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This publication contains the results of the second year of research done at EDHEC-Risk Institute as part of the EDHEC-Deutsche Bank research chair on asset-liability management (ALM) techniques for sovereign wealth fund management.

Under the responsibility of Lionel Martellini, the scientific director of EDHEC-Risk Institute, this chair examines optimal allocation policies for sovereign wealth funds (SWFs). SWFs have become a dominant force in international financial markets, and a better understanding of optimal investment policy and risk management practices for these long-term investment funds is needed.

Authored by Bernd Scherer, this publication incorporates the economic balance sheet of the sovereign sponsor into the optimal asset allocation problem of the SWF in a way that is reminiscent of recent advances in corporate pension fund investing that consider the fund an integral part of the corporate balance sheet and jointly analyse capital structure and pension fund allocation choices.

Moving from an SWF-centric framework to an asset-liability approach integrating sovereign assets and liabilities offers interesting insights into optimal asset allocation given different drivers of economic risks and sheds light on the impact of sovereign leverage—determined by the ratio of existing debt and contingent liabilities to foreign reserves and sovereign assets—on optimal investment choices.

This work shows in particular that leverage reduces speculative investment demand while leaving hedging demand against fluctuations in the net fiscal position of the sovereign state unchanged.

The paper also shows that dynamic asset allocation methods increase the amount of risk an SWF can withstand, while narrow tactical asset allocation ranges reduce a fund’s ability to manage risks.

This analysis has important potential implications for the provision of ALM and risk management advice and solutions for sovereign wealth funds.

We would like to express our gratitude to our partners at Deutsche Bank for their support for our research.

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Director, EDHEC Risk Institute–Asia
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Abstract

Sovereign wealth funds (SWFs) typically have no direct earmarked liabilities. Nor should they, as the financial asset they represent is only part of total sovereign assets, which in turn guarantee all sovereign liabilities. The objective of this paper is to incorporate the economic balance sheet of the sovereign sponsor into the optimal asset allocation problem of the SWF. This paper outlines an easy to implement solution that nests well in the literature on SWFs. We show that economic leverage will reduce speculative demand but leave hedging demand (against fluctuations in the net fiscal position of the sovereign state) unchanged. We also show how to extend our one-period methodology to a multi-period context by solving a dynamic stochastic programme. Allowing for optimal dynamic decision making increases the amount of equity risk an SWF can take. The advantage is greatest for large values of economic leverage. Finally, we conclude that narrow tactical asset allocation ranges limit the SWF’s ability to manage its risks.
Executive Summary
Asset allocation for sovereign wealth funds (SWFs) focuses predominantly on optimal portfolio choice with non-tradable wealth (usually such resource-based wealth as underground oil). In this framework, sovereign wealth funds will—depending on their preferences—allocate to a combination of minimum-variance portfolio, speculative demand portfolio,1 and hedging-demand (against oil shocks) portfolio. The generic advice from these models is to invest less in assets with strong positive oil price correlation than in exposure to such recession-hedging assets as government bonds, which pay well if oil prices are down. Sovereign wealth funds taking this advice would have avoided the large losses they incurred in 2008. This advice rests essentially on a risk management argument grounded in economic theory—oil price movements, after all, are unpredictable and volatile and have extremely wide confidence intervals. Little research has been done into SWF investments related to foreign exchange (FX) reserves. Resource-based SWF assets are financed through foreign currency earnings on commodity exports. These assets represent sovereign wealth that can be used to manage macroeconomic risks or intergenerational distribution. However, many Asian SWFs are instead financed from FX reserves after periods of significant reserve accumulation. Reserve accumulation in managed exchange rate regimes is usually accompanied by sterilisation (i.e., the domestic currency created to purchase foreign assets is sterilised through local currency debt issuance). Since we can think of these funds as being financed through borrowed funds (local currency debt), it is not always clear they represent net sovereign wealth. In fact, both assets and liabilities (bonds issued for sterilisation purposes) grow. As a consequence, increased economic leverage should lead to more conservative asset allocation policies. This example illustrates the need to move from an SWF-centric framework to an asset/liability approach integrating sovereign liabilities (monetary base, local and foreign debt). Instead of focusing on SWF assets and liabilities in isolation, the SWF is now integrated into total sovereign assets and liabilities. This integration is analogous to modern pension fund investing, in which a pension fund is made an integral part of the corporate balance sheet and capital structure (enterprise-wide risk management) rather than managed in isolation. The size of local- and foreign-currency-denominated debt (or contingent liabilities towards pension systems or industries) relative to foreign reserves and sovereign assets will, for example, determine sovereign leverage and is expected to have a material impact on optimal sovereign asset management. Our research shows how to derive the optimal dynamic asset allocation (DAA) for sovereign assets given different drivers of economic risks as well as varying degrees of debt; it likewise discusses the impact of allocation constraints on the optimal policy.

So far SWF asset allocation has not taken liabilities into account. In fact, it is a widespread belief in the SWF literature that SWFs lack dedicated liabilities. Although this is true from the bottom-up view of an SWF portfolio manager, it is not true from the top-down view of a sovereign risk manager (sponsoring country). The approach taken in this article is to look at sovereign assets and liabilities in the same way as we would look at corporate assets and liabilities. We present a simplified

1 - Speculative demand increases with an improved risk/return tradeoff, whereas hedging demand is independent of return expectations. It is driven entirely by hedging out liability, reinvestment, or total wealth risks.
version of a sovereign balance sheet (see exhibit 1). All values are in foreign currency. The left side of the sovereign balance sheet contains sovereign assets. They contain FX reserves, the SWF, as well as the present value of the primary budget. The latter can be thought of as the present value of future taxes minus future expenditures. It reflects the present value of economic surpluses from running a country. The right-hand side of the sovereign balance sheet describes how the economy is financed. We view the monetary base and local debt as equivalent to shares such that its local currency value multiplied by the current exchange rate resembles sovereign market capitalisation. Foreign currency debt is treated as a senior claim. Sovereign default occurs if sovereign assets fall below foreign debt (in foreign currency). As default is costly (i.e., it comes with frictional bankruptcy costs in the form of social unrest, capital flight, and so on) we treat foreign currency debt as a hard threshold. If the Modigliani/Miller proposition (of capital structure irrelevance) applied at the sovereign level, the asset allocation decision would be irrelevant. Moreover, the above approach offers only an approximate summary of the economic position of a sovereign sponsor. For example, we left out the sovereign sponsor’s contingent liabilities against such key industries as banks.

An economy with zero foreign debt exhibits leverage of 0%. Suppose our sovereign state desires to maximise the long-term growth of net sovereign wealth (sovereign assets minus senior sovereign liabilities). Economic leverage can be thought of as introducing (pseudo) risk aversion. This interpretation shows that economic leverage leaves hedging demand unchanged but has a material effect on speculative demand. High economic leverage leads to greatly reduced speculative demand. An SWF is not a standalone investment vehicle. Second, an SWF should find it desirable to invest in assets that have low correlation with changes in the sovereign state’s primary budget. Assets that offer insurance in bad states of the world for the particular sovereign sponsor (tail hedge) are even more desirable. Economies differ, and so should SWF asset allocation.

The main risk factor for the primary budget in China, for example, is a slowdown in US consumer demand. So a Chinese SWF should not hold US retail stocks (unless return expectations are high enough to generate enough speculative demand to offset negative hedging demand). In fact, a Chinese SWF might want to short Wal-Mart stocks (the biggest US retail stock highly dependent on Chinese exports and the main distribution channel for cheap Chinese goods to Americans). In addition to hedging a fall in US consumer demand, it will hedge appreciation of the Chinese currency. The situation in Russia is different. The Russian budget depends heavily on oil price growth and on economic balance sheet leverage. This situation calls for modest aggressiveness, with a stronger focus on bonds and possibly hedge funds.
Finally, the GCC countries share the dependence of Russia on oil revenues, but with much less economic balance sheet leverage (GCC countries have little outstanding foreign debt). They can thus allocate more aggressively than Russia.

We then extend our model by adding a time dimension. Suppose the objective of the SWF is to maximise long-term (multi-period) portfolio growth of net sovereign wealth. Other things being equal, net wealth decreases as economic leverage increases. The sterilisation of FX reserves arising from undervalued exchange rates results in increased sovereign leverage and should thus lead to less aggressive investment policies. To model sovereign assets, we use equity (MSCI US), bonds (Barclays Long Treasury Index), and cash (rolling one-month US T-bills). We assume that the primary budget is driven by commodity prices (GSCI). All data are quarterly and run from 1970:Q1 to 2009:Q4.

We assume that $n = 12$ (three-year time horizon with quarterly rebalancing). Exhibit 9 shows the optimal equity allocation for a sovereign sponsor with 50% of sovereign wealth ($\theta = 0.5$) tied up in sovereign financial assets for a one-period, four-period, and eight-period investor.

Given the high volatility of the primary budget (approximated by GSCI commodities), even the most aggressive sovereign sponsor will invest only 25% in US equities. This amount falls as economic leverage increases. The remainder of the portfolio will be invested in bonds. For an investor with four periods to go the optimal asset allocation in the first period is greater than for a one-period (myopic) investor. This reflects the advantage of multi-period solutions. We can afford to invest a larger portion of current wealth in equities because we are aware that the optimal asset allocation can be reduced if period-one returns disappoint. This logic
carries over to the eight-period investor. For low leverage there is virtually no difference between the optimal dynamic and the rollover myopic investor.

Investment guidelines designed by SWF risk management committees often impose target ranges for SWF asset allocation. For example, if the target allocation to equities is set to 50%, tactical boundaries...
of 40% and 60% are set. Although initially deemed conservative, fixed-mix allocation will reduce the benefit of purely risk-based dynamic decision making. Exhibit 10 shows the impact of a (70%/30%) range on the optimal equity allocation over time. The equity allocation is smaller over all time horizons than in the unconstrained case, as risk management effectiveness is greatly reduced. Our dynamic stochastic programme is not allowed to invest 100% in cash if leverage becomes very high.

Constraints have an effect on the extent of the rebalancing advantage (the advantage of dynamic decision making over that of a buy-and-hold investor’s static decision making). The extent (measured in security equivalents, i.e., 0.01, is equivalent to a 1% return advantage) is summarised in exhibit 11. We see that the rebalancing advantage is small for investors with little leverage. In this case, the difference from a constrained solution is also small, as the constraints do not bind. However, as we increase leverage the difference between the optimal dynamic asset allocation and the myopic one-period solution grows and so does the advantage of having limited constraints. Dynamic investment policies are even more desirable for highly leveraged SWFs.

By using an easy-to-implement one-period model to incorporate the economic balance sheet of the sovereign sponsor into the SWF optimal asset allocation problem, we show that economic leverage reduces speculative demand while leaving hedging demand (against fluctuations in the sovereign net fiscal position) unchanged. We also show how to extend our one-period methodology into a multi-period context by solving a dynamic stochastic programme. Allowing optimal dynamic decision making is found to increase the amount of equity risk a SWF can take. The advantage is greatest for high economic leverage. Finally, we observe that narrow tactical asset allocation ranges limited an SWF’s ability to manage its risks.

Executive Summary
1. Introduction
1. Introduction

Asset allocation for sovereign wealth funds (SWFs) focuses predominantly on optimal portfolio choice with non-tradable wealth. This non-tradable wealth is usually resource-based wealth such as underground oil, as in Gintschel and Scherer (2004, 2008) or Scherer (2009a, 2011). In this framework, sovereign wealth funds will—depending on their preferences—allocate their resources to a combination of minimum-variance portfolio, speculative-demand portfolio, and hedging demand (against oil price shocks). The generic advice from these models is to invest less in assets with strong positive oil price correlation and more in such recession-hedging assets as government bonds, which pay well if oil prices fall. As Setser and Ziemba show (2009), SWFs taking this advice would have avoided the large losses they incurred in 2008. This view is essentially a risk management argument backed by economic theory. We have evidence (Aghion, Bachetta, and Ranciere 2006) that managing macroeconomic risks increases growth. Oil price movements are unpredictable and volatile and have extremely wide confidence intervals. This unpredictability is, as noted in Scherer (2010c), a powerful argument for oil-rich states to reduce the volatility of oil-related revenues. The usual routes to consumption smoothing, i.e., borrowing funds or hedging revenue risk, are unavailable as a result of limited access to international debt markets (precautionary savings motive) or incomplete markets for instruments for hedging oil prices (size, liquidity, contract choice). There is also a hidden economic diversification argument. An economy can diversify away from its vulnerability to oil price shocks either by developing a competitive non-oil sector or by setting up an international SWF. To an economist, macroeconomic diversification (developing another competitive sector) makes it hard to profit from the advantages of specialisation and takes too long. Investment diversification (by creating an SWF) is faster and easier.

Scherer (2011) adds resource uncertainty to the SWF asset allocation problem. He shows that if resource uncertainty is high (as a result of neighbourhood conflict, disputed oil fields, or geophysical uncertainty), optimal SWF asset allocation should be less aggressive, as the extent of desirable leverage is uncertain. Scherer (2009b) and Martellini and Milhau (2010) make inter-temporal hedging demand an integral part of SWF asset allocation. In their models, SWFs exhibit three-fund separation: a portfolio of speculative demand (the same for every investor with identical return and risk expectations), a portfolio of demand for hedging reinvestment risk (hedges against deteriorating investment opportunities), and a portfolio of demand for hedging oil price risk (primary sovereign budget risk). All of these results rely heavily on predictability implied by in-sample regressions. Recent research by Goyal and Welch (2003, 2008) indicates that out-of-sample predictability might be poor. Only if out-of-sample predictability creates term structures of risk and hence asset allocation dependent on time horizons will this research offer additional insight. In other words, if variations in returns can be predicted they are not risk. If predictability rises over time, risk falls.

A more currency-focused approach is taken by Kosowski and Breeden (2010). Countries that run a current account surplus
1. Introduction

Accumulate foreign assets, i.e., lend money to the rest of the world. So they should be concerned mainly about maintaining the purchasing power of their assets (claims against the rest of the world). Kosowski and Breeden (2010) recommend that currency allocation track import shares (the shares of imports from particular countries). The easiest solution would be the purchase of inflation-linked bonds in target countries. Countries that import Swiss chocolate or German cars should buy Swiss and German inflation-linked bonds.

Finally, there is practitioner (commercial) interest in the hedging of portfolio tail “risk” after the events of 2008. Tail risk is usually defined as an increase in systematic (macroeconomic) risk that leads to rising volatility, rising correlation of risk assets, and a dominance of the equity factor that runs through many assets and strategies (credit, carry, short volatility hedge fund strategies, private equity, and so on). But it is not clear that sovereign wealth funds would always prefer to insure against tail risks. Not only are these tail risks unusually large but they also tend to arise at the worst possible time—that is, they add to portfolio losses when a portfolio is already down in times of rising risk aversion. Insofar as SWFs are true long-term investors (intergenerational wealth transfer funds rather than stabilisation funds), one might argue the reverse—that is, that SWFs could provide insurance against tail risks. Leland (1980) has shown that investors whose risk aversion rises less rapidly with decreases in wealth than that of the average investor will want to sell portfolio insurance. These investors, in other words, will invest in tail risk rather than buy protection from it. SWF investments in foreign exchange (FX) reserves have not been widely studied. Resource-based SWF assets are financed through (owned/taxed) foreign currency earnings on commodity exports. These assets represent sovereign wealth that can be used to manage macroeconomic risks or intergenerational distribution. However, many Asian SWFs are instead financed from FX reserves after periods of significant reserve accumulation. Reserve accumulation in managed exchange rate regimes is usually accompanied by sterilisation (the domestic currency created to purchase foreign assets is sterilised by issuing debt in local currency). As these funds can be seen as being financed by borrowing (local-currency debt), it is not always clear they represent net sovereign wealth. In fact, not only do assets grow but so do liabilities (bonds issued for sterilisation). As a consequence, increased economic leverage should lead to more conservative asset allocation policies. This example illustrates the need to move from an SWF-centric framework to an asset/liability approach integrating sovereign liabilities (monetary base, local and foreign debt).

First, instead of focusing on SWF assets and liabilities in isolation, the SWF is now integrated into total sovereign assets and liabilities. This integration is analogous to modern pension fund investing, in which a pension fund is made a component of the corporate balance sheet and capital structure (enterprise-wide risk management) rather than managed in isolation. We therefore suggest extending the existing work towards the inclusion of other sovereign assets as well as sovereign liabilities. The size of local- and foreign-currency-denominated debt (or
contingent liabilities towards pension systems or industries) relative to foreign reserves and sovereign assets will, for example, determine sovereign leverage and is expected to have a material impact on optimal sovereign asset management. Second, the academic obsession with achieving closed-form solutions comes at large practical costs. The multi-period framework of Campbell and Viceira (2002), the workhorse of many asset allocation research applications, is only partially a multi-period framework, as it does not allow dynamic decision making. Horizon-dependent portfolio choice is driven by term structures of risk that rely heavily on significant out-of-sample predictability, whereas all of these models are examples of perfect in-sample arguments. This paper thus tries to address the following questions:

• How can we derive the optimal dynamic asset allocation (DAA) for sovereign assets given different drivers of economic risks as well as varying levels of debt? Should more leveraged economies invest more carefully?

• What is the impact of constraints on the advantage of dynamic decision making?

Section 3 reviews the classic model for portfolio choice with shadow assets (exogenous and non-tradable wealth). We extend the literature by outlining how to deal with resource uncertainty in a truly Bayesian context and how implied returns can be interpreted as a weighted combination of marginal risk contributions. Section 4 incorporates economic leverage into SWF asset allocation decisions, whereas section 5 draws parallels with pension funds. Section 6 adds the time dimension to SWF asset allocation with results presented in section 7. Section 8 introduces the impact of constraints and section 9 concludes.
2. One-Period Model of SWF Portfolio Choice
2. One-Period Model of SWF Portfolio Choice

Gintschel and Scherer (2004) integrate the SWF into the asset side of an economy in an early Deutsche Bank research paper. Throughout the literature, this model is the intellectual framework for analysing SWF portfolio choice. Reviewing their setup will contribute to an understanding of the SWF asset allocation problem and the route taken in this paper.

Let us assume that returns from financial wealth amount to a fraction $q$ of sovereign wealth. The remaining fraction $(1 - q)$ is the result of returns from exogenously given sovereign assets (non-tradable underground oil reserves, tax receipts from the underlying economy, and so on). The parameter $q$ catches the idea of shadow assets, i.e., assets that are not tradable such as underground oil (that cannot be extracted instantaneously for delivery in a futures contract) or human capital. Another application of this framework could be endowments. In this instance, shadow assets would amount to the hitherto unpaid contribution (its present value) of a school’s alumnus to the university endowment.\(^2\)

The present value of future contributions would also exhibit equity beta: if the economy is doing well, alumni will be in a better position to contribute to the school’s endowment.\(^2\) The present value of future contributions would also exhibit equity beta: if the economy is doing well, alumni will be in a better position to contribute to the school’s endowment.\(^2\) The present value of future contributions would also exhibit equity beta: if the economy is doing well, alumni will be in a better position to contribute to the school’s endowment.\(^2\)

We define the relevant first and second moments according to

$$E\{r_a\} - r_f = \mu_a, E\{w^T r_a\} = \Omega_{aa}, E\{w^T r_o\} = \Omega_{ao}$$

where $\Omega_{aa}$ is the $k \times k$ covariance matrix of asset returns and $\Omega_{ao}$ the $k \times 1$ covariance of assets with the government’s primary budget and $r_o$ the risk-free rate. $r_o$ is not limited to oil revenues. In general, it can reflect government (net) tax receipts on the country’s industrial base. Applying the expectations and variance operator on (1) and gathering only those terms relevant to SWF decision making (terms that are multiplied with $w$) into a standard mean-variance objective, we arrive at

$$w^* = \arg \max_w \{w^T \mu_a - \frac{1}{2} \theta^2 w^T \Omega_{aa} w + 2 \theta (1 - \theta) w^T \Omega_{ao} \}$$

The solution to this asset allocation problem becomes

$$w^* = \left(\frac{1}{\theta}\right) w_{spec}^* + \left(1 - \frac{1}{\theta}\right) w_{hedge}^*$$

$$= \left(\frac{1}{\theta}\right) \lambda^{-1} \Omega_{aa}^{-1} \mu_a + \left(1 - \frac{1}{\theta}\right) \Omega_{aa}^{-1} \Omega_{ao}$$

Total demand for risky assets can be decomposed into a weighted combination of speculative demand $w_{spec}^*$ and hedging demand, $w_{hedge}^*$. In the case of assets uncorrelated with the primary budget, the optimal solution is equivalent to a leveraged (with factor $\frac{1}{\theta}$) position in the asset-only maximum Sharpe ratio portfolio, or, in other words, $w_{spec}^*$. The second component represents hedging demand. In other words, the desirability of the risky asset depends not only on the Sharpe ratio but also on its ability to hedge unexpected shocks to the government budget. Hedging demand...
2. One-Period Model of SWF Portfolio Choice

equals individual asset beta $\beta_{a,a} = \frac{\omega_{a}}{\omega}$ with respect to other government revenue. The latter is equivalent to the slope coefficient of a regression of (demeaned) asset returns against (demeaned) budget returns

$$\left( \tau - \bar{\tau} \right) = \beta_{a,a} \left( \tau - \bar{\tau} \right) + \varepsilon .$$  

Speculative demand will be an important driver of total asset demand only if risk aversion is low or if financial wealth is very large relative to the underlying economy. In general, speculative demand is uninteresting to look at as it is the same for every investor (with identical risk-return expectations), i.e. it is the same for individuals, pension funds, endowments, etc. What makes investors different and hence asset/liability management an interesting subject is investors’ hedging demand.

A numerical example might help to understand equation (4). Let us assume the sovereign entity’s balance sheet contains one monetary unit of financial wealth (the SWF) and five monetary units of underground oil. In this example $\theta = \frac{1}{1+5} = \frac{1}{6}$ and $1 - \frac{1}{6} = \frac{5}{6}$. Hence speculative demand needs to be leveraged by a factor $\frac{1}{\theta} = \frac{6}{1} = 6$. Although this amount of leverage might be on the high side for some practical applications, the logic is clear. The sovereign entity can (and should) invest more aggressively in its tradable wealth (financial wealth or SWF), as it is much wealthier than the size of its SWF in isolation might suggest. We can capture this idea best by realising that (4) is independent of sovereign wealth. If a sovereign needs (depending on its risk aversion) to create a 20% allocation to risky assets as a fraction of total wealth (tradable and non-tradable) but has only one monetary unit of tradable wealth, this monetary unit needs to be invested 120% (six times 20%) in equities. This could be well achieved by investing 100% in equities with a beta of 1.2 or by (preferably) taking on some modest portfolio leverage of 20%. The same applies to hedging demand. Assume a beta of -0.15 between US government bonds and oil—the hedge demand for bonds would amount to $\left( 1 - \frac{1}{\theta} \right) \beta = (1 - 0.15) \beta = (5) \beta = 0.15 \times 5 = 75\%$. Futures contracts could be used to take both positions.

Let us stick with this example. Practitioners often criticise this solution as unrealistic. It calls for a massive long position in US government bonds of 75% of financial portfolio wealth at a time of quantitative easing and rapidly deteriorating creditworthiness of the US. However, this argument confuses speculative and hedging demand. Although the rationale for hedging demand is independent of the future expected path of US long-term bond yields, that for speculative demand is not. Hedging demand is unambiguous, but total demand is not. What is clear, however, is that any bond portfolio allocation of less than 75% contains a speculative element.

A second concern of (4) is that it is difficult to implement. There are simply many parameters to estimate. First, we need to distinguish between conceptual problems and implementation problems. Estimation error is an implementation problem. Implementation problems are ubiquitous in portfolio choice and do not invalidate our framework. There is a wide array of statistical (Bayesian statistics and robust statistics) and optimisation (robust optimisation, diversification constraints) techniques to deal with these issues.
2. One-Period Model of SWF Portfolio Choice

The only additional problem in (4) is $\theta$. We suggest the following remedy. Start with a model for the empirical likelihood of shadow assets. Suppose we take oil. Green (2005) uses USGS (US Geological Survey) data to describe the uncertainty in oil reserves.

The data comes in the form of 95th, 50th, and 5th percentiles ($F_{95} = 255$, $F_{50} = 675$, $F_{5} = 1094$) in billions of barrels. Taking logs and approximating mean and variance by

$$
\mu = \frac{1}{3}(F_{95} + F_{50} + F_{95})
$$

$$
\sigma = \frac{1}{4}(F_{5} - F_{95})
$$

leads us to the (lognormal) empirical likelihood function. Combining these data with a lognormal prior in a Bayesian regression framework as in Scherer (2010) allows us to plot the predictive posterior distribution of oil reserve as in exhibit 6.

The posterior distribution is generated from a linear Bayesian regression of log returns on a constant with a non-informative prior on the regression constant (distribution mean) and an inverse Chi-square prior (three degrees of freedom) on the variance of the regression residuals. To run this regression we use (6) to simulate a large set of (10,000) data points.

How could we go from here? First, we sample values for $\theta$ from the posterior predictive distribution. Combining these simulations with draws from the distribution of portfolio returns, we come to highly leptokurtic returns on sovereign wealth. Feeding this scenario return matrix into a utility function of your choice will result in portfolios that deal with background risk arising from resource uncertainty.

Finally, we can use (3) to calculate the implied returns of a given SWF portfolio. Implied returns are also called breakeven returns—that is, they represent the return requirements that an investor must be willing to assume to arrive at the asset allocation implied returns have been derived.
from. In other words: show me how you invested and I will show you what you must have been thinking when you made the investment. We start to derive the first-order condition for (3)

\[ d \left[ \theta w^T \mu - \frac{1}{2} \left( \theta^2 w^T \Omega_{aa} w + 2 \theta \left( 1 - \theta \right) w^T \Omega_{ao} \right) \right] = 0 \]  (7)

and solve it for \( \mu \) instead of \( w \):

\[ \theta \mu - \lambda \theta^2 \Omega_{aa} - \lambda \theta \left( 1 - \theta \right) \Omega_{ao} = 0 \]  (8)

\[ \mu_{imp} = \lambda \left[ \theta \Omega_{ao} w + \left( 1 - \theta \right) \Omega_{ao} \right] \]  (9)

Implied returns for the SWF investor are a linear combination of implied returns for a mean-variance investor \( \lambda \Omega_{aa} w \) and a covariance term of risky assets and oil \( \lambda \Omega_{ao} \) with weighting factors \( q \), \( 1 - q \).

The idea here is that we can derive return expectations from observed portfolios if we are willing to assume investors have known mean-variance preferences and we know the investors' risk forecasts.3 In general, even 0% holdings have implied returns different from zero. In this case implied returns provide the return expectation that make an asset mean-variance spanned (that is, redundant) in the investor's portfolio choice problem. We close this section with an additional example. Exhibit 7 describes the assets involved.

Exhibit 7: Historical mean, volatility correlation for bonds (Barclays Long Treasury Index), equities (MSCI US), and commodities (GSCI) from 1970 to 2009: Q4.

<table>
<thead>
<tr>
<th></th>
<th>Excess return</th>
<th>Volatility</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bonds</td>
<td>2.81%</td>
<td>10.60%</td>
<td>1.00</td>
</tr>
<tr>
<td>Equities</td>
<td>3.72%</td>
<td>18.35%</td>
<td>0.12</td>
</tr>
<tr>
<td>Commodities</td>
<td>2.88%</td>
<td>22.39%</td>
<td>-0.17</td>
</tr>
</tbody>
</table>

Suppose the SWF represents 50% of sovereign wealth, i.e., \( \theta = 0.5 \). Assume, too, that it is invested 40% in bonds (proxied by the Barclays Long Treasury Index) and 60% in equities (proxied by the MSCI US). Based on the risk-return and correlation characteristics of bonds, equities, and commodities (our proxy for the non-tradable shadow asset) over the period from 1970 to 2009, the volatility of sovereign wealth is 17.86% per annum.4 In order to fix the level of return (up to arbitrary scaling) we assume that the expected return for the SWF portfolio manager equals the historical excess return—2.98% per annum. This calculation allows us to fix \( \lambda = 2.98\% / 17.86\%^2 = 0.93 \). Our first impression is that the risk aversion of the SWF is very low. The reason is the willingness of the SWF to hold a position with a high-volatility impact on sovereign wealth despite its small return for sovereign wealth. Using (9), we can back out implied returns

\[ \mu_{imp} = 0.93 \cdot 0.25 \cdot \Omega_{ao} \left[ \begin{array}{c} 0.4 \\ 0.6 \end{array} \right] + \left( 1 - 0.25 \right) \cdot \Omega_{ao} \]

\[ = \left[ \begin{array}{c} 0.16\% \\ 0.56\% \end{array} \right] + \left[ \begin{array}{c} -0.32\% \\ -0.11\% \end{array} \right] = \left[ \begin{array}{c} -0.16\% \\ 0.45\% \end{array} \right] \]  (10)

By our calculations, the sovereign is expecting small returns on both equities and bonds. One reason is the low implied risk aversion. Equally important, however, is the low marginal contribution of equities and bonds to total portfolio risk. The implied return for a shadow wealth position in commodities is

\[ \theta \Omega_{ao} w + \left( 1 - \theta \right) \Omega_{ao} = 2.97\% \]  (11)

This result is the direct consequence of this unbalanced portfolio structure.
2. One-Period Model of SWF Portfolio Choice

The biggest bet on sovereign wealth is still in commodities. To obtain more efficient portfolios we need to leverage up the SWF so that its holdings make a meaningful contribution to the sovereign’s risk/return tradeoff. In the above example, virtually all of the sovereign’s expected growth in wealth (2.98%) arises from the returns—2.97%—of the economy’s underlying shadow asset (commodity returns). One might as well not have a SWF in the first place. The difference would be trivial.
3. Integrating Economic Leverage into the One-Period Model
3. Integrating Economic Leverage into the One-Period Model

So far SWF asset allocation has not taken liabilities into account. In fact, it is a widespread belief in the SWF literature that SWFs lack dedicated liabilities. Although this is true from the bottom-up view of an SWF portfolio manager, it is not true from the top-down view of a sovereign risk manager (the sponsoring country). The approach taken in this section is to look at sovereign assets and liabilities in the same way as we would look at corporate assets and liabilities, or as Cochrane (2005, 502) put it, “nominal debt, including the monetary base, is a residual claim on government surpluses, just as Microsoft stock is a claim on Microsoft earnings. If surpluses are not sufficient, the government must default or inflate away the debt.” Intellectually, we borrow from macro-finance as developed by Merton, Gray, and Bodie (2007) and Gray and Malone (2008).

We present (see exhibit 8) a simplified version of a sovereign balance sheet. All figures are in foreign currency. The left side of the sovereign balance sheet contains sovereign assets. These contain FX reserves, the SWF, and the present value of the primary budget. The latter can be thought of as the present value of future taxes minus future expenditures. It reflects the present value of economic surpluses from running a country. The right-hand side of the sovereign balance sheet describes how the economy is financed. We view the monetary base and local debt as equivalent to shares such that its local currency value multiplied by the current exchange rate resembles the sovereign (state) market capitalisation. Foreign currency debt is treated as a senior claim. Sovereign default occurs if sovereign assets fall below foreign debt (in foreign currency). As default is costly (i.e., it comes with frictional bankruptcy costs in the form of social unrest, capital flight, and so on), we treat foreign currency debt as a hard threshold. If Modigliani and Miller’s capital structure irrelevance proposition applied at the sovereign level the asset allocation decision would be irrelevant. Moreover, the above approach offers only an approximate summary of the economic position of a sovereign sponsor. For example, we left out the sovereign sponsor’s contingent liabilities against key industries (banks, for example).

We next standardise the left and right side of the sovereign balance sheet (without loss of generality) to one and let $\theta = fx + swf$. Economic leverage can then be expressed as

$$l = \frac{\theta + (1 - \theta) - f}{\theta + (1 - \theta) - f} - 1 = (1 - f)^{-1} - 1 \tag{12}$$

An economy with zero foreign debt ($f = 0$) exhibits a leverage of $0\%$. Suppose our sovereign desires to maximise the long-term growth of net sovereign wealth (sovereign assets minus senior sovereign liabilities): $1 - f$. This approach is a direct application of Fabozzi and Wilcox (2009) and relates strongly to the notion of “Kelly” betting as described in McLean and Ziemba (2006) or Thorp (2006).
Maximising geometric returns (median wealth) is equivalent to maximising log utility. For our leveraged sovereign investor, this maximisation amounts to

\[ w^* = \arg \max w \left[ \log (1 + (1 + l) r) \right] \quad (13) \]

However, we can approximate (13), assuming normally distributed returns, by

\[ w^* \approx \arg \max \left( \mu - \frac{1 + l}{2} \sigma^2 \right) \quad (14) \]

which implies standard mean-variance portfolio choice with pseudo-risk aversion \( l \). Higher leverage implies higher pseudo-risk aversion and hence less aggressive asset allocation. In fact, (14) allows us to derive mean-variance preferences by simply defining a threshold beneath which wealth must not fall. A sovereign sponsor with 80% foreign currency debt is implicitly leveraged by a factor \( 4 = (1 - 0.8)^{-1} - 1 \). The implied risk aversion coefficient is 5. Optimal one-period portfolio choice becomes

\[ w^* = \left( \frac{1}{\sigma} \right)^{-1} \Omega_{\mu}^{-1} \mu_a + \left( 1 - \frac{1}{\sigma} \right)^{-1} \Omega_{\sigma}^{-1} \Omega_{\mu} \quad (15) \]

From equation (15) we can derive some general principles of SWF asset allocation. First, economic leverage leaves hedging demand unchanged but has a material effect on speculative demand. High economic leverage leads to strongly reduced speculative demand. An SWF is not a standalone investment vehicle. Second, an SWF should find it desirable to invest in assets that have low correlation with changes in the sovereign’s primary budget. Assets that offer insurance in bad states of the world for the particular sovereign sponsor (tail hedge) are even more desirable. Economies differ, and so should SWF asset allocation.

The main risk factor for the primary budget in China, for example, is a slowdown in US consumer demand. So a Chinese SWF should not hold US retail stocks (unless return expectations are high enough to generate enough speculative demand to offset negative hedging demand). In fact, a Chinese SWF might want to short Wal-Mart stocks (the biggest US retail stock highly dependent on Chinese exports and the main distribution channel for cheap Chinese goods to Americans). In addition to hedging a fall in US consumer demand, it will serve as hedge against appreciation of the renminbi. The Russian budget, by contrast, is heavily dependent on oil price growth and economic balance sheet leverage. This combination calls for modest aggressiveness with a stronger focus on bonds and possibly hedge funds. Finally, the GCC countries share the dependence of Russia on oil revenues, but with much less economic balance sheet leverage (GCC countries have little outstanding foreign debt). They can thus allocate more aggressively.

Our framework leads to dynamic-portfolio-insurance-like trading. As economic leverage increases (for example, via an outflow of economic reserves) the need to dynamically de-risk the SWF increases. However, relative to a classic CPPI strategy, log utility implicitly sets a multiplier that is dependent both on portfolio return expectations and all higher moments. This leaves less room for overtrading (lower volatility costs).

The main beauty of equation (15) is that we can use it to apply well developed optimisation (quadratic programming, robust optimisation, regularisation...).
3. Integrating Economic Leverage into the One-Period Model

constraints, integer constraints), and statistical tools (Bayesian econometrics), as reviewed in Scherer (2010b). In the following section, we no longer assume that liabilities are non-stochastic, that, in other words, they exhibit no co-variation with assets.
4. SWF Asset Allocation as Pension Fund Allocation
Instead of implicitly assuming that sovereign debt has cash-like properties (as above), we now use the well-known pension fund framework by Sharpe and Tint (1990). In this framework, we look at the spread return between assets and liabilities and have the difference between assets and liabilities be a surplus. If, in addition, we normalise surplus changes with current sovereign assets (SWF assets plus other resource assets) we arrive at surplus growth as

\[ r_s = \left[ \theta w^T r_a + \left(1 - \theta \right) r_0 \right] - \frac{1}{f} \eta \]  \hspace{1cm} (16)

where \( f \) denotes the sovereign’s funding ratio (total sovereign assets over sovereign liabilities. For large \( f \) (very rich countries), equation (16) will converge to (1) and so will all solutions. We now need to look simultaneously at the liability hedging properties of assets as well as their stabilising effect on sovereign wealth. We write the problem of trading of surplus return against surplus risk as

\[ \mu_s - \frac{\lambda}{2} \sigma_s^2 \]  \hspace{1cm} (17)

Collecting the terms for only such expressions that involve \( w \), we can write the objective function as

\[ w^* = \arg \max_{w} w^T \mu_s - \frac{\lambda}{2} \left( \theta w^T \Omega_{aa} w + 2 \theta \left(1 - \theta \right) \Omega_{ao} w - 2 \theta \left( \frac{1}{f} \right) \Omega_{al} w \right) \]  \hspace{1cm} (19)

The solution to (19) is given by

\[ w^* = \left( \frac{1}{\theta} \right) \lambda^{-1} \Omega_{aa}^{-1} \mu_a + (1 - \frac{1}{\theta}) \Omega_{ao}^{-1} \Omega_{ao} + \left( \frac{1}{\theta} \right) (1 - \frac{1}{\theta}) \Omega_{al}^{-1} \Omega_{al} \]  \hspace{1cm} (20)

To keep track of what is going on we make use of matrix algebra.

\[
\sigma_s^2 = \begin{bmatrix}
\omega \theta & \Omega_{aa} & \Omega_{ao} & \Omega_{al} \\
1 - \theta & \Omega_{ao} & \Omega_{ao} & \Omega_{al} \\
-\frac{1}{f} & \Omega_{ao} & \Omega_{al} & \Omega_{al} \\
\end{bmatrix}
\]

\[
\mu_s = \begin{bmatrix}
\omega \theta \\
1 - \theta \\
-\frac{1}{f} \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
\omega \theta & \Omega_{aa} & \Omega_{ao} & \Omega_{al} \\
1 - \theta & \Omega_{ao} & \Omega_{ao} & \Omega_{al} \\
-\frac{1}{f} & \Omega_{ao} & \Omega_{al} & \Omega_{al} \\
\end{bmatrix}
\begin{bmatrix}
\omega \theta \\
1 - \theta \\
-\frac{1}{f} \\
\end{bmatrix}
\]

\[
\mu_s = \begin{bmatrix}
\mu_a \\
\mu_a \\
\mu_l \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
\omega \theta & \Omega_{aa} & \Omega_{ao} & \Omega_{al} \\
1 - \theta & \Omega_{ao} & \Omega_{ao} & \Omega_{al} \\
-\frac{1}{f} & \Omega_{ao} & \Omega_{al} & \Omega_{al} \\
\end{bmatrix}
\begin{bmatrix}
\mu_a \\
\mu_a \\
\mu_l \\
\end{bmatrix}
\]

\[
\mu_s = \begin{bmatrix}
\mu_a \\
\mu_a \\
\mu_l \\
\end{bmatrix}
\]

Assets will enter the solution if they are part of the speculative demand or the two hedging demands. The first (shadow assets) hedging demand attempts to mitigate fluctuations in combined sovereign assets, whereas the second (liability hedging demand) addresses the co-variation of SWF assets and earmarked sovereign liabilities. As before, we can also derive implied returns from the first order condition:

\[ \theta \mu_a - \lambda \theta^2 \Omega_{aa} w - \lambda \left(1 - \theta \right) \theta \Omega_{ao} + \lambda \theta \frac{1}{f} \Omega_{al} = 0 \]  \hspace{1cm} (21)

From this we can solve for the implied asset returns

\[ \mu_{a,\text{impl}} = \lambda \left( \theta \Omega_{aa} w + (1 - \theta) \Omega_{ao} - \frac{1}{f} \Omega_{al} \right) \]  \hspace{1cm} (22)

An asset will demand a higher return in an (optimal) investor portfolio if it exhibits a high marginal contribution to asset portfolio risk, if it co-varies positively with shadow assets, or if it co-varies negatively with liabilities. The lower the sovereign funding ratio, the greater this last effect. We can apply (22) for our data set in

4. SWF Asset Allocation as Pension Fund Allocation
exhibit 7. To keep matters simple, we assume that liabilities behave as bonds do and that $f = 1$.

\[
\begin{pmatrix}
0.11\
0.39
\end{pmatrix} + 
\begin{pmatrix}
-0.3\% \\
-0.1\%
\end{pmatrix} + 
\begin{pmatrix}
-1.05\% \\
-0.22\%
\end{pmatrix} = 
\begin{pmatrix}
-1.24\% \\
0.29\%
\end{pmatrix}
\]

Implied returns now consist of three building blocks. Although the first two are unchanged, the last one catches the co-variation with liabilities. The adjustment for bonds is large (-1.05%), as $\Omega_{al} = \Omega_{ll} = 10.6\%^2$ and hence it amounts to $-\lambda \Omega_{al} = 0.93 \cdot 10.6\%^2 = 1.05\%$.

**4. SWF Asset Allocation as Pension Fund Allocation**
4. SWF Asset Allocation as Pension Fund Allocation
5. The Multi-period Model of SWF Portfolio Choice
5. The Multi-period Model of SWF Portfolio Choice

The framework in the previous sections integrated an SWF into the sovereign economic balance sheet but essentially still presents a one-period model. Suppose the objective of the SWF is to maximise long-term portfolio growth of net sovereign wealth. Net wealth decreases as economic leverage increases.

The sterilisation of accumulated FX reserves arising from undervalued exchange rates results in increased sovereign leverage and should hence lead to less aggressive investment policies. For the framework outlined in the previous section, this amounts to maximising expected log utility (for a leveraged investor) over \( n \) decision periods.

\[
\max_{w_i, \ldots, w_n} E \left( \sum_{i=1}^{n} \log \left( 1 + (1 + l_i) r_i \right) \right) r_i \quad (24)
\]

For each period \( i = 1, \ldots, n \) we choose a portfolio \( w_i \) of \( k \) assets. Assuming that financial wealth amounts to a fraction \( \theta \) of sovereign wealth, we can again write the return on sovereign assets as

\[
r_i = \theta w_i^T r_{a,i} + (1 - \theta) r_{o,i} \quad (25)
\]

Here, \( w_i \) is a \( k \times 1 \) vector of portfolio weights at period \( i \), whereas \( r_{a,i} \) is the corresponding asset return. The returns generated by the primary budget are given by \( r_{o,i} \).

We solve (24) numerically in a finite-time, discrete-action, and discrete-time setup,7 imposing a long-only constraint on portfolio weights. Our state variable (a variable that describes the state of the system known to the decision maker) is sovereign leverage \( l_i \). It evolves according to

\[
l_{i+1} = \frac{l_i (1 + r_i)}{l_i (1 + r_i) - (l_i - 1)} \quad (26)
\]

The suggested solution starts from the last period. For period \( n \) we solve the asset allocation problem for a one-period investor. This provides us with a policy function for the last period, i.e., the optimal asset allocation for given leverage with only one period to go.

\[
U_n(l_n) = \max \log \left( 1 + (1 + l_n) r_n \right) \quad (27)
\]

For period \( n - 1 \) we need to work backwards one period. Here we need to find the optimal asset allocation according to

\[
U_{n-1}(l_{n-1}) = \max \left[ u_{n-1} \left( l_{n-1}, w_{n-1} \right) + \sum_{s=1}^{S} \text{prob}_{l_{n-1}, s} \left( w_{n-1} \right) U_n(l_{n,s}) \right] \quad (28)
\]

We see from (28) that we can no longer solve a one-period asset allocation problem. Instead, we need to take into account that every decision \( (w_{n-1}) \) today will impact both current portfolio growth \( u_{n-1} \left( l_{n-1}, w_{n-1} \right) \) and future portfolio growth via leverage. Leverage will in turn trigger the optimal policy as calculated from the optimisation problem for period \( n \). This policy is denoted by \( U_n(l_{n,s}) \), where we look at \( s = 1, \ldots, S \) scenarios. The result is maximum utility (two-period portfolio growth) for an investor in period \( n - 1 \): \( U_{n-1}(l_{n-1}) \). To achieve greater precision and reduce the computational effort when calculating \( U_n(l_{n,s}) \), we first run an OLS regression of optimal period \( n \) utility against a constant, portfolio leverage (our state variable), and squared leverage. The estimated regression coefficients help us to interpolate utility for each level of sovereign leverage:

\[
\hat{U}_n(l_{n,s}) = \hat{\alpha} + \hat{\beta} \cdot l_{n,s} + \hat{\gamma} \cdot l_{n,s}^2 \quad (29)
\]
5. The Multi-period Model of SWF Portfolio Choice

Hence, the second term in (28) becomes

\[
\sum_{\sigma=1}^{g} \prod_{t=1}^{n} \left( \frac{u_{t,n-1}}{u_{t-1,n-1}} \right) U_n(t,\sigma) = \frac{1}{S} \sum_{s=1}^{S} \left( \hat{\alpha} + \hat{\beta} \cdot \left( \frac{t_{n-1,1} (1 + r_{n-1,s})}{l_{n-1} (1 + r_{n-1,s}) - (l_{n-1} - 1)} \right) \right. \\
+ \hat{\gamma} \cdot \left. \left( \frac{t_{n-1,1} (1 + r_{n-1,s})}{l_{n-1} (1 + r_{n-1,s}) - (l_{n-1} - 1)} \right)^{2} \right)
\]

(30)

For \( n - 2, n - 3, n - 4, ... \) we repeat (28) to (30) and iterate backwards until we reach period 1.
5. The Multi-period Model of SWF Portfolio Choice
6. Time Horizon, Economic Leverage, and Optimal Asset Allocation Policies
6. Time Horizon, Economic Leverage, and Optimal Asset Allocation Policies

This section applies our framework to a hypothetical example. To model sovereign assets we use equity (MSCI US), bonds (Barclays Long Treasury Index), and cash (rolling one-month US T-bills). We assume that the primary budget is driven by commodity prices (GSCI). All data are quarterly and run from 1970:Q1 to 2009:Q4. We assume that $n = 12$ (three-year time horizon with quarterly rebalancing). Exhibit 9 shows the optimal equity allocation for a sovereign sponsor with $\theta = 0.5$ for a one-period, four-period, and eight-period investor.

Given the high volatility of the primary budget (approximated by GSCI commodities), even the most aggressive sovereign sponsor ($l + 1 = 1$) will invest only 25% in US equities. This amount falls as economic leverage increases. The remainder of the portfolio will be invested in bonds. For an investor with four periods to go, the optimal asset allocation in the first period shows a larger allocation to equity than for a one-period (myopic) investor. This result reflects the advantage of our dynamic stochastic programme. We can afford to invest a larger share of current wealth in equities because we are aware that the optimal asset allocation can be reduced if period-one returns disappoint. This logic carries over to the eight-period investor. For low leverage there is virtually no difference between the optimal dynamic and the rollover myopic investor. This similarity is not surprising, as our objective function effectively exhibits constant relative risk aversion (CRRA) for $l = 0$ (log utility). In this case, it is well known, as shown by Samuelson (1969), that there is no time-horizon effect.

Exhibit 9: Optimal asset allocation for a one-, four-, and eight-period unconstrained investor ($\theta = 0.5$)
7. The Impact of Constraints
7. The Impact of Constraints

This section explores the cost of imposing constraints on dynamic asset allocation. Investment guidelines designed by SWF risk management committees often impose target ranges for SWF asset allocation. If, for example, the target allocation to equities is 50%, tactical boundaries of 40% and 60% are set. Although initially deemed conservative, investment constraints will reduce the benefit of purely risk-based dynamic decision making. Exhibit 10 shows the impact of a [70%/30%] range on the optimal equity allocation over time. The equity allocation is smaller over all time horizons than in the unconstrained case, as risk management effectiveness is greatly reduced. Our dynamic stochastic programme is not allowed to invest 100% in cash if leverage becomes very high.

Exhibit 10: Optimal asset allocation for a one-, four-, and eight-period constrained investor (70%/30% range for equities and bonds)

Exhibit 11: Impact of constraints
7. The Impact of Constraints

This result has consequences for the extent of the rebalancing advantage (the advantage of dynamic decision making versus that of static decision making, i.e., that of a buy-and-hold investor) summarised in exhibit 11. We see that the rebalancing advantage is small for investors with little leverage. Here, the optimal solution is close to the log utility (CRRA) case in which there is little dynamic variation in asset weights. In this case, the difference from a constrained solution is also small as the constraints do not bind. As leverage increases, however, the difference between optimal and myopic solutions grows and so does the advantage of having few constraints. For an SWF with \( l + 1 = 2 \), the return advantage per annum amounts to 125 basis points.\(^9\) Dynamic investment policies are even more desirable for highly leveraged countries.

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\(^9\) In the context of rising trading costs for less liquid (but potentially diversifying) assets, 125bps is a modest advantage. In reality, SWF investors with potentially illiquid holdings would be more inclined to use a more conventional framework.
7. The Impact of Constraints
8. Conclusion
8. Conclusion

This paper describes a method, easy to put into place, of making economic leverage an integral part of the SWF optimal asset allocation problem. Our insight is valuable, as it allows us to draw on existing solutions to one-period mean-variance-based asset allocation problems. In practice, these solutions are robust and easy to communicate and provide us with poor man’s portfolio insurance. We have also shown that economic leverage will reduce speculative demand while it leaves macro-based hedging demand (against fluctuations in the sovereign net fiscal position) unchanged. Likewise, by solving a dynamic stochastic programme, we have shown how to extend the existing one-period method to a multi-period context. Allowing optimal dynamic decision making increases the amount of equity risk an SWF can take. The advantage is greatest for high leverage. Finally, we conclude that narrow ranges of tactical asset allocation limit an SWF’s ability to manage its risks.
References
References

References


• —. 2010b. SWF asset allocation. Q-Group presentation, Phoenix, Arizona.


References
About EDHEC-Risk Institute
About EDHEC-Risk Institute

The Choice of Asset Allocation and Risk Management

EDHEC-Risk structures all of its research work around asset allocation and risk management. This issue corresponds to a genuine expectation from the market.

On the one hand, the prevailing stock market situation in recent years has shown the limitations of diversification alone as a risk management technique and the usefulness of approaches based on dynamic portfolio allocation.

On the other, the appearance of new asset classes (hedge funds, private equity, real assets), with risk profiles that are very different from those of the traditional investment universe, constitutes a new opportunity and challenge for the implementation of allocation in an asset management or asset-liability management context.

This strategic choice is applied to all of the Institute’s research programmes, whether they involve proposing new methods of strategic allocation, which integrate the alternative class; taking extreme risks into account in portfolio construction; studying the usefulness of derivatives in implementing asset-liability management approaches; or orienting the concept of dynamic “core-satellite” investment management in the framework of absolute return or target-date funds.

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In an attempt to ensure that the research it carries out is truly applicable, EDHEC has implemented a dual validation system for the work of EDHEC-Risk. All research work must be part of a research programme, the relevance and goals of which have been validated from both an academic and a business viewpoint by the Institute’s advisory board. This board is made up of internationally recognised researchers, the Institute’s business partners, and representatives of major international institutional investors. Management of the research programmes respects a rigorous validation process, which guarantees the scientific quality and the operational usefulness of the programmes.

Six research programmes have been conducted by the centre to date:

- Asset allocation and alternative diversification
- Style and performance analysis
- Indices and benchmarking
- Operational risks and performance
- Asset allocation and derivative instruments
- ALM and asset management

These programmes receive the support of a large number of financial companies. The results of the research programmes are disseminated through the EDHEC-Risk locations in London, Nice, and Singapore.

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- Asset-Liability Management and Institutional Investment Management, in partnership with BNP Paribas Investment Partners
- Risk and Regulation in the European Fund Management Industry, in partnership with CACEIS
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• The Case for Inflation-Linked Corporate Bonds: Issuers’ and Investors’ Perspectives, in partnership with Rothschild & Cie
• Advanced Investment Solutions for Liability Hedging for Inflation Risk, in partnership with Ontario Teachers’ Pension Plan
• Exploring the Commodity Futures Risk Premium: Implications for Asset Allocation and Regulation, in partnership with CME Group
• Structured Equity Investment Strategies for Long-Term Asian Investors, in partnership with Société Générale Corporate & Investment Banking
• The Benefits of Volatility Derivatives in Equity Portfolio Management, in partnership with Eurex
• Solvency II Benchmarks, in partnership with Russell Investments

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EDHEC-Risk Institute: Key Figures, 2009-2010

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The philosophy of the Institute is to validate its work by publication in international journals, as well as to make it available to the sector through its position papers, published studies, and conferences.
Research for Business

The Institute’s activities have also given rise to executive education and research service offshoots. EDHEC-Risk’s executive education programmes help investment professionals to upgrade their skills with advanced risk and asset management training across traditional and alternative classes.

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www.edhec-risk.com/indexes/efficient

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EDHEC-Risk Alternative Indexes

www.edhec-risk.com/indexes/pure_style

The different hedge fund indexes available on the market are computed from different data, according to diverse fund selection criteria and index construction methods; they unsurprisingly tell very different stories. Challenged by this heterogeneity, investors cannot rely on competing hedge fund indexes to obtain a “true and fair” view of performance and are at a loss when selecting benchmarks. To address this issue, EDHEC Risk was the first to launch composite hedge fund strategy indexes as early as 2003. The thirteen EDHEC-Risk Alternative Indexes are published monthly on www.edhec-risk.com and are freely available to managers and investors.

2011
• Sender, S. The elephant in the room: Accounting and sponsor risks in corporate pension plans (March).
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• Martellini, L., and V. Milhau. Capital structure choices, pension fund allocation decisions and the rational pricing of liability streams (July).
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- Real estate indexing and the EDHEC IEIF Commercial Property (France) Index (February).

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- Amenc, N., F. Goltz, and D. Schroeder. Reactions to an EDHEC study on asset-liability management decisions in wealth management (September).
- Le Sourd, V. Hedge fund performance in 2007 (February).

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• Amenc, N., P. Schoefler, and P. Lasserre. Organisation optimale de la liquidité des fonds d’investissement (March).
• Lioui, A. Spillover effects of counter-cyclical market regulation: Evidence from the 2008 ban on short sales (March).

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• Till, H. Has there been excessive speculation in the US oil futures markets? (November).
• Amenc, N., and S. Sender. A welcome European Commission consultation on the UCITS depositary function, a hastily considered proposal (September).
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• Giraud, J.-R. MiFID: One year on (May).
• Lioui, A. The undesirable effects of banning short sales (April).

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• Amenc, N., and S. Sender. Assessing the European banking sector bailout plans (December).
• Amenc, N., and S. Sender. Les mesures de recapitalisation et de soutien à la liquidité du secteur bancaire européen (December).
• Amenc, N., F. Ducoulombier, and P. Foulquier. Reactions to an EDHEC study on the fair value controversy (December). With the EDHEC Financial Analysis and Accounting Research Centre.
• Amenc, N., and V. Le Sourd. Les performances de l’investissement socialement responsable en France (December).
• Amenc, N., and V. Le Sourd. Socially responsible investment performance in France (December).
• Amenc, N., B. Maffei, and H. Till. Les causes structurelles du troisième choc pétrolier (November).
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Deutsche Bank is a leading global investment bank with a substantial private clients franchise. Its businesses are mutually reinforcing. A leader in Germany and Europe, the bank is continuously growing in North America, Asia and key emerging markets.

With more than 100,000 employees in 73 countries, Deutsche Bank offers unparalleled financial services throughout the world. The bank competes to be the leading global provider of financial solutions, creating lasting value for its clients, shareholders, people and the communities in which it operates.

Despite turbulence in financial markets, Deutsche Bank maintained its capital strength. This gives the bank a firm foundation from which to focus on its responsibilities: responsibilities to its clients, who continue to look to the bank as a dependable business partner; responsibilities to its shareholders and staff, to whom the bank seeks to remain attractive in future; and finally, the responsibilities to the financial system, of which the bank is a part, and which now needs to be rigorously analysed and re-engineered.

Deutsche Bank comprises three Group Divisions: Corporate and Investment Bank (CIB); Private Clients and Asset Management (PCAM) and Corporate Investments (CI). Within Asia Pacific all of the bank’s key business divisions are long and well established with over 18,000* staff located within 17 markets. The region was recently highlighted in the fourth phase of the bank’s Management Agenda, announced in late 2009, as a key driver of additional revenue growth.

While this is a significant statement of intent, Deutsche Bank already has a world-class operation across Asia of significant scale, having invested in the region throughout many financial cycles. The strength of its CIB business is nonetheless a standout in terms of scale and success. As a result, the region has made a major contribution to the bank for a number of years, with EUR 3.77 billion in revenues derived from Asia Pacific in 2010. The bank recently celebrated 30 years of business in India, where it has a large and highly successful franchise; while in China, a priority growth market, the bank holds all the operating licenses required to compete in its core global business lines.

<table>
<thead>
<tr>
<th>The Group at a Glance</th>
<th>2010 in € m.</th>
<th>2009 in € m.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total net revenues</td>
<td>28,567</td>
<td>27,952</td>
</tr>
<tr>
<td>Total non-interest expenses</td>
<td>23,318</td>
<td>20,120</td>
</tr>
<tr>
<td>Income (loss) before income taxes</td>
<td>3,975</td>
<td>5,202</td>
</tr>
<tr>
<td>Net income (loss)</td>
<td>2,330</td>
<td>4,958</td>
</tr>
</tbody>
</table>

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<thead>
<tr>
<th>Dec 31, 2010 in € m.</th>
<th>Dec 31, 2009 in € m.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total assets</td>
<td>1,906</td>
</tr>
<tr>
<td>Shareholders’ equity</td>
<td>48.8</td>
</tr>
<tr>
<td>Tier 1 capital ratio</td>
<td>12.3%</td>
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</table>

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<thead>
<tr>
<th>Long-term rating</th>
<th>Moody’s Investors Service</th>
<th>Standard &amp; Poor’s</th>
<th>Fitch Ratings</th>
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<tr>
<td></td>
<td>Aa3</td>
<td>A+</td>
<td>AA-</td>
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