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Foreword

The present publication is drawn from the first-year research work of the EDHEC/ORTEC Finance research chair on Private ALM.

This chair, under the responsibility of Lionel Martellini, the Scientific Director of EDHEC-Risk, is based on the principle that suitable extensions of portfolio optimisation techniques used by institutional investors can be transposed to private wealth management, precisely because these techniques have been engineered to incorporate in the portfolio construction process an investor’s specific context, objectives, and horizon.

In the present study we argue that asset-only asset allocation models fail to account for the presence of investment and/or consumption goals and objectives, such as preparing for retirement or acquiring property. We instead provide a formal framework suggesting that asset-liability management can ensure that private wealth managers are able to offer their clients investment programmes and asset allocation advice that truly meet their needs.

I would like to thank my co-authors, Lionel Martellini, Vincent Milhau and Volker Ziemann for the quality of their work on this report. We hope that you will find our analysis and conclusions valuable and will continue to monitor our research in this area and contribute to our work through your feedback.

Finally, we would like to extend our warm thanks to our partners at ORTEC Finance for their close involvement with our research and their commitment to the Private ALM research chair.

Wishing you an informative and thought-provoking read,

Noël Amenc
Professor of Finance
Director of EDHEC-Risk
About the Authors

Noël Amenc is professor of finance and director of research and development at EDHEC Business School, where he heads the EDHEC Risk and Asset Management Research Centre. He has a masters in economics and a PhD in finance and has conducted active research in the fields of quantitative equity management, portfolio performance analysis, and active asset allocation, resulting in numerous academic and practitioner articles and books. He is a member of the editorial board of the Journal of Portfolio Management, associate editor of the Journal of Alternative Investments and a member of the scientific advisory council of the AMF (French financial regulatory authority).

Lionel Martellini is professor of finance at EDHEC Business School and scientific director of the EDHEC Risk and Asset Management Research Centre. He has graduate degrees in economics, statistics, and mathematics, as well as a PhD in finance from the University of California at Berkeley. Lionel is a member of the editorial board of the Journal of Portfolio Management and the Journal of Alternative Investments. An expert in quantitative asset management and derivatives valuation, Lionel has published widely in academic and practitioner journals and has co-authored textbooks on alternative investment strategies and fixed-income securities.

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Volker Ziemann is a senior research engineer at the EDHEC Risk and Asset Management Research Centre. He has master’s degrees in economics and statistics and a PhD in finance. His research focus is on the econometrics of financial markets and optimal asset allocation decisions involving non-linear payoffs.
While the private banking industry is in general relatively well equipped on the tax planning side, with tools that can allow private bankers to analyse the situation of high net worth individuals operating offshore or in multiple tax jurisdictions, the software packages used on the financial simulation side often suffer from significant limitations and cannot satisfy the needs of a sophisticated clientele. In fact, most financial software packages used by private bankers to generate asset allocation recommendations rely on single-period mean-variance asset-portfolio optimisation, a tactic that, for at least two reasons, cannot lead to proper strategic allocation. For one, optimisation parameters (expected returns, volatilities, and correlations) are defined as constant across time, a practice which is contradicted by empirical observation and does not make it possible to take into account the length of the investment horizon. For another, and most importantly perhaps, liability constraints and risk factors affecting them, such as inflation risk on targeted spending, are neither modelled nor explicitly taken into account in the portfolio construction process.

Overall, dealing with a private client usually involves a detailed analysis of the client’s objectives, constraints, and risk-aversion parameters, sometimes on the basis of rather sophisticated approaches. Yet it is striking that once this information has been collected, and sometimes formalised, very little is done to tailor a portfolio to the client’s specific needs. In general, several profiles, expressed in terms of volatility or drawdown, are provided; in some instances a distinction in how the capital will eventually be accessed (annuities or lump-sum payment) is made, but the client’s specific objectives, constraints, and associated risk factors are simply not taken into account in the design of the optimal allocation.

The objective of this paper is to shed light on the ways new forms of welfare-improving financial innovation inspired by the use of asset-liability management techniques, originally developed for institutional money management, can be used in private wealth management. Asset-liability management (ALM) refers to the adaptation of the portfolio management process to the presence of constraints relating to the commitments represented by the investor’s liabilities. We argue that suitable extensions of portfolio optimisation techniques used by institutional investors, e.g., pension funds, could be transposed to private wealth management, precisely because these techniques have been engineered to incorporate in the portfolio construction process an investor’s specific constraints, objectives, and horizon, all of which can be summarised in a single state variable, the value of the "liability" portfolio. As such, our paper can be seen as an attempt to merge two somewhat separate strands of the literature, that is, the literature on long-term financial decisions for private investors, which has focused mostly on an asset-only perspective, and the literature on asset-liability management decisions, which have been analysed mostly from an institutional perspective (pension funds, insurance companies, or endowments). We do so by casting the long-horizon life-cycle investment problem in an asset-liability management framework suitable for the private wealth management context, which allows us to show that pursuing an asset-only strategy usually involves a substantial opportunity cost.
Broadly, taking an ALM approach leads to defining risk and return relative to a liability portfolio, a critical improvement on asset-only asset allocation models that fail to account for the presence of investment and/or consumption goals and objectives, such as preparing for retirement or for a real estate acquisition. As a result, taking an ALM approach leads to a focus on the liability-hedging properties of various asset classes, a focus that would, by definition, be absent from an asset-only perspective. We also present a series of numerical illustrations suggesting that the model introduced in this paper could be applied in several situations typical of private wealth management.

Overall, it is not the performance of a particular fund or that of a given asset class that will be the determinant in the ability to meet a private investor’s expectations. Satisfaction of the investor’s long-term objectives is fundamentally dependent on an ALM exercise whose aim is to determine the proper strategic inter-class allocation as a function of the investor’s specific objectives, constraints, and time-horizon. In other words, what will prove decisive is the ability to design an asset allocation programme that depends on the particular risks to which the investor is exposed. Similarly, the very concept of a risk-free asset depends on the investor’s time-horizon and on his objectives. Hence, a five-year zero-coupon Treasury bond will not prove a perfectly safe investment for a private investor interested in a real estate acquisition in five years. The actual risk-free asset in this context (which we call below the liability-hedging portfolio) would instead be an asset perfectly correlated with real estate prices. More generally, an investor whose objective is the acquisition of property is likely to accept low and even negative returns when real estate prices are falling significantly but will not be satisfied with relatively high returns if these returns do not match dramatic increases in real estate prices. In such circumstances, a long-term investment in stocks and bonds, with a performance weakly correlated with real estate prices, would not be the right investment. Likewise, in a pension context, absolute returns, often perceived as a natural choice in private wealth management, would not be a satisfactory response to the needs of a private investor facing long-term inflation risk, where the concern is capital preservation in real terms. In other words, the first benefit of the ALM approach is perhaps its impact on the menu of asset classes, with a focus on including an asset that exhibits the highest possible correlation with the liability portfolio.

While ours is obviously a fairly stylised model, and while important effects such as taxes or mortality risk are not explicitly taken into account at this stage, we believe it is a significant first normative step towards a better understanding of private wealth management decisions. Our main contribution is to show that a significant fraction of the complexity of optimal asset allocation decisions for private investors can be captured through the introduction of a single additional state variable, the liability value, which can account in a parsimonious way for investors’ specific constraints and objectives.

Our analysis has great potential implications for the wealth management industry. Indeed, it is often said that proximity to investors is the main raison d’être of private wealth managers and a key source of competitive advantages. Building on
this proximity, private bankers should be ideally placed to better account for their clients' specific liability constraints when engineering an investment solution for them. Most private bankers actually implicitly promote an ALM approach to wealth management. In particular, they claim to account for the investor's goals and constraints. The technical tools involved, however, are often inappropriate. While the private client is routinely asked all kinds of questions about his current situation, goals, preferences, constraints, etc., the resulting service and product offering mostly boil down to a rather basic classification in terms of risk profiles. In this paper, we provide a formal framework suggesting that asset-liability management can ensure that private wealth managers are able to offer their clients investment programmes and asset allocation advice that truly meet their needs.
Introduction
Introduction

Over the past decade, private wealth management has become a profitable business for banks and asset managers around the globe. According to the private banking and wealth management survey conducted by Euromoney (2008), global private banking assets rose from USD 3.3 trillion in 2007 to USD 7.6 trillion in 2008. This increase is currently driving a larger wealth management market, creating greater opportunities for wealth advisors to leverage new technology to acquire new clients and grow profits. As a result, competition to find ways to improve existing client relationships and provide new tools to improve advisor effectiveness is increasing. While the private banking industry is in general relatively well equipped on the tax planning side, with tools that can potentially allow private bankers to analyse the situation of high net worth individuals operating offshore or across multiple tax jurisdictions, the software packages used on the financial simulation side typically suffer from significant limitations and cannot satisfy the needs of a sophisticated clientele.

In fact, most existing financial software packages used by private bankers to generate asset allocation recommendations rely on single-period mean-variance asset-portfolio optimisation, which cannot yield a proper strategic allocation for at least two reasons. For one, optimisation parameters (expected returns, volatilities and correlations) are defined as constant across time, a practice which is contradicted by empirical observation and does not make it possible to take into account the length of the investment horizon. For another, and most importantly perhaps, liability constraints and risk factors affecting them, such as inflation-risk on targeted spending, are neither modelled nor explicitly taken into account in the portfolio construction process. Overall, dealing with a private client usually leads to a detailed analysis of the client’s objectives, constraints, and risk-aversion parameters, sometimes on the basis of rather sophisticated approaches. Yet it is striking that once this information has been collected, and sometimes formalised, very little is done in terms of customising a portfolio solution to the specific needs of the client. In general, the approach consists of providing several profiles, expressed in terms of volatility or drawdown; in some instances a distinction in how the capital will eventually be accessed (annuities or lump-sum payment) is made, but the client’s specific objectives, constraints, and associated risk factors are simply not taken into account in the design of the optimal allocation. While some industry players have recently developed planning tools that model assets in a multi-period stochastic framework, asset-liability matching for individuals remains an area for exploration. The objective of this paper is to shed light on the ways new forms of welfare-improving financial innovation inspired by the use of asset-liability management techniques, originally developed for institutional money management, can be used in private wealth management.

Asset-liability management (ALM) refers to the adaptation of the portfolio management process to the presence of constraints relating to the commitments represented by the investor’s liabilities. In what follows, we argue that suitable extensions of portfolio optimisation
techniques used by institutional investors, e.g., pension funds, could be transposed to private wealth management, precisely because these techniques have been engineered to incorporate in the portfolio construction process an investor’s specific constraints, objectives and horizon, all of which can be summarised in a single state variable, the value of the “liability” portfolio. While ours is obviously a fairly stylised model, and while important effects such as taxes or mortality risk are not explicitly taken into account at this stage (see suggestions for further research), we believe it is a significant first normative step towards a better understanding of private wealth management decisions. In this context, our paper can be regarded as an attempt to take a first step towards a rational framework for private investors’ financial decisions that extends standard portfolio optimisation techniques by recognising that the aforementioned factors seriously affect the optimal allocation decision.

Our main contribution is to show that a significant fraction of the complexity of optimal asset allocation decisions for private investors can be captured through the introduction of a single additional state variable, the liability value, which can account in a parsimonious way for investors’ specific constraints and objectives. At this point, in the framework of private wealth management, we use a broad definition of “liabilities”, which encompasses any commitment or spending objective, usually self-imposed (as opposed to exogenously imposed, as in a pension fund).

For example, an investor committed to a real estate acquisition will perceive such an expense as a future commitment or soft liability for which money should be available. Overall, it is not the performance of a particular fund or that of a given asset class that will be the determinant in the ability to meet a private investor’s expectations. Satisfaction of the investor’s long-term objectives is fundamentally dependent on an ALM exercise whose aim is to determine the proper strategic inter-class allocation as a function of the investor’s specific objectives, constraints, and time-horizon. In other words, what will prove decisive is the ability to design an asset allocation programme that depends on the particular risks to which the investor is exposed. Similarly, the very concept of a risk-free asset depends on the investor’s time-horizon and on his objectives. Hence, a five-year zero-coupon Treasury bond will not prove a perfectly safe investment for a private investor interested in a real estate acquisition in five years. The actual risk-free asset in this context (which we call below the liability-hedging portfolio) would instead be an asset perfectly correlated with real estate prices. More generally, an investor whose objective is the acquisition of property would accept low and even negative returns in situations when real estate prices fall significantly, but will not be satisfied with relatively high returns if these returns do not match dramatic increases in real estate prices. In such circumstances, a long-term investment in stocks and bonds, with a performance weakly correlated with real estate prices, would not be the right investment. Likewise, in a pension context, absolute returns, often perceived as a natural choice in private wealth management, would not be a satisfactory response to the needs of a private investor facing long-term inflation risk, where the concern is capital preservation in real terms.
In other words, the first benefit of the ALM approach is perhaps its impact on the menu of asset classes, with a focus on including an asset that exhibits the highest possible correlation with the liability portfolio.

Our paper is related to the literature on long-term financial decisions, which starts with the seminal work of Merton (1969; 1971) and was further specialised to encompass either uncertain interest rates (Campbell and Viceira 2001; Brennan and Xia 2002; Wachter 2003), uncertain risk premia (Kim and Omberg 1996; Campbell and Viceira 1999), or both (Brennan et al. 1997; Lynch 2001; Campbell et al. 2003). These early papers highlight important aspects of life-cycle investing, including the usefulness of real bonds for inflation-hedging purposes. On the other hand, they mostly abstract away from some of the key complexities of private financial decisions. A large number of more recent papers have subsequently focused on integrating salient features of private wealth management including the impact of human capital (Bodie et al. 1992; Heaton and Lucas 2000; Viceira 2001; Cocco et al. 2005), illiquid real estate allocation (Sinai and Souleles 2005) or borrowing constraints on optimal allocation decisions. However, because they usually rely on standard expected utility maximisation of terminal wealth, these papers fail to integrate a key dimension of private wealth management, that is that investment decisions should be designed to help investors achieve certain pre-determined objectives such as preparing for retirement or, earlier in the life cycle, for real estate acquisition. In sum, we argue that the literature on household finance has mostly taken an asset management perspective, as opposed to an asset-liability management perspective. In parallel, several authors have attempted to extend intertemporal selection analysis to account for the presence of liability constraints in the asset allocation policy. An initial attempt to introduce liability constraints in optimal portfolio selection theory was made by Merton (1993), who studies the allocation decision of a university that manages an endowment. In this particular strand of the finance literature, are papers by Rudolf and Ziemba (2004), who have formulated a continuous-time dynamic programming model of pension fund management in the presence of a time-varying opportunity set, and Sundaresan and Zapatero (1997), whose work also involves an endogenous retirement decision.

Our paper can be seen as an attempt to merge these two somewhat separate strands of the literature, that is, the literature on long-term financial decisions for private investors, which has focused mostly on an asset-only perspective, and the literature on asset-liability management decisions, which have been analysed mostly from an institutional perspective (pension funds, insurance companies, or endowments).

We do so by casting the long-horizon life cycle investment problem in an asset-liability management framework suitable for the private wealth management context, which allows us to show that pursuing an asset-only strategy usually involves a substantial opportunity cost. Broadly, taking an ALM approach leads to defining risk and return relative to a liability portfolio, a critical improvement on asset-only asset allocation models that fail to account for the presence of investment and/or consumption goals and
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objectives, such as preparing for retirement or for a real estate acquisition. As a result, taking an ALM approach leads to a focus on the liability-hedging properties of various asset classes, a focus that would, by definition, be absent from an asset-only perspective.

The rest of the paper is organised as follows. In section 2, we introduce a formal stylised model of asset-liability management for household-financial decisions. In section 3, we present a series of illustrations of the usefulness of asset-liability management techniques in a household-finance context, with a focus on a pension objective, as well as a real estate acquisition objective. A conclusion as well as suggestions for further research can be found in section 4.
Introduction
1. A Formal Model of Asset-Liability Management in Private Wealth Management
1. A Formal Model of Asset-Liability Management in Private Wealth Management

In what follows, we introduce a formal model of asset-liability management and discuss its application in private wealth management. This analytical approach to ALM is appealing in spite of its highly stylised nature because it leads to a tractable solution, allowing one to understand fully and explicitly the various mechanisms affecting the optimal allocation strategy. In particular, we argue that the three-fund separation theorem we obtain, typical of optimal asset allocation decisions in the presence of stochastic state variables, is a parsimonious way to capture some of the complexity involved in private wealth management decisions.

1.1 Stochastic Model for Risk Factors Impacting Asset and Liability Values

We used a suitable econometric model to provide estimates for the short- and long-term dependencies between the return on a set of asset classes and the factors impacting liability values. To do so, we rely on a stationary vector-autoregressive (VAR) approach, extended to ALM to model the joint asset and liability return dynamic distributions. Our approach is similar to that of Campbell et al. 2003 and Campbell and Viceira 2005 (see also Hoevenaars et al. 2009):

\[ z_{t+1} = \Phi_0 + \Phi_1 z_t + \varepsilon_t \]  

(1)

where \( z_t \) is a vector of risk factors impacting asset and liability values, and \( \varepsilon_t \) an error term whose distribution must be specified.

More specifically, on the asset side, our empirical analysis focuses on a set of traditional and alternative asset classes. Stock returns are represented by the CRSP value-weighted stock index. Commodities are proxied by the S&P Goldman Sachs Commodity index (GSCI). Real estate investments are represented by the FTSE NAREIT real estate index, which is a value-weighted basket of REITs listed on NYSE, AMEX, and NASDAQ. We thus limit the opportunity set to liquid and publicly traded assets.

On the liability side, we consider two main examples. The first has to do with a pension objective; the focus is to preserve purchasing power after retirement. As far as this objective is concerned, the natural proxy for liability returns is the return on a treasury inflation-protected security (TIPS), since the payoff of the TIPS is given by the cumulative
inflation over the time-horizon. Unfortunately, an empirical time series with a sufficiently long history is not available for TIPS. We therefore construct a time series for a constant-maturity treasury inflation-protected bond by using the following ingredients: constant maturity nominal bond returns from CRSP ($r^b_t$), the median inflation forecast ($\hat{\pi}_t$) from the Survey Research Center (University of Michigan) and realised inflation ($\pi_t$) as proxied by the US consumer price index. The return on the TIPS portfolio is denoted by $r^r_t$ and given by:

$$r^r_t = r^b_t - \hat{\pi}_t + \pi_t - \Lambda_\pi$$

where $\Lambda_\pi$ is the inflation risk premium that we assume at fifty annual basis points, which is consistent with Kothari and Shanken (2004). The constructed nominal return incorporates an interest rate risk premium since the nominal bond return $r^b_t$ accounts for the presence of a term spread. The constant time to maturity for the bond is set to five years and, consistently, the five-year-ahead inflation forecast is used from the inflation survey.

The second example has to do with real estate acquisition. In this case, we assume that the liability return coincides exactly with the return on the FTSE NAREIT index. Obviously, this is a simplification since the value of a given piece of property that a private investor considers acquiring will not be perfectly correlated with the return on a broad REIT index, which is well known to be exposed to both equity and real estate markets. We use an investable proxy for real estate returns because it will prove useful in the empirical section to analyse a complete market setting, where risk factors in liability returns are entirely spanned by existing securities.

In total, the vector of endogenous variables $z$ in (1) contains ten elements. Table 1 presents summary statistics of the corresponding time series of quarterly returns from Q1 1962 to Q4 2005. We see that Sharpe ratios for all asset classes are somewhat similar over the sample period, with the notable exception of long bonds, which are dominated on a risk-adjusted basis. These summary statistics refer to arithmetic averages over the empirical sample, while the analysis that follows is designed to derive horizon-dependent moments for the different asset classes.

To estimate the VAR model, we follow the procedure of Hoevenaars et al. (2009), who impose restrictions on the parameters. We first define a core system of equations in the VAR: consider the sub-vector $z_1$ containing the excess returns on the stock, the long bond, the return on the T-Bill, the inflation rate, the credit spread, the term spread, and the dividend yield. We estimate an unrestricted VAR model for $z_1$:

$$z_{1,t+1} = \Phi_{0,1} + \Phi_{1,1} z_{1,t} + \varepsilon_{1,t}$$

As a consequence, the variables that are not contained in the core model have no impact on core variables. We then define $z_2$ as the complement of $z_1$: $z_2$ thus contains the excess returns on commodities, real estate, and the TIPS. We then estimate the following regression system:

$$z_{2,t+1} = \Phi_{0,2} + A z_{1,t+1} + B z_{2,t} + \Phi_{1,2} z_{2,t} + \varepsilon_{2,t}$$

where the matrix $\Phi_{1,2}$ is restricted to be diagonal and so is the covariance matrix of the error term $\varepsilon_2$. These restrictions imply that non-core variables have no effect on each other and that their innovations are independent. The matrices $A$ and $B$ are not restricted. In particular, $A$ accounts
for contemporaneous correlations of non-core variables and core variables, so we can assume that the error terms $\varepsilon_1$ and $\varepsilon_2$ are uncorrelated. Finally, we can write the restricted parameters of the VAR system (1) as:

$$\Phi_1 = \begin{pmatrix} \Phi_{0,1} & 0 \\ \Phi_{1,1} & \Phi_{1,2} \end{pmatrix}$$

These restrictions allow us to make use of the full history for core variables, which covers the period from Q1 1962 to Q4 2005, even if the sample for non-core variables is much smaller. Indeed, our series of returns on commodities starts in Q2 1970, while the series of returns on real estate starts in Q2 1972, and the series for the TIPS starts in Q2 1979.\(^2\)

We present the estimated matrix $\Phi_1$ in table 2 and the residual correlations in table 3. The VAR-implied dynamics for asset and liability returns are indeed governed by these two parameter matrices. The VAR modelling framework is particularly convenient in a portfolio context, since it generates analytic expressions for time-dependent variances and expected returns. The model-implied first and second moments of compounded returns can in fact be written as:

$$E_t \left( \sum_{k=1}^{T} H z_{t+k} \right) = \left( \mu_T^A \right)$$

$$V_t \left( \sum_{k=1}^{T} H z_{t+k} \right) = \left( \Sigma_T^A \right)$$

Here $\mu_T^A$ is the vector of expected returns on assets, $\mu_T^L$ is the scalar return on the liability portfolio (either inflation-linked or real-estate related) for a horizon equal to $T$ years. $\Sigma_T^A$ is the asset covariance matrix for the $T$-year horizon, $\Sigma_T^{AL}$ is the vector of covariances between the asset classes and the liability portfolio, and $\Sigma_T^L$ is the liability variance for a horizon equal to $T$ years as well. $\Sigma_T^e$ is the residual covariance matrix. Finally, $H$ is a matrix that selects the vector of excess returns on the assets and the liability from the state vector $z$.

Figure 1 depicts the horizon-dependent annualised volatilities as derived from the fitted parameters and implied by the VAR system (see formula (4)). Consistent with the findings in Campbell and Viceira (2001), we find that equity markets are less risky for the long-term. This effect is explained by the presence of implied mean-reversion in stock returns. Indeed, dividend yields are widely documented to exhibit significant predictive power for stock returns (Campbell and Viceira 2001). On the one hand, innovations to dividend yields and stock returns are negatively correlated (see table 3). On the other
hand, lagged dividend yields are positively correlated with contemporaneous stock returns (see table 2), which leads to smoothing of the past innovations. In contrast, we find that investing in T-Bills generates higher annualised volatility as the investment horizon increases, which is due to the uncertainty involved in rolling over short-term debt in the presence of stochastic interest rates. Figure 1 also shows that the term structure of volatility implied by the VAR model is also upward-sloping for real estate and commodity investments.

Figure 2 shows the correlations of the various asset classes with the consumer price index as functions of time horizon. The inflation-hedging properties of the various asset classes depend on the horizon. For instance, T-Bills have negative correlation with realised inflation over short horizons, but this correlation becomes positive and relatively high over horizons exceeding ten years. In addition, we find that stocks and inflation are negatively correlated over the short term, but that they have good inflation-hedging properties over horizons that exceed twenty years. Our finding of a negative short-term relationship between expected stock returns and expected inflation is consistent with previous empirical findings on the subject (Fama and Schwert 1977; Gultekin 1983; Kaul 1987) and is also consistent with the intuition that higher inflation leads to lower economic activity, thus depressing stock returns (Fama 1981). On the other hand, higher future inflation leads to higher dividends and thus higher returns on stocks (Campbell and Shiller 1988), and thus equity investments should offer significant inflation protection over longer horizons, as confirmed by a number of recent empirical academic studies (Boudoukh and Richardson 1993; Schotman and Schweitzer 2000). Overall, these findings suggest that using a standard one-period optimisation model, as is customary in private wealth management, is a great over-simplification that does not allow investors to benefit from the life-cycle effects induced by time-varying opportunity sets. We will formally confirm that optimal allocation decisions are a function of time-horizon, a fact that cannot be captured in the context of a static portfolio optimisation exercise. We will also argue that not taking into account the presence of liabilities leads to a substantial opportunity cost.

1.2 Life-Cycle Investment Decisions in Asset-Liability Management

In a dynamic asset allocation model, it is customary to assume that the preferences of the investor are expressed in terms of expected utility of terminal asset value:

$$\max_{\omega} E[u(A_T)]$$

(5)

where $T$ is the investor’s time-horizon. One key problem with this objective is precisely that it fails to recognise that targeted (liability) payments are scheduled beyond the horizon $T$. One natural approach to tackling this problem is to recognise that terminal wealth at date $T$ is made of a long position in the asset portfolio with value $A_T$ and a short position in the liability portfolio with value $L_T$. Another related approach to accounting for the presence of liability payments beyond the horizon is to introduce a related state variable, the funding ratio, defined as the ratio of assets to liabilities:

$$F_t = \frac{A_t}{L_t}$$

(6)
1. A Formal Model of Asset-Liability Management in Private Wealth Management

which is well defined as long as $L_t$ is not zero. This quantity is commonly used in pension fund practice, where a fund is said to be overfunded when the funding ratio is greater than 100%, to be fully funded when the funding ratio equals 100%, and to be underfunded when the funding ratio is lower than 100%. From an interpretation standpoint, focusing on the funding ratio amounts to using the liability value process $(L_t)_{t \geq 0}$ as opposed to the bank account, as a numeraire, an approach that has been taken by van Binsbergen and Brandt (2007), and which we also take in this paper.

We take $u$ to be the constant relative risk aversion (CRRA) utility function, defined as:

$$u(x) = \frac{x^{1-\gamma}}{1-\gamma} \quad \text{for } x > 0$$

$$u(x) = -\infty \quad \text{for } x \leq 0$$

where $\gamma$ lies in $[1, \infty]$. If $\gamma = 1$ we obtain the logarithmic utility function.

If one further makes the additional assumption of log-normal return distributions, the portfolio choice model collapses into a mean-variance problem:

$$\max_{\omega} \left\{ \mathbb{E} \left[ \frac{F_T^{1-\gamma}}{1-\gamma} \right] \right\}$$

$$\Rightarrow \max_{\omega} \left\{ \mathbb{E} \left[ r_T^F \right] + \frac{1-\gamma}{2} \mathbb{V} \left[ r_T^F \right] \right\}$$

with $r_T^F$ the $T$-period forward-looking log-funding-ratio return such that $F_T = \exp (r_T^F)$ and $\gamma$ the relative risk aversion. In the absence of log-normal returns, this approach can also be justified by a second-order approximation (Campbell 1993; 1996 or Campbell et al. 2003 for details).

Analysing the above programme for different degrees of relative risk aversion coincides with analysing the mean-risk efficient frontier of the terminal funding ratio or the $T$-period funding ratio return. Portfolios on the efficient frontier are thus the solutions to the following programme:

$$\min_{\omega} \frac{1}{2} \mathbb{V} \left( r_T^F \right)$$

s.t. $\mathbb{E} \left( r_T^F \right) = \mu_T^F$ \hspace{1cm} (8)

where $\mu_T^F$ is an achievable target funding ratio return. The Lagrangian for this programme is given as:

$$L = \frac{1}{2} \mathbb{V} \left( r_T^F \right) - \lambda \left( \mathbb{E} \left( r_T^F \right) - \mu_T^F \right)$$

As transparent from equations (3) and (4), first and second moments in (9) can be derived from the VAR model and exhibit an explicit dependency with respect to time-horizon. Assuming a fixed-mix allocation $\omega$ for a given time-to-maturity $T$, and discretising Ito’s lemma (Campbell et al. 2003), we can write the log-funding ratio return as:

$$r_T^F = \omega^T \left( r_T^A + \frac{1}{2} \sigma_T^A \right) - \frac{1}{2} \omega^T \Sigma_T^A \omega - r_T^L$$

with $\Sigma_T^A$ the covariance matrix, $\sigma_T^A$ its diagonal, $r_T^A$ the vector of the asset log-returns, $r_T^L$ the log-liability return and $\omega$ denotes the vector of the asset portfolio weights. All returns here are expressed as excess returns over the T-Bills.

Using (10) we obtain the VAR–implied annualised expected funding ratio returns:
1. A Formal Model of Asset-Liability Management in Private Wealth Management

We may compare these portfolio allocations to mean-variance-efficient portfolios of an asset-only investor who does not take the presence of liability streams into account. As a consequence, the investor focuses on asset return only, and in (10) becomes:

\[ r_T = \omega' \left( r_T^A + \frac{1}{2} \sigma_T^A \right) - \frac{1}{2} \omega' \Sigma_T^A \omega + r_R \]

with \( r_R \) the return of the risk-free asset (T-Bills in this setting). The expectation in (11) thus becomes:

\[ E(r_T) = \omega' \left( r_T^A + \frac{1}{2} \sigma_T^A \right) - \frac{1}{2} \omega' \Sigma_T^A \omega + \mu_R^R \]

while the variance in (12) is now given by:

\[ \nu(r_T) = \omega' \Sigma_T^A \omega + 2 \omega' \sigma_T^{AR} + \sigma_T^R \]

where \( \sigma_T^{AR} \) is the covariance vector of the assets with the risk-free asset at horizon \( T \) and \( \mu_R^R \) and \( \sigma_T^R \) the corresponding mean and variance of the risk-free asset. Accordingly, we can solve the Lagrangian for the asset-only problem and obtain the set of mean-variance-efficient portfolios \( \tilde{\omega} \) as:

\[ \tilde{\omega} = \alpha (\Sigma_T^A)^{-1} \left( \mu_T^A + \frac{1}{2} \sigma_T^A \right) - (1-\alpha) (\Sigma_T^A)^{-1} \sigma_T^{AR} \]

Again, we obtain a separation result involving the PSP as well as a portfolio capturing hedging demand against unexpected changes in interest rates (this is the second term in the right side of (18)). This demand was not present in (14); indeed, the funding ratio is the ratio of assets to liabilities, so the impacts of changes in interest rates...
1. A Formal Model of Asset-Liability Management in Private Wealth Management

on the numerator and the denominator cancel out, as can be seen from the fact that the risk-free rate does not appear in (10). In the empirical section (3), we study the behaviour of sub-optimal strategies (both in the AM and in the ALM sense), in which the hedging demand in (18) is simply ignored, to analyse the opportunity costs of a purely myopic portfolio strategy. Moreover, the portfolios in (18), while efficient in an asset-management sense, will not be efficient in the ALM space, and in the following section, we provide ample evidence of the efficiency/opportunity cost of not taking liabilities into account in private wealth management.

As argued before, our analysis of the life-cycle component of long-term investment decisions consists of allowing the asset allocation decisions to depend on the investor’s time horizon, a dimension that cannot be captured by standard static optimisation problems. While a significant improvement, this approach, which directly follows the seminal work by Campbell et al. (2003), does not provide the most general form of asset allocation strategies. There are in fact three different levels of extensions of standard static portfolio allocation models. They involve: i.) allowing for time-horizon dependencies, ii.) allowing for (purely deterministic) time dependencies, and iii.) allowing for time- and state-dependencies. These advances were made possible by the pioneering work of Merton (1969; 1971), who opened a world of opportunities for more subtle dynamic asset allocation decisions, involving intertemporal adjustments to the asset mix as time goes by. The calculation of optimal intertemporal portfolios is, however, very challenging, whether analytically or numerically, as soon as the number of state variables exceeds one or two. In this paper, we therefore stick to the first level of extension and focus on allowing investors with different time-horizons to hold different optimal portfolios. The second level of extension (time-dependency without state-dependency), while seemingly offering a good compromise between the conflicting objectives of generality and tractability, and while often used in the context of so-called target-date funds, simply cannot be rationalised within a formal asset allocation model (Viceira 2007).
2. Empirical Illustrations of the Benefits of an ALM Approach to Private Wealth Management
2. Empirical Illustrations of the Benefits of an ALM Approach to Private Wealth Management

In the empirical applications, we distinguish between a pension-related objective and a real estate acquisition objective. The idea of this distinction is to highlight the importance of properly identifying the appropriate benchmark liability and its impact on optimal portfolios. It is also worth noting that the nature of the liability stream raises the question of whether or not a perfect hedge against unexpected shocks in the liability asset is available and how to model the liabilities. For the pension-related objective, it is appropriate to assume that the contractual pension is written in real terms with an actual payment indexed to inflation. For the acquisition of real estate, several real estate indices are suitable candidates for the liability benchmark. To distinguish explicitly between the complete and the incomplete market case, we choose (as explained above) to proxy real estate prices with an investable real estate investment trust index.

The illustrations that follow are highly stylised, and a number of additional dimensions would have to be addressed in the context of a real-world application of the framework developed in this paper. Among these additional dimensions is, in particular, the necessity to account for the tax regimes to which different forms of investment are subject. It is to be expected that accounting for tax treatments will impact the optimal allocation decisions in a complex manner. It would also be desirable to account for a variety of constraints (e.g., maximum drawdown limits) and objectives (e.g., bequest motives) that extend beyond standard expected utility maximisation framework used in this paper and that are relevant to private investors. Finally, one would also need to take into account the presence of flexible contribution as well as consumption schedules, as opposed to assuming, as is done below, a pre-defined contribution and withdrawal or even more than one. Designing a very general asset allocation model incorporating a realistic tax treatment, a variety of risk budgets, as well as flexible endogenous contribution and liability schedules, would certainly be a very desirable objective, but not one that could be achieved in an analytical model such as the one proposed in this paper. While our results are derived under a number of simplifying assumptions, a number of useful insights can still be provided by this stylised analysis, in particular related to the fact that failing to take an ALM approach to long-term investment decisions and sticking to the sub-optimal asset-only perspective will generate very substantial opportunity costs for the private investor.

2.1 Pension-Related Objective

In this section, we focus on a pension objective and consider a wealthy sixty-five-year-old individual who is already retired. His/her goal is to ensure inflation-protected pension payments, which we normalised at 100 with no loss of generality, at a given horizon date $T$ (we consider $T =$ one, five, ten, and twenty-five years). To achieve this goal the individual is prepared to invest a fixed amount, and we assume that the funding ratio at retirement date is 100%. For each given time-horizon, we will derive four different efficient frontiers corresponding to i) the AM objective in (18), where we assume that the menu of asset classes

---

5 - Similar results could be obtained for a younger investor, who would prepare for retirement by making contributions to the pension portfolio until retirement date.
does not include the perfect liability-hedging asset; the investor mistakenly uses a short-term one-year horizon—the actual horizon is \( T \) years—and ignores the hedging demand against interest rate risk (a case we term “AM SH” and to which we refer as “AM with short horizon”); ii) the AM objective in (18), where we still assume that the menu of asset classes does not include the perfect liability-hedging asset (a case we term “AM LH” and to which we refer as “AM with long horizon”) but the investor uses the true horizon \( T \) when computing the parameters; iii) the ALM objective (14), with a proper treatment of the horizon \( T \), but without the perfect liability-hedging asset in the menu of asset classes (a case we term “ALM–” and refer to as the “incomplete market case”); and iv) the ALM objective (14) with a proper treatment of the horizon \( T \) and with the perfect liability-hedging asset (in this case an inflation-linked bond) in the menu of asset classes (a case we term “ALM+” and refer to as the “complete market case” since the menu of tradable assets is sufficiently rich to allow for a perfect hedge of liability risk).

The first asset-only approach is consistent with the static approach used in standard asset allocation exercises. The second efficient frontier is an improvement based on allowing for time-horizon dependencies, but it still fails to account for the presence of liabilities. The third efficient frontier depicts the case in which both the time-horizon and the presence of liabilities are taken into account, but no effort is made to integrate new asset classes with specific liability-hedging properties. The fourth efficient frontier corresponds to the final improvement, with an asset allocation decision that takes into account time-horizon effects and the presence of liabilities, and with a specific liability-hedging asset in the asset mix. Analysing separately the ALM– and ALM+ portfolios allows us to disentangle the two main benefits of the ALM approach: namely the benefits that can be put down to assessing risk and return relative to the liability benchmark and those that can be put down to a liability-hedging instrument.

We focus first on the benefits obtained by taking into account the investment horizon as opposed to using a standard static model in an asset-only context. To do so, we compute for allocations i) and ii) the expected value and the variance of the log return on the asset portfolio, following (16) and (17) respectively. We then let \( \alpha \) vary over the interval \([0, 1]\), so as to obtain a representation of these strategies in the AM space, as shown in figure 3. By definition, the AM LH strategy dominates the AM SH strategy in the mean-variance sense, except, of course, in a one-year time to horizon, where they are mathematically equivalent. In fact, for \( T \) = five or ten years, the performance of the AM strategy with a short horizon is similar to that of the AM strategy with a long horizon. On the other hand, when the horizon is very long (\( T \) = twenty-five years), the opportunity cost of using a static optimisation model with a short-term objective is substantial.

We then move on to the analysis of the additional benefits by accounting for the presence of liabilities, in addition to the investment horizon. To do so, we obtain expected (log) funding ratios according to (11) and variances for (log) funding ratios from (12) for each set of efficient
portfolios corresponding to the situations ii), iii), and iv). Assuming normality of the log-returns in the system (1) and letting \( \mu^F = \mathbb{E}(r^F) \) and \( \sigma^F = \mathbb{V}(r^F) \) we can derive the expected value and the variance of the funding ratio as:

\[
\mathbb{E}(FR) = e^{\mu^F + \frac{1}{2} \sigma^F} \quad (19)
\]

\[
\mathbb{V}(FR) = \left(e^{\sigma^F} - 1 \right) \left(e^{2\mu^F + \sigma^F} \right) \quad (20)
\]

We let \( \alpha \) vary in (14) and (18) from 0 to 1 and use (19) and (20) to plot the efficient frontiers for various investment horizons. Figure 4 shows the three frontiers (ALM with TIPS, ALM without TIPS, and AM SH – without TIPS) when the investor’s goal is based on pension payments related to the consumer price index, and this for four different time-horizons (one year, five years, ten years, and twenty-five years).

As the figure shows, the myopic strategy “AM SH” is strongly dominated by ALM-efficient portfolios, especially when the menu of asset classes available to the ALM investor includes inflation-linked bonds. For a better sense of the magnitude by which ALM-efficient portfolios outperform asset-only efficient portfolios, table 4 presents summary statistics for three portfolios on the efficient frontiers in figure 4, corresponding to low, medium, and high target expected returns. As it happens, we consider three levels of annualised expected return on the funding ratio, 5%, 10%, and 15%.

Shortfall probabilities \( P(\mathbb{R} < 1) \) are simply derived from the relation \( P(\mathbb{R} < 1) = P(r^F < 0), \) which under the assumption of Gaussian returns can also be written:

\[
P(r^F < 0) = \Phi \left( -\frac{\mu^F}{\sqrt{\sigma^F}} \right) \quad (21)
\]

with \( \Phi \) being the cumulative density function of the standard normal distribution. Expected shortfalls can in fact be derived in analytical form from the Black-Scholes formula as follows. First, we can decompose the expression for the put price as:

\[
\mathbb{E} \left[ e^{-rT} \max(K - S_T, 0) \right] = e^{-rT} K \Phi(S_T < K) - e^{-rT} \mathbb{E} (S_T 1_{\{S_T < K\}})
\]

\[
= e^{-rT} K \Phi(-d_2) - S_0 \Phi(-d_1) \quad (22)
\]

Secondly, we know that the expected shortfall ES can be written:

\[
ES = 1 - \mathbb{E} (S_T | S_T < K)
\]

\[
= 1 - \frac{\mathbb{E} (S_T 1_{\{S_T < K\}})}{P(S_T < K)} \quad (23)
\]

Noting that \( \mu^F \) and \( \sigma^F \) are annualised parameters and setting \( S_T \equiv \mathbb{R}, S_0 \equiv 1, K \equiv 1, r \equiv \mu^F + \frac{1}{2} \sigma^F, T \equiv 1 \) we obtain the expected annualised shortfalls from (22) and (23) as:

\[
ES = 1 - \mathbb{E} (FR | FR < 1)
\]

\[
= 1 - \frac{e^{\mu^F + \frac{1}{2} \sigma^F} \Phi(-d_1)}{\Phi(-d_2)} \quad (24)
\]

with:

\[
d_1 = \frac{\mu^F + \sigma^F}{\sqrt{\sigma^F}}; \quad d_2 = \frac{\mu^F}{\sqrt{\sigma^F}} \quad (25)
\]

Table 4 shows that for a given expected funding ratio the optimal allocation from the ALM sense dominates optimal allocations from an AM sense in terms of the risk perspective, and this holds whether risk is measured in terms of funding ratio volatility, expected shortfall or probability of a shortfall. This result is more pronounced for the more risk-averse investor, which is not surprising, since an AM perspective cannot

6 - One key difference is that probabilities are taken under the risk-adjusted measure in an option-pricing context, while they are taken under the historical measure in the context of risk parameter estimation.
2. Empirical Illustrations of the Benefits of an ALM Approach to Private Wealth Management

deliver a sound risk-minimisation strategy with respect to the liability benchmark. We also obtain that the effect is more pronounced for longer horizons, which is the typical case in a pension-related context. We confirm that ALM strategies in complete market environments (i.e., when the inflation-linked bonds are included in the asset mix) strongly dominate the corresponding ALM strategies in the absence of a perfect liability-hedging instrument. For example, in case of a twenty-five-year horizon and a high (15%) expected return target, the volatility of the funding ratio is 21.16% for the ALM strategy in complete markets, versus 29.49% in an incomplete market setting, and 35.49% for the AM strategy.

Table 5 gives the composition of the efficient portfolios for the three strategies ALM+, ALM–, and AM SH, when the target expected funding ratio is low, high, or medium. Overall, we find a substantial amount of leverage in some cases, an effect which is larger when a high expected funding ratio, implying a more aggressive strategy is targeted. When comparing the ALM– and AM SH allocations, we find that the allocation to stocks is smaller in ALM– than in AM SH for the short horizon \( T = \) one year. This is in line with the findings of figure 2, which suggest that stocks have poor hedging properties against inflation in the short term, while having substantial volatility (see figure 1). Overall, this makes them relatively undesirable in the liability-hedging portfolio. In contrast, when the horizon lengthens, the inflation-hedging properties of stock investment improve while the annualised volatility decreases, which makes the allocation to stocks higher in ALM– compared to AM SH. This effect is most visible for the longest horizon \( (T = \) twenty-five years) and the least risk-averse investor, where the weight allocated to stocks is multiplied by three.

Again, these results are likely to underestimate the inflation-hedging properties of asset classes such as stocks, and quite different results would be obtained in the context of a more general model allowing for the presence of long-term cointegration relationships. Using a vector error correction model (VECM), Amenc et al. (2009) actually show that novel forms of investment, including equities, commodities, and real estate, in addition to inflation-linked securities, can be designed to decrease the cost of inflation insurance for long-horizon investors. As a result, it is important to emphasise that investing in inflation-linked instruments is neither the only nor necessarily the most cost-efficient means of obtaining protection from inflation uncertainty. For one, the capacity of the inflation-linked securities market is not sufficient to meet the collective demand of institutional and private investors. And the over-the-counter inflation derivatives market, which is used in institutional money management, is probably not a natural alternative to inflation-linked bonds for private investors. In addition, real returns on inflation-protected securities, negatively impacted by the presence of a significant inflation risk premium, are usually very low, which implies that investing in inflation-linked securities, when feasible, is costly.

In general, our results strongly suggest that failing to take an ALM approach to
2. Empirical Illustrations of the Benefits of an ALM Approach to Private Wealth Management

2.2 Real Estate Acquisition Objective
We now take an investor who wishes to invest either a single contribution or a stream of contributions for a future expenditure, e.g., to buy a property the current value of which is normalised at 100 in \( T \) years, and the future value of which evolves according to some stochastic property index value. For simplicity, as explained above, we assume that house price increases are perfectly correlated with the REIT index returns. Of course, because real estate prices are only imperfectly correlated with a broad-based REIT index, it would be more accurate to introduce a separate series for the dynamics of real estate prices. This, however, would induce a degree of market incompleteness.

Again, we take four sets of portfolios: the AM-efficient portfolios with short horizons of one year and long horizons \( T = \) five, ten, or twenty-five years with an asset mix that does not include a real estate-related investment vehicle, the ALM-efficient portfolios assuming a focus on the terminal funding ratio, as opposed to terminal wealth, but also in the absence of the REIT investable index (ALM− efficient frontier), and finally the ALM-efficient portfolios assuming a complete market setting with the REIT index available in the menu of asset classes (ALM+ efficient frontier). To generate comparable portfolios, we consider the improvement in surplus volatility for a given expected surplus. We find that the presence of assets allowing investors to span real estate price uncertainty proves to be a key element in improving the efficient frontiers obtained from an ALM perspective. We also confirm that a portfolio efficient in an AM sense is not necessarily efficient in an ALM sense, and vice-versa, as was done in the context of the pension-related objective.

Figure 5 shows the three efficient frontiers (ALM+, ALM−, and AM SH) when the liability benchmark is given by the real estate index, which reflects the investor’s intention to acquire property. The improvement in efficient frontiers obtained from taking an ALM perspective is even more spectacular than in the pension-related case. That there is more uncertainty about future house prices than about future inflation rates presumably accounts for this greater improvement. These results are confirmed by table 6, where summary statistics for some portfolios on the efficient frontiers in figure 5 are presented. We find that for a given expected funding ratio the decrease in funding ratio risk (measured in terms of funding ratio volatility, expected shortfall, or shortfall probability) is substantial, especially for more risk-averse investors and longer time-horizons. For example, for a twenty-five-year horizon and the most risk-averse investor, the volatility of the funding ratio is almost four times higher with the AM SH objective than that obtained with the ALM objective in the presence of the REIT index in the menu of asset classes!

As a general comment, we find that adding the liability-hedging instrument to the menu of asset classes (going from ALM− to ALM+) generates a marginal benefit
greater than the benefit obtained from replacing the AM objective with an ALM objective without introducing a proper liability-hedging instrument (going from AM to ALM–). Finally, table 7 gives the composition of the optimal portfolios in the ALM+, ALM–, and AM SH cases, which again, in general involves a substantial amount of leverage.
2. Empirical Illustrations of the Benefits of an ALM Approach to Private Wealth Management
Conclusion
In this paper, we have shown that a significant fraction of the complexity inherent to optimal asset allocation decisions in private wealth management can be addressed by adding a single additional state variable, the value of the household liability portfolio, which accounts in a parsimonious and tractable way for investors' specific constraints and objectives. We have also presented a series of numerical illustrations suggesting that the model introduced in this paper could be applied in several situations typical of private wealth management.

Our analysis has great potential implications for the wealth management industry. Indeed, it is often said that proximity to investors is the main raison d'être of private wealth managers and a key source of competitive advantages. Building on this proximity, private bankers should be ideally placed to better account for their clients' specific liability constraints when engineering an investment solution for them. Most private bankers actually implicitly promote an ALM approach to wealth management. In particular, they claim to account for the investor's goals and constraints. The technical tools involved, however, are often non-existent or ill-adapted. While the private client is routinely asked all kinds of questions about his current situation, goals, preferences, constraints, etc., the resulting service and product offering mostly boil down to a rather basic classification in terms of risk profiles. In this paper, we have provided a framework suggesting that asset-liability management is an essential improvement in private wealth management that allows private bankers to provide their clients investment programmes and asset allocation advice that truly meet their needs.

Broadly speaking, our analysis has shown that taking an ALM approach to private wealth management generates two main benefits. First, it has a direct impact on the selection of asset classes. In particular, it leads to a focus on the liability-hedging properties of various asset classes, a focus that would, by definition, be absent from an asset-only perspective. Second, it leads to defining risk and return in relative rather than absolute terms, with the liability portfolio used as a benchmark or numeraire. This is a critical improvement on asset-only asset allocation models, which fail to recognise that changes to asset values must be analysed in comparison to changes in liability values. In other words, private investors are not seeking terminal wealth per se so much as they are seeking terminal wealth whose purchasing power enables them to achieve such goals as preparing for retirement or buying property.

Our research can be extended in a number of directions. First, it would be desirable to incorporate the impact of taxes in the analysis of optimal asset allocation decisions. While tax optimisation is arguably one of the key sources of added value in private wealth management, it is usually very challenging to account for the detailed features of tax regulations that vary significantly across countries in the context of a formal optimal allocation model, especially given that tax treatment can depend on trading behaviour and portfolio turnover in a complex manner. Other previously mentioned elements that are left for further research are the introduction of flexible contribution/withdrawal decisions as well as the extension of the asset allocation model to more general forms of state-dependent optimal allocation strategies.
Finally, and perhaps more importantly, it would be interesting to try to cast the ALM approach to private wealth management in a context in which the investor has a behavioural objective. One challenge here is that recent advances in behavioural finance, while they provide very useful insights into investors’ behaviour, do not provide much guidance to the design of a formal normative analysis of optimal asset allocation decisions. A possible approach would involve capturing some of this complexity by adding a set of suitably specified investor-dependent goals and constraints to the standard expected utility maximisation paradigm. These questions, as well as other possible extensions, are also left for further research.
Conclusion
Appendix

Figure 1: Term structure of risk

This figure plots the volatility implied by the VAR model of the nominal return on each asset class, as a function of investment horizon.

Figure 2: Term structure of correlations with realised inflation

This figure plots the correlation implied by the VAR model between nominal returns on the asset classes and realised inflation, as a function of the investment horizon.
Appendix

Figure 3: Mean-variance-efficient portfolios in the AM space

This figure plots the efficient frontiers in the \([\mathbb{E}[r_T], \sigma(r_T)]\)-diagram, where \(r_T\) is the return on the asset portfolio after \(T\) years, as computed in (16) and (17). These frontiers are obtained by letting \(\alpha\) vary from 0 to 1 in (18). In the “ALM LH” situation, the investor has a “long-term” horizon equal to \(T\) years, while in the “AM SH” case she applies (18) by mistakenly assuming that \(T = \text{one year}\).
Figure 4: Mean-variance-efficient portfolios in the ALM space—CPI indexed returns

This figure plots the efficient frontiers in the $(\mathbb{E}[r^F_T], \mathbb{V}[r^F_T])$-diagram, where $\mathbb{E}[r^F_T]$ and $\mathbb{V}[r^F_T]$ respectively are the annualised expected value and standard deviation of the log funding ratio after $T$ years, as computed in (11) and (12). “ALM+” and “ALM–” are obtained by letting $\alpha$ vary from 0 to 1 in (14). “ALM+” refers to the complete market case where TIPS are available for trading, while “ALM–” refers to the incomplete market setting. “AM SH” is obtained by mistakenly assuming that $T = \text{one year}$ and letting $\alpha$ vary from 0 to 1 in the myopic portfolio rule.
Appendix

Figure 5: Mean-variance-efficient portfolios in the ALM space—RE indexed returns

This figure plots the efficient frontiers in the \( (\mathbb{E}[r_F^T], \sigma[r_F^T]) \)-diagram, where \( \mathbb{E}[r_F^T] \) and \( \sigma[r_F^T] \) are the annualised expected value and standard deviation respectively of the log funding ratio after \( T \) years, as computed in (11) and (12). “ALM+” and “ALM–” are obtained by letting \( \alpha \) vary from 0 to 1 in (14). “ALM+” refers to the complete market case where real estate is available for trading, while “ALM–” refers to the incomplete market setting. “AM SH” is obtained by mistakenly assuming that \( T = \) one year and letting \( \alpha \) vary from 0 to 1 in the myopic portfolio rule.
### Appendix

#### Table 1: Summary statistics

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<th>Average return</th>
<th>Volatility</th>
<th>Sharpe ratio</th>
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<tbody>
<tr>
<td>Stocks</td>
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<td>0.71</td>
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<td>Commodities</td>
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<td>16.95</td>
<td>0.71</td>
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<tr>
<td>TIPS</td>
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</tr>
<tr>
<td>3M T-Bill</td>
<td>6.82</td>
<td>1.51</td>
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</tr>
</tbody>
</table>

Summary statistics are calculated from quarterly log-returns from Q1 1962 to Q4 2005 for stocks, bonds, and T-Bills, from Q2 1970 through Q4 2005 for commodities, from Q2 1972 through Q4 2005 for real estate, and from Q2 1979 through Q4 2005 for TIPS. All returns are in excess of T-Bills except for the T-Bill returns. Average returns and volatilities are corrected for Jensen’s inequality and in annualised percentages.

#### Table 2: Estimated VAR—Φ₁

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<th></th>
<th>1</th>
<th>2</th>
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<td>−0.11</td>
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<tr>
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<td>0.89</td>
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<td>−0.01</td>
<td>−0.28</td>
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</table>

Quarterly returns from Q1 1962 to Q4 2005 are fitted to the VAR system (1). Blank elements are zero by construction of the VAR. All returns related to tradable assets 1, 2, 8, 9, and 10 are in excess of T-Bills.

#### Table 3: Correlation matrix of residuals Ɛ

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<td>−0.15</td>
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<td>0.06</td>
<td>−0.02</td>
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<tr>
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<td>0.10</td>
<td>0.11</td>
<td>0.02</td>
<td>0.09</td>
<td>0.07</td>
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<td>−0.14</td>
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Quarterly returns from Q1 1962 to Q4 2005 are fitted to the VAR system (1). Off-diagonal elements are correlations, and diagonal elements are standard deviations. All returns related to tradable assets 1, 2, 8, 9, and 10 are in excess of T-Bills.
### Table 4: Efficient portfolios (CPI-indexed returns)—Summary statistics

<table>
<thead>
<tr>
<th>Horizon (years)</th>
<th>LOW</th>
<th>MEDIUM</th>
<th>HIGH</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ALM+</td>
<td>ALM-</td>
<td>AM SH</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E[(r^F)]</td>
<td>5.00</td>
<td>5.00</td>
<td>5.00</td>
</tr>
<tr>
<td>(\sigma[(r^F)])</td>
<td>6.71</td>
<td>10.89</td>
<td>10.98</td>
</tr>
<tr>
<td>P(FR &lt; 1)</td>
<td>23.77</td>
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<td>32.43</td>
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<tr>
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<td>3.76</td>
<td>6.71</td>
<td>6.77</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E[(r^F)]</td>
<td>5.00</td>
<td>5.00</td>
<td>5.00</td>
</tr>
<tr>
<td>(\sigma[(r^F)])</td>
<td>6.83</td>
<td>11.64</td>
<td>12.21</td>
</tr>
<tr>
<td>P(FR &lt; 1)</td>
<td>23.18</td>
<td>33.38</td>
<td>34.15</td>
</tr>
<tr>
<td>ES</td>
<td>3.85</td>
<td>7.23</td>
<td>7.62</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E[(r^F)]</td>
<td>5.00</td>
<td>5.00</td>
<td>5.00</td>
</tr>
<tr>
<td>(\sigma[(r^F)])</td>
<td>6.57</td>
<td>11.48</td>
<td>12.32</td>
</tr>
<tr>
<td>P(FR &lt; 1)</td>
<td>22.41</td>
<td>33.16</td>
<td>34.23</td>
</tr>
<tr>
<td>ES</td>
<td>3.67</td>
<td>7.12</td>
<td>7.69</td>
</tr>
<tr>
<td>25</td>
<td></td>
<td></td>
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<tr>
<td>E[(r^F)]</td>
<td>5.00</td>
<td>5.00</td>
<td>5.00</td>
</tr>
<tr>
<td>(\sigma[(r^F)])</td>
<td>6.43</td>
<td>11.23</td>
<td>12.30</td>
</tr>
<tr>
<td>P(FR &lt; 1)</td>
<td>21.72</td>
<td>32.85</td>
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</tr>
<tr>
<td>ES</td>
<td>3.56</td>
<td>6.95</td>
<td>7.69</td>
</tr>
</tbody>
</table>

Expected returns \(E[\(r^F\)]\) and return volatilities \(\sigma[\(r^F\)]\) are annualised percentage values. Shortfall probabilities \(P(FR < 1)\) and expected shortfalls \(ES\) are derived from closed form formulas presented in (21) and (23) and are expressed as percentage values. Nine different setups are studied. “ALM+” refers to the ALM objective function under complete market conditions. “ALM−” denotes the same objective function in incomplete markets, that is, without TIPS, and “AM SH” relates to a situation in which the investor chooses the weights following the myopic portfolio rule, assumes that \(T = 1\) year, and ignores the presence of a liability. LOW, MEDIUM and HIGH refer to ex-ante fixed annualised expected returns on funding ratio of 5%, 10%, and 15%. 

---

**Appendix**
Table 5: Efficient portfolios (CPI-indexed returns)—Allocations

<table>
<thead>
<tr>
<th>Horizon (years)</th>
<th>LOW ALM+</th>
<th>AM SH</th>
<th>MEDIUM ALM+</th>
<th>AM SH</th>
<th>HIGH ALM+</th>
<th>AM SH</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stocks</td>
<td>0.16</td>
<td>0.07</td>
<td>0.14</td>
<td>0.33</td>
<td>0.22</td>
<td>0.28</td>
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<tr>
<td>Long Bond</td>
<td>0.09</td>
<td>0.28</td>
<td>0.20</td>
<td>0.20</td>
<td>0.46</td>
<td>0.29</td>
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<tr>
<td>Commodities</td>
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<td>0.41</td>
<td>0.63</td>
<td>0.83</td>
<td>0.81</td>
</tr>
<tr>
<td>Real Estate</td>
<td>0.04</td>
<td>0.08</td>
<td>0.07</td>
<td>0.09</td>
<td>0.14</td>
<td>0.13</td>
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<tr>
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<td>0.00</td>
<td>2.06</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>T-Bills</td>
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<td>0.14</td>
<td>0.18</td>
<td>−2.30</td>
<td>−0.65</td>
<td>−0.62</td>
</tr>
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<td>0.14</td>
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<tr>
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<td>0.09</td>
<td>0.19</td>
<td>−0.18</td>
<td>0.07</td>
<td>0.39</td>
</tr>
<tr>
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<td>0.38</td>
<td>0.40</td>
<td>0.53</td>
<td>0.74</td>
<td>0.81</td>
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<tr>
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<td>0.22</td>
<td>0.06</td>
<td>0.23</td>
<td>0.41</td>
<td>0.13</td>
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<td>0.00</td>
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<tr>
<td>T-Bills</td>
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<td>−0.65</td>
<td>−0.61</td>
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<tr>
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<td>0.26</td>
<td>0.14</td>
<td>0.67</td>
<td>0.61</td>
<td>0.28</td>
</tr>
<tr>
<td>Long Bond</td>
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<td>0.01</td>
<td>0.19</td>
<td>−0.25</td>
<td>−0.08</td>
<td>0.39</td>
</tr>
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<td>0.36</td>
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<td>0.69</td>
<td>0.81</td>
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<td>0.06</td>
<td>0.23</td>
<td>0.42</td>
<td>0.13</td>
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<tr>
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<td>0.00</td>
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<td>0.00</td>
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<tr>
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<td>−0.64</td>
<td>−0.61</td>
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<td>0.14</td>
<td>0.91</td>
<td>0.92</td>
<td>0.28</td>
</tr>
<tr>
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<td>−0.03</td>
<td>0.19</td>
<td>−0.24</td>
<td>−0.13</td>
<td>0.39</td>
</tr>
<tr>
<td>Commodities</td>
<td>0.22</td>
<td>0.32</td>
<td>0.40</td>
<td>0.45</td>
<td>0.62</td>
<td>0.81</td>
</tr>
<tr>
<td>Real Estate</td>
<td>0.11</td>
<td>0.23</td>
<td>0.06</td>
<td>0.23</td>
<td>0.41</td>
<td>0.13</td>
</tr>
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<td>0.00</td>
<td>2.27</td>
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</table>

This table displays the optimal weights allocated to the various asset classes in different contexts. LOW, MEDIUM, and HIGH refer to target annualised expected returns of 5%, 10%, and 15%. "ALM+" refers to the complete market case, in which the investor chooses the weights according to formula (14) and has access to TIPS as an asset class only in the hedging part of her portfolio, "ALM−" refers to the incomplete market case, in which she chooses the weights according to the same formula but has no access to TIPS, and "AM SH" refers to the case in which she chooses the weights following the myopic portfolio rule, assumes that \( T = \) one year, and cannot trade in TIPS.
Table 6: Efficient portfolios (RE-indexed returns)—Summary statistics

<table>
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<th>MEDIUM</th>
<th>HIGH</th>
</tr>
</thead>
<tbody>
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<td>ALM-</td>
<td>AM SH</td>
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<td>11.21</td>
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</table>

Expected returns $\mathbb{E}[r^F_t]$ and return volatilities $\sigma[r^F_t]$ are annualised percentage values. Shortfall probabilities $\mathbb{P}(FR < 1)$ and annualised expected shortfalls $ES$ are derived from closed-form formulas presented in (21) and (23). Nine different setups are studied. "ALM+" refers to the complete market case, in which the investor chooses the weights according to formula (14) and has access to real estate as an asset class only in the hedging part of her portfolio, "ALM–" denotes the same objective function in incomplete markets, that is, without real estate, and "AM SH" relates to a situation in which the investor chooses the weights following the myopic portfolio rule, assumes that $T = 1$ year, and ignores the presence of a liability. LOW, MEDIUM, and HIGH refer to ex-ante fixed annualised expected returns on funding ratios of 5%, 10%, and 15%. 

Appendix
## Appendix

### Table 7: Efficient portfolios (RE-indexed returns)—Allocations

<table>
<thead>
<tr>
<th>Horizon (years)</th>
<th>ALM+</th>
<th>ALM-</th>
<th>AM SH</th>
<th>ALM+</th>
<th>ALM-</th>
<th>AM SH</th>
<th>ALM+</th>
<th>ALM-</th>
<th>AM SH</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LOW</td>
<td>MEDIUM</td>
<td>HIGH</td>
<td></td>
<td></td>
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<td></td>
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<td></td>
<td></td>
</tr>
<tr>
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<td>0.19</td>
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<td>0.61</td>
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<td>0.56</td>
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<td>0.89</td>
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<td>0.00</td>
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<td>0.50</td>
<td>1.17</td>
<td>1.27</td>
<td>1.14</td>
<td>1.86</td>
<td>1.96</td>
<td>1.84</td>
</tr>
<tr>
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<td>-0.41</td>
<td>0.02</td>
<td>-2.17</td>
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<td>-1.21</td>
<td>-3.45</td>
<td>-2.93</td>
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</tr>
<tr>
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<td>0.24</td>
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<td>0.14</td>
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<td>0.31</td>
<td>0.80</td>
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<td>0.64</td>
<td>0.10</td>
<td>-0.17</td>
<td>0.56</td>
<td>0.21</td>
<td>-0.27</td>
<td>0.47</td>
<td>0.35</td>
</tr>
<tr>
<td>Commodities</td>
<td>0.24</td>
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<td>0.26</td>
<td>0.51</td>
<td>0.38</td>
<td>0.58</td>
<td>0.81</td>
<td>0.69</td>
<td>0.94</td>
</tr>
<tr>
<td>Real Estate</td>
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<td>0.00</td>
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This table displays the optimal weights allocated to the various asset classes in different contexts. LOW, MEDIUM, and HIGH refer to target annualised expected returns of 5%, 10%, and 15%. "ALM+" refers to the complete market case, in which the investor chooses the weights according to formula (14) and has access to real estate as an asset class only in the hedging part of her portfolio. "ALM–" refers to the incomplete market case, in which she chooses the weights according to the same formula but has no access to real estate, and “AM SH” refers to the case in which she chooses the weights following the myopic portfolio rule, assumes that $T$ = one year, and cannot trade in real estate.
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About EDHEC-Risk

The choice of asset allocation and risk management

EDHEC-Risk structures all of its research work around asset allocation and risk management. This issue corresponds to a genuine expectation from the market. On the one hand, the prevailing stock market situation in recent years has shown the limitations of diversification alone as a risk management technique and the usefulness of approaches based on dynamic portfolio allocation. On the other, the appearance of new asset classes (hedge funds, private equity, real assets), with risk profiles that are very different from those of the traditional investment universe, constitutes a new opportunity and challenge for the implementation of allocation in an asset management or asset-liability management context. This strategic choice is applied to all of the centre’s research programmes, whether they involve proposing new methods of strategic allocation, which integrate the alternative class; taking extreme risks into account in portfolio construction; studying the usefulness of derivatives in implementing asset-liability management approaches; or orienting the concept of dynamic “core-satellite” investment management in the framework of absolute return or target-date funds.

An applied research approach

In an attempt to ensure that the research it carries out is truly applicable, EDHEC has implemented a dual validation system for the work of EDHEC-Risk. All research work must be part of a research programme, the relevance and goals of which have been validated from both an academic and a business viewpoint by the centre’s advisory board. This board is made up of internationally recognised researchers, the centre’s business partners and representatives of major international institutional investors. The management of the research programmes respects a rigorous validation process, which guarantees the scientific quality and the operational usefulness of the programmes.

Six research programmes have been conducted by the centre to date:

- Asset allocation and alternative diversification
- Style and performance analysis
- Indices and benchmarking
- Operational risks and performance
- Asset allocation and derivative instruments
- ALM and asset management

These programmes receive the support of several financial companies, both from within France and abroad, representing some thirty different sponsors (including AXA Investment Managers, Barclays Global Investors, BNP Paribas Investment Partners, Citigroup, CACEIS, Deutsche Bank, FBF, Fortis, Eurex, LODH, NYSE Euronext, HSBC, Pictet, Robeco, Morgan Stanley, NewEdge, SG CIB, State Street, UBS, UFG, and many others).

In addition, EDHEC has developed a close partnership with a small number of

[Graph showing asset allocation percentages]

Source EDHEC (2002) and Ibbotson, Kaplan (2000)
sponsors within the framework of research chairs. These research chairs correspond to a commitment over three years from the partner on research themes that are agreed in common. The following research chairs have been endowed to date:

• ‘Regulation and Institutional Investment’, in partnership with AXA Investment Managers (AXA IM)
• ‘Asset-Liability Management and Institutional Investment Management’ in partnership with BNP Paribas Investment Partners
• ‘Risk and Regulation in the European Fund Management Industry’, in partnership with CACEIS
• ‘Structured Products and Derivative Instruments’, sponsored by the French Banking Federation (FBF)
• ‘Financial Engineering and Global Alternative Portfolios for Institutional Investors’, in partnership with Morgan Stanley Investment Management (MSIM)
• ‘Private Asset-Liability Management’ in partnership with ORTEC Finance
• ‘Dynamic Allocation Models and New Forms of Target-Date Funds’ in partnership with UFG
• ‘Advanced Modelling for Alternative Investments’ in partnership with Newedge Prime Brokerage
• ‘Asset-Liability Management Techniques for Sovereign Wealth Fund Management’ in partnership with Deutsche Bank
• ‘Core-Satellite and ETF Investment’ in partnership with CASAM.

The philosophy of the centre is to validate its work by publication in international journals, but also to make it available to the sector through its Position Papers, published studies and conferences. Each year, EDHEC organises two conferences for professionals with a view to presenting the results of its research: EDHEC Alternative Investment Days (London) and EDHEC Institutional Days (Paris), attracting more than 2,000 professional delegates.

EDHEC also provides professionals with access to its website, www.edhec-risk.com, which is entirely devoted to international asset management research. The website, which has more than 30,000 regular visitors, is aimed at professionals who wish to benefit from EDHEC’s analysis and expertise in the area of applied portfolio management research. Its monthly newsletter is distributed to more than 300,000 readers.

EDHEC’s distinguished international faculty includes renowned researchers and professors in finance and economics whose work has appeared in the major academic journals worldwide, including Professor Noël Amenc, Professor René Garcia, Professor Pierre Mella-Barral, Professor Lionel Martellini, Professor Florencio Lopez de Silanes and Professor Joëlle Miffre.

Research for Business
The centre’s activities have also given rise to the business offshoot EDHEC Asset Management Education.

EDHEC Asset Management Education helps investment professionals to upgrade their skills with advanced risk and asset management training across traditional and alternative classes.
About EDHEC-Risk

The EDHEC PhD in Finance
The PhD in Finance at EDHEC Business School is designed for professionals who aspire to higher intellectual levels and aim to redefine the investment banking and asset management industries.

It is offered in two tracks: a residential track for high-potential graduate students, who hold part-time positions at EDHEC Business School, and an executive track for practitioners who keep their full-time jobs.

Drawing its faculty from the world’s best universities and enjoying the support of the research centre with the greatest impact on the European financial industry, the EDHEC PhD in Finance creates an extraordinary platform for professional development and industry innovation.
EDHEC Position Papers and Publications from the Last Four Years

EDHEC-Risk

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About ORTEC Finance

ORTEC Finance is a leading, global provider of solutions for holistic risk/return management for pension funds, insurance companies and asset managers. The company was founded in 1981 and has more than 150 employees with expertise in areas such as actuarial sciences, economics, econometrics, investment management and information technology. The philosophy of ORTEC Finance is to provide highly specialised decision making support in a transparent and truly independent manner. Our clients are active in the pension, investment management, insurance and private wealth management markets in Europe, Asia, Australia and North America.

ORTEC Finance delivers tailored solutions in Asset Liability Management (ALM), risk management and investment consulting. Our ALM is founded on a methodology that has been developed over the last 20 years and is based on stochastic analysis of both assets and liabilities. ORTEC Finance’s Risk Management Solution starts where ALM finishes and provides a continued ex ante and ex post overview of the risks in a holistic, objective and transparent way.

ALM solutions for Private Wealth Management (PWM)
Besides our expertise with pension funds and insurance companies, we are pioneers in making the institutional ALM techniques available to advisors focused on private individuals. Our client-centric methodology allows for an efficient but tailored approach for both the initial advice and for continuous monitoring.

Wealth managers strive to give the best possible investment advice to their clients in view of their clients’ own, personal, financial goals. Therefore ORTEC Finance’s PWM solutions seek to satisfy the most important objectives set for Private ALM such as establishment of client-specific and individual objectives, taking into account varying risk tolerances, a high level of transparency, and the applicable regulatory frameworks (for example KYC and MiFID).

An important characteristic of the ORTEC Finance business model is the central use of models as means for communication, transfer of knowledge, and discipline for analysis and decision making. Rather than just providing our clients with our recommendations, we deliver them the tools and experience to gain intuition for good strategies under different circumstances: this unique combination of models and advice into appropriate solutions for each client has been the key to ORTEC Finance’s success.

Using the institutional industry experiences and techniques for the benefit of individuals, ORTEC Finance Private ALM solutions integrate the following methodologies:
1) Economic scenario generation including the underlying economic theories
2) Portfolio and asset allocation optimisation techniques
3) Risk profiling and behaviour of individuals (behavioural finance)
4) Financial Life Planning (objectives and cash flows)
5) Legislative framework
6) Monitoring the objectives on a regular basis
In this client-centric approach, scenario analysis is used as a basis for advising each individual client, with particular emphasis on their specific level of risk tolerance, including multi-period and multi-target planning.

In order to accomplish the above, our methodology covers the entire advising process from the first assessment of the client’s needs to the optimised portfolio/asset allocation selection. The focus of Private ALM is on optimising the probability of reaching the client’s objectives whilst giving insight into risk, return and the development of the client’s wealth.

Besides the planning phase, the reaching of the goals needs to be managed. This part is covered in the monitoring phase in which the goals vs. actual situation of total clientele can be included, an important dimension required for efficient firm-wide planning.

In 2009 over 4,000 advisors, predominantly in Holland, but also in other European countries, such as Belgium, Luxembourg and Switzerland, use ORTEC Finance solutions for PWM. These professionals work for more than 20 asset managers, family offices and (private) banks, but also include independent financial advisors, and financial and wealth planners. ORTEC Finance offers customised solutions for each market segment composed of consultancy, software models and training.

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