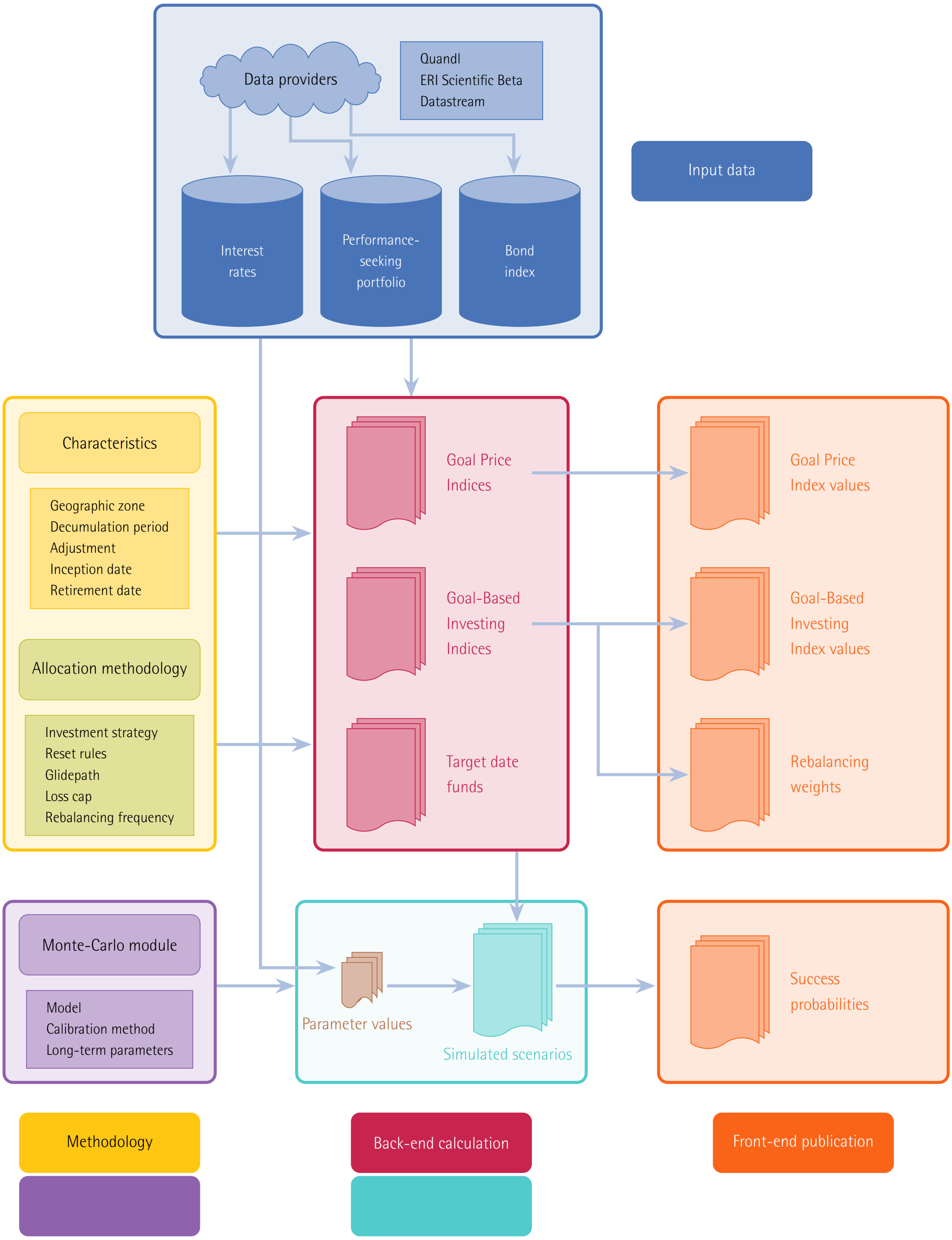
Details on the construction of the index universe and the index calculation can be found on ERI Scientific Beta website at

`http://www.scientificbeta.com/index/download/UniverseGroundRules/ADU-xxxx-wCx`

`http://www.scientificbeta.com/index/download/CalculationGroundRules/ADU-xxxx-wCx`

6. Data Sources

Summary of Index Construction



This figure summarizes the index construction process. Input data is obtained from external providers, and is used in the valuation of Goal Price Indices, Goal-Based Investing

Summary of Index Construction

Indices and target date funds. These portfolios also depend on the five index characteristics and on the allocation methodology described in this document. The Monte-Carlo model is solely used for the estimation of selected parameters (interest rate parameters) and for the generation of stochastic scenarios for state variables. Market data is also involved in the estimation of parameters. Portfolio values and weights are used as initial conditions in the simulation of future portfolio values. The right part of the plot displays the data reported on EDHEC-Risk Institute website, that is index values, weights of Goal-Based Investing Indices and the success probabilities for these indices and for their benchmark target date funds.

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For more information, please send an email to research@edhec-risk.com
or call + 33 (0)4 93 18 78 87

EDHEC-Risk Institute
393-400 promenade des Anglais
BP 3116 - 06202 Nice Cedex 3
France
Ph. + 33 (0)4 93 18 78 87

EDHEC-Risk Institute – Europe
10 Fleet Place, Ludgate
London EC4M 7RB
United Kingdom
Ph. +44 (0)207 332 5600

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