Introducing a New Form of Volatility Index: The Cross-Sectional Volatility Index

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Executive Summary
We introduce a new form of volatility index, the cross-sectional volatility index. Through formal central limit arguments, we show that the cross-sectional dispersion of stock returns can be regarded as an efficient estimator for the average idiosyncratic volatility of stocks within the universe under consideration. Among the key advantages of the cross-sectional volatility index measure over currently available measures are its observability at any frequency, its model-free nature, and its availability for every region, sector, and style of the world equity markets, without the need to resort to any auxiliary option market. We also provide some interpretation of the cross-sectional volatility index as a proxy for aggregate economic uncertainty, which suggests that the cross-sectional volatility index should be intimately related to option-based implied volatility measures. We confirm this intuition by reporting high correlation between the VIX index and the corresponding cross-sectional volatility index based on the S&P500 universe. We also find the high correlation between the two volatility measures to be robust with respect to changes in sample period, changes in market conditions, and changes in the region under consideration. Overall, these results suggest that the cross-sectional volatility index is intimately related to other volatility measures where and when such measures are available, and that it can be used as a reliable proxy for volatility when such measures are not available.

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1. Introduction
Recent market turmoil, as well as the presence of ever stricter regulatory constraints, has led investors and asset managers to monitor with increased scrutiny the volatility and downside risk of their equity holdings. In this context, market participants have shown an increasing interest in volatility indices, which are not only heavily used as sentiment indicators, but also serve as underlyings for a number of derivatives contracts that can be used for obtaining long or short exposure to volatility.

While their existence is critical to allow investors to measure and/or trade in volatility, current volatility indices, suffer from a number of drawbacks. In particular, they are based not on actual stock index returns but on auxiliary option markets. As a result, the volatility estimates they provide tend to be polluted by option market factors that have little to do with the underlying stock or stock index returns. Besides, and perhaps more importantly, such measures can only made available only on those rare occasions when liquid option markets exist for the given universe under consideration. As a result, very few volatility indices are available, for example, for Asian markets (they have been introduced—recently—only in Japan and Korea) or emerging markets beyond Asia (they have been introduced—recently—only in Mexico and India). Similarly, when it comes to developed equity markets, volatility indices, when they exist, are available only at the broad market level, so no information is currently available regarding volatility for specific sectors or styles for a given region of the world.

To address these concerns, we introduce a new form of volatility index, the cross-sectional volatility index. Through formal central limit arguments, we show that the cross-sectional variance of stock returns can be regarded as an efficient estimator for the average idiosyncratic variance of stocks within the universe under consideration. This measure of idiosyncratic risk is found to be highly correlated to standard measures of systematic risk when they exist, which further justifies its use in the context of equity volatility measurement. Key advantages of this measure over currently available measures such as sample-dependent historical volatility measures or option-based implied volatility measures are: its observability at any frequency, its model-free nature, and its availability for every region, sector, and style of the world equity markets, without the need to resort to any auxiliary option market.

The reminder of this note is structured as follows. In section 2, we review the basic definitions of volatility, emphasizing the key distinction between historical and implied volatility measures, and also introduce volatility indices. In section 3, we review the motives for trading in volatility, and also the instruments that have been made available for obtaining access to volatility as an investable asset class. In section 4, we introduce a new form of volatility index, which is meant to alleviate existing concerns over currently available volatility indices, and we document the properties of a suitably designed cross-sectional dispersion measure as a model-free unbiased and efficient estimate for specific volatility. In section 5, we provide some interpretation of the cross-sectional volatility index as a proxy for aggregate economic uncertainty, which suggests that the correlation between systematic volatility and average idiosyncratic volatility should be high, which we confirm empirically. Finally, section 6 concludes.

2. Volatility and Volatility Indices
Volatility is a statistical measure of the dispersion of returns for a given security or market index. In other words, volatility refers to the amount of uncertainty or risk about the size of changes in an underlying security or index value. Higher volatility means that the underlying value can potentially be spread out over a larger range of values, signaling higher risk for investors holding the security or index. Indeed, high volatility means that the price of the security can change dramatically in either direction over a given interval of time. Lower volatility means that a security
or index value does not fluctuate dramatically, but changes in value at a steadier pace over a period of time.

2.1. Historical versus Implicit Volatility
While volatility is unambiguously formally defined as the standard deviation or variance of returns from a given stock or market index, a key distinction exists between two different kinds of volatility measures, namely historical volatility measures and implicit volatility measures.

**Historical volatility measures**
Historical volatility measures are obtained by estimating the standard deviation of returns from a past sample of equity returns. One advantage about these measures is that they can be estimated directly from time-series of individual stock or stock index returns. One drawback of these measures is that they are not directly observable, and are dependent on a sample of past returns, which makes historical volatility estimates backward-looking measures of current volatility. Besides, historical volatility measures are artificially smooth, in that tomorrow's estimate for historical volatility will only differ from today's estimate for historical volatility only by a single data point in a rolling-window analysis. More forward-looking estimates for historical volatility are available through GARCH-type models, which make it possible to put more weight on more recent observations, but the sample-dependency and artificial smoothing problems remain unsolved.

**Implied volatility measures**
More recently, implicit volatility estimates have been obtained from option prices. Indeed, volatility is a key variable in option pricing formulas, showing the extent to which the return of the underlying asset will fluctuate between now and the option expiration date. One advantage of these measures is that they are more forward-looking than historical volatility measures since they reflect market's expectations about future volatility. One drawback is that they are based not on actual stock index returns but on auxiliary option markets. As a result, the volatility estimate is polluted by any factor impacting supply and demand in option markets that have little to do with the underlying stock or stock index returns. Besides, such measures can only be made available in those rare cases when there are liquid option markets for the given universe under consideration.

2.2. Systematic versus Specific Volatility
Regardless of the method used in estimating volatility, another key distinction exists between systematic and specific volatility. For any given stock, total volatility can be decomposed into systematic volatility, driven by the stock exposure with respect to systematic risk factors, and specific volatility, which is driven by the uncertainty impacting a particular company.

The recent financial literature has paid considerable attention to idiosyncratic volatility. Campbell et al. (2001) and Malkiel and Xu (2002) document that idiosyncratic volatility increased over time, while Brandt et al. (2009) show that this trend completely reversed itself by 2007, falling below pre-1990s levels, and suggest that the increase in idiosyncratic volatility through the 1990s was not a time trend but rather an "episodic phenomenon". Bekaert et al. (2008) confirm that there is no trend either for the United States or for other developed countries. A second fact about idiosyncratic volatility is also a bone of contention. Goyal and Santa-Clara (2003) posit that idiosyncratic volatility has forecasting power for future excess returns, while Bali et al. (2005) and Wei and Zhang (2005) find that the positive relationship is not robust to the sample chosen.

While representing two different underlying risk measures, one expects systematic and average specific volatility risk indicators to be highly correlated, since they both reflect the aggregate
uncertainty faced by investors at a given point in time regarding economic fundamentals (see section 5.1).

In what follows, we confirm this intuition and find high correlation between the VIX index, a measure of systematic risk based on option prices, and a model-free measure for specific risk, which we introduce in this note (see section 5.2). This high correlation is found to be robust to different regions and time-periods, and remains stable across market conditions.

2.3. Volatility Indices
Because information about changes in volatility is of a critical importance to market participants, a number of initiatives have been launched to make volatility measures available to investors in form of volatility indices, which are designed to track the aggregate volatility of an asset market. Such indices are typically calculated based on option prices, and as such are based on implicit as opposed to actual volatility measures.

Volatility indices can now be found on such major stock indices as the S&P 500, EURO STOXX 50, DAX, Dow Jones Industrial Average and lately (November 2010) the Nikkei 225. The most popular volatility index is the VIX, which is built from prices of equity index options on the S&P 500. The index was introduced by the options exchange CBOE in 1993 and was originally designed to measure the market’s expectation of thirty-day volatility implied by prices of at-the-money S&P 100 index options. The implied volatility was based on the Black-Scholes model. Ten years later, CBOE, together with Goldman Sachs, updated the VIX methodology to reflect a new way to measure expected volatility. The implied volatility is now derived from the implied risk-neutral distribution that can be extracted from call and put options with different strike prices. This volatility is model-free: rather than assuming that Black-Scholes holds, the only necessary assumption is that of an absence of arbitrage opportunities.

Table 1: Volatility indices currently available and their underlying indices

<table>
<thead>
<tr>
<th>Index</th>
<th>Ticker</th>
<th>Underlying</th>
<th>Index Provider</th>
</tr>
</thead>
<tbody>
<tr>
<td>AMEX Volatility Index</td>
<td>QQV</td>
<td>QQQ</td>
<td>AMEX</td>
</tr>
<tr>
<td>CBOE Volatility Index®</td>
<td>VIX</td>
<td>SPX</td>
<td>CBOE</td>
</tr>
<tr>
<td>CBOE DJIA Volatility Index</td>
<td>VXD</td>
<td>DJX</td>
<td>CBOE</td>
</tr>
<tr>
<td>CBOE NASDAQ-100 Volatility Index</td>
<td>VXN</td>
<td>NDX</td>
<td>CBOE</td>
</tr>
<tr>
<td>CBOE Russell 2000 Volatility Index</td>
<td>RXV</td>
<td>RUT</td>
<td>CBOE</td>
</tr>
<tr>
<td>CBOE S&amp;P 500 Volatility Index</td>
<td>VXO</td>
<td>OEX</td>
<td>CBOE</td>
</tr>
<tr>
<td>CBOE S&amp;P 500 3-Month Volatility Index</td>
<td>VVX</td>
<td>SPX</td>
<td>CBOE</td>
</tr>
<tr>
<td>CBOE VIX Premium Strategy Index</td>
<td>VPD</td>
<td>VIX</td>
<td>CBOE</td>
</tr>
<tr>
<td>CBOE Capped VIX Premium Strategy Index</td>
<td>VPN</td>
<td>VIX</td>
<td>CBOE</td>
</tr>
<tr>
<td>CBOE Crude Oil Volatility Index</td>
<td>OVX</td>
<td>USD</td>
<td>CBOE</td>
</tr>
<tr>
<td>CBOE Gold Volatility Index</td>
<td>GVZ</td>
<td>GLD</td>
<td>CBOE</td>
</tr>
<tr>
<td>CBOE EuroCurrency Volatility Index</td>
<td>EVZ</td>
<td>FXE</td>
<td>CBOE</td>
</tr>
<tr>
<td>AEX Volatility</td>
<td>VAEX</td>
<td>AEX</td>
<td>Euronext</td>
</tr>
<tr>
<td>BEL 20 Volatility</td>
<td>VBEL</td>
<td>BEL 20</td>
<td>Euronext</td>
</tr>
<tr>
<td>CAC 40 Volatility</td>
<td>VCAC</td>
<td>CAC 40</td>
<td>Euronext</td>
</tr>
<tr>
<td>FTSE 100 Volatility</td>
<td>VFTE</td>
<td>FTSE 100</td>
<td>Euronext</td>
</tr>
<tr>
<td>DAX Volatility</td>
<td>VDAX-NEW</td>
<td>DAX</td>
<td>Deutsche Borse AG</td>
</tr>
<tr>
<td>SMI Volatility</td>
<td>VSMI</td>
<td>SMI</td>
<td>SIX Swiss Exchange AG</td>
</tr>
<tr>
<td>EURO STOXX 50 Volatility</td>
<td>VSTOXX</td>
<td>EURO STOXX 50</td>
<td>STOXX Limited</td>
</tr>
<tr>
<td>NIKKEI Volatility Index</td>
<td>VNKY</td>
<td>NIKKEI 225</td>
<td>Nikkei Inc.</td>
</tr>
<tr>
<td>India NSE VIX</td>
<td>INVIXN</td>
<td>NIFTY 50</td>
<td>India NSE</td>
</tr>
<tr>
<td>KOSPI 200 Volatility Index</td>
<td>VKOSPI</td>
<td>KOSPI 200</td>
<td>Korea Exchange</td>
</tr>
<tr>
<td>Mexico Volatility Index</td>
<td>VIMEX</td>
<td>IPC</td>
<td>MexDer</td>
</tr>
</tbody>
</table>
Other exchanges have developed volatility indices that are similar in methodology to those provided by the CBOE (see table 1 for the list of currently available volatility indices). For example, Euronext has developed volatility indices on major equity indices like the AEX, BEL20 and CAC40. Volatility indices on the DAX and SMI indices are also developed by the respective exchanges.

The volatility indices are typically computed directly by the options exchange. The notable exception is the VSTOXX which indicates the volatility index based on Dow Jones EURO STOXX 50 options prices and is computed by STOXX, the index provider, rather than by an options exchange. Volatility indices typically use options with a short time period to expiry. They thus indicate market expectations over, e.g., the next month. Some indices also exist for different time horizons. For the VSTOXX for example, calculations of eight sub-indices are done for 1, 2, 3, 6, 9, 12, 18, and 24 months to expiry.

3. Volatility as an Asset Class
Recent market turbulence, as well as the presence of ever stricter regulatory constraints, has led investors and asset managers to monitor with increased scrutiny the volatility and downside risk of their equity holdings. In this context, investors have shown an increased interest in regarding volatility as an asset class that can be traded, as opposed to a mere statistical indicator measuring stock return uncertainty.

3.1. Motives for Trading Volatility
One of the main motivations for trading in volatility is to diversify equity risk through a long implied volatility exposure (see Hill (2004) or Szado (2009) for recent references). A key point to note is that volatility of equity returns and equity returns tend to move in opposite directions, i.e., they are strongly negatively correlated. In addition, negative correlation and high volatility are particularly pronounced in stock market downturns, offering protection against stock market losses when it is needed the most and when other forms of diversification are not very effective.

One possible explanation for the negative correlation of equity volatility to the equity market is the “leverage effect” (Black (1976), Christie (1982), Schwert (1989)): a decrease (respectively, an increase) in equity prices increases (respectively, decreases) the company’s leverage, thereby increasing (respectively, decreasing) the risk to equity holders and increasing (respectively, decreasing) equity volatility. Another alternative explanation (French et al. (1987), Bekaert et Wu (2000), Wu (2001), Kim et al. (2004)) is the “volatility feedback effect”: assuming that volatility is incorporated in stock prices, a positive volatility shock increases the future required return on equity and stock prices are expected to fall simultaneously.

The presence of sensible economic reasons that explain the inverse relationship between equity return and volatility is a comforting indication of the robustness of the diversification benefits to be expected, which stands in contrast with the well-known lack of robustness of portfolio diversification within the equity universe, where diversification is known to fail precisely when it is most needed because of the convergence of all correlations to one in periods of high market turbulence.

Of course, it should be expected that the risk diversification benefits of long volatility exposure may come at a cost. Recent academic research has found that there is a positive risk premium over time to being short volatility or conversely, that there is a negative risk premium to being long volatility. Because of the negative correlation between market index returns and market index volatility, buyers of options may be willing to pay a premium because a long position in volatility helps hedge market-wide risk (Bakshi and Kapadia (2003)). In other words, because volatility
is negatively correlated with the returns to equities, investors are willing to pay a premium to hold this asset. In a recent paper, Carr and Wu (2010) find that the negative correlation between stock index returns and the return variance generates a strongly negative beta, but this negative beta explains only a small portion of the negative variance risk premiums. Other risk factors identified by the recent literature, such as size, book-to-market, and momentum, cannot explain the strongly negative variance risk premiums, either, and they conclude that the majority of the market variance risk premium is generated by an independent variance risk factor.

Other motives for trading in volatility include speculative, arbitrage and hedging demands.

- **Directional speculative** bets on volatility changes, implemented by going long volatility exposure when volatility is expected to rise and going short volatility exposure when volatility is expected to fall (see Dash (2005) or Jacob (2009)).
- **Non-directional speculative arbitrage** bets, so as to benefit from mean-reversion to more normal levels in a number of key spreads such as implied volatility versus historical volatility, three-month implied volatility vs. one-month implied volatility, etc.
- **Volatility exposure hedging** by hedge funds and mutual funds that are often implicitly short volatility. In particular, benchmarked equity fund managers are short volatility because portfolio tracking error and rebalancing costs increase with an increase in the volatility of equity markets (Hill (2004)).

Obviously, the ability for market participants to implement trades on volatility depends on the availability of instruments that can be used to make volatility indices investable quantities. This is what we turn to next.

### 3.2. Instruments for Trading Volatility

Since volatility is the key determinant of option prices, trading in options is one possible way to get volatility exposure, but this is not a clean bet on volatility alone: while option prices are sensitive to volatility, they are not sensitive only to volatility because of non-trivial exposure to changes in the value of the underlying asset. In this context, derivatives instruments on volatility are becoming more popular. With these instruments the investors can have a pure play on volatility and can implement their views more precisely.

In particular, investors can invest in volatility products though exchange-traded futures and options on the volatility indices. Also, other products on volatility are available like exchange-traded notes and OTC variance swaps. Table 2 shows the products currently available on volatility indices.

<table>
<thead>
<tr>
<th>Index</th>
<th>Ticker</th>
<th>Institution (Owner)</th>
<th>Futures Ticker</th>
<th>Options on Futures</th>
</tr>
</thead>
<tbody>
<tr>
<td>CBOE Volatility Index®</td>
<td>VIX</td>
<td>CBOE</td>
<td>VX (VIX Futures), VM (Mini-VIX Futures)</td>
<td>Available</td>
</tr>
<tr>
<td>CBOE DJIA Volatility Index</td>
<td>VXD</td>
<td>CBOE</td>
<td>DV</td>
<td>-</td>
</tr>
<tr>
<td>CBOE NASDAQ-100 Volatility Index</td>
<td>VXN</td>
<td>CBOE</td>
<td>VN</td>
<td>Available</td>
</tr>
<tr>
<td>CBOE Russell 2000 Volatility Index</td>
<td>RVX</td>
<td>CBOE</td>
<td>VR</td>
<td>Available</td>
</tr>
<tr>
<td>DAX Volatility</td>
<td>VDAX-NEW</td>
<td>Deutsche Borse AG</td>
<td>FDAX (delisted)</td>
<td>-</td>
</tr>
<tr>
<td>SMI Volatility</td>
<td>VSMI</td>
<td>SIX Swiss Exchange AG</td>
<td>FVSM (delisted)</td>
<td>-</td>
</tr>
<tr>
<td>EURO STOXX 50 Volatility</td>
<td>VSTOXX</td>
<td>STOXX Limited</td>
<td>FVSX (delisted), FVS (mini-futures)</td>
<td>-</td>
</tr>
</tbody>
</table>

Overall, trading in volatility can be done through a number of alternative investment vehicles: futures on volatility indices, options on volatility indices, exchange-traded notes on volatility indices, and variance swaps as well as forward variance swaps.
Futures on Volatility Indices
Futures are available on four major CBOE volatility indices which track the volatility of the S&P 500, NASDAQ 100, Russell 2000 and Dow Jones Industrial Average. Futures on volatility indices for European market are also available. But recently (July 1, 2009) some of these were delisted from the exchanges, including the futures on the volatility indices for the DAX, SMI, and STOXX which had a contract size of 1000 euros per index point. Dow Jones STOXX Limited continues to offer mini-futures (FVS) on the volatility index (VSTOXX) for the EURO STOXX 50. Mini-futures on VSTOXX have been available since June 2, 2009, with a contract value of 100 euros per index point.

One of the biggest problems with volatility related products is liquidity. Most of the futures on these volatility indices have low trading volume and open interest. The most liquid are VIX futures, which have an open interest of around 50,000 contracts and a daily volume of several thousand contracts. The next most liquid product is the Mini-VIX futures, which has very low open interest (~100s) compared to VIX futures and a low daily volume.

Options on Volatility Indices
CBOE also offers options on VIX, RVX, and VXN which can be used for asymmetric exposure to volatility in these markets. The CBOE Volatility Index Option has the largest volume and open interest compared to other option products on volatility indices. Table 3 shows the dollar volume of options traded each year on these instruments.

<table>
<thead>
<tr>
<th>Option Type</th>
<th>2009</th>
<th>2008</th>
<th>2007</th>
<th>2006</th>
</tr>
</thead>
<tbody>
<tr>
<td>RVX (Russell 2000 Volatility Index Options)</td>
<td>$108,980</td>
<td>$6,693,381</td>
<td>$6,274,162</td>
<td>-</td>
</tr>
<tr>
<td>VIX (CBOE Volatility Index)</td>
<td>$5,224,807,164</td>
<td>$5,199,374,615</td>
<td>$3,343,053,366</td>
<td>$714,252,679</td>
</tr>
<tr>
<td>VXN (NASDAQ 100 Volatility Index Options)</td>
<td>$2,750</td>
<td>$589,685</td>
<td>$952,775</td>
<td>-</td>
</tr>
</tbody>
</table>

Exchange-Traded Notes (ETNs)
Investors can also get exposure to VIX using ETNs. Barclays Bank PLC issues two different types of ETNs which hold VIX future contracts. The iPath S&P 500 VIX Short-Term Futures and iPath S&P 500 VIX Mid-Term Futures provide exposure to short and medium term volatility of the S&P 500.

Variance Swaps and Forward-Start Variance Swaps
Variance swaps and forward-start variance swaps are also used for pure exposure to either realized or implied volatility. These are OTC products. Usually the maturities of these swaps are similar to option expiry dates so that they can be used in hedging applications for option traders. A forward-start variance swap is similar to a variance swap starting at a later date. Unlike VIX futures, variance swaps can provide long term exposure to volatility term structures.

4. Cross-Sectional Volatility Index as a New Volatility Index
Because they play a central role as market uncertainty measures and because they are used as a basis for investable volatility products, informative and robust volatility indices are critically important to a large number of market participants.

As we have seen, existing volatility indices suffer from a number of shortcomings. On the one hand, volatility indices are not available for an extensive set of markets, because they require the presence of a liquid option market; for example, no volatility index exists for small cap stocks, growth/value stocks, or various sectors for developed markets, and volatility indices do not exist even at the broad market level in most emerging markets. On the other hand, where and when they exist, implied volatility estimates are plagued by option-market problems that have little to do with underlying equity markets.
In what follows, we introduce a new set of volatility indices, which are meant to be based on observable and model-free volatility measures, obtained using equity market data alone and available for all markets/sectors at all frequencies. These indices are based on the cross-sectional dispersion of observed stock returns on a given date, a readily measurable quantity for any equity universe. Using formal central-limit arguments, we show that under mild simplifying assumptions this cross-sectional measure provides a very good approximation of average idiosyncratic variance.

The conceptual and technical foundations for these indices have been outlined by Garcia, Mantilla-Garcia, and Martellini (2010), and we propose below a short summary of the formal arguments they present to motivate the use of cross-sectional dispersion as a measure of volatility.

4.1. Cross-Sectional Dispersion and Specific Volatility

We first assume without any loss of generality the following single conditional factor model for (excess) stock returns: 

\[ r_{it} = \beta_{it} F_t + \varepsilon_{it}, \]

where \( F_t \) is the factor excess return at time \( t \), \( \beta_{it} \) is the beta of stock \( i \) at time \( t \), and \( \varepsilon_{it} \) is the residual or specific return on stock \( i \) at date \( t \), with \( E(\varepsilon_{it}) = 0 \) and \( \text{cov}(F_t, \varepsilon_{it}) = 0 \). We assume that the factor model under consideration is a strict factor model, that \( \text{cov}(\varepsilon_{it}, \varepsilon_{jt}) = 0 \) for \( i \neq j \).

We further make the following two simplifying assumptions (we will argue below that they come with very little loss of generality):

- Homogenous beta assumption: \( \beta_{it} = \beta_i \) for all \( i \);
- Homogenous residual variance assumption: \( E(\varepsilon_{i,t}^2) = \sigma^2_{\varepsilon}(t) \) for all \( i \).

Under these assumptions, Garcia, Mantilla-Garcia and Martellini (2010) show that cross-sectional variance (or CSV in short) converges towards specific variance in the limit of an increasingly large number of constituents:

\[
\text{CSV}^{(w_i)}(t) = \frac{\sum_{i=1}^{N_t} w_i (r_{it} - \bar{r}_t)^2}{N_t} \rightarrow s^2(t)
\]

where \( r_{it}^{(w_i)} \) is the weighted-return on the portfolio with weights \( w_i \) at date \( t \), where CSV is the cross-sectional variance and where \( N_t \) is the number of constituents in the universe for a given date \( t \). The proof of this result is based upon central limit arguments, and we refer the interested reader to Garcia, Mantilla-Garcia, and Martellini (2010) for more details.

This result is important because it draws a formal relationship between the dynamics of the cross-sectional dispersion of realized returns and the dynamics of idiosyncratic variance.\(^\dagger\)

Note that this asymptotic result holds for any (non-trivial) weighting scheme. Of course, for a finite number of constituents \( N_t \), different weighting schemes will generate different proxies for idiosyncratic variance. In fact, it can be shown that the equally-weighted CSV is the best estimator for idiosyncratic variance within the class of CSV estimators obtained under a strictly positive weighting scheme. To see this, we report the following result from Garcia, Mantilla-Garcia, and Martellini (2010) about the first two moments of the CSV estimator for a finite number of constituents \( N_t \):

\[
E[\text{CSV}_{t}^{(w_i)}] = s^2(t) \left( 1 - \sum_{i=1}^{N_t} w_i^2 \right)
\]

\[
\text{Var}[\text{CSV}_{t}^{(w_i)}] = 2 s^4(t) \left( \sum_{i=1}^{N_t} w_i^4 - 2 \sum_{i=1}^{N_t} w_i^2 \right)
\]

Hence the CSV appears to be a biased estimator for idiosyncratic variance, with a bias given by the multiplicative factor \( 1 - \sum_{i=1}^{N_t} w_i^2 \), which can be easily corrected for (see section 4.2).

\(^\dagger\) In the practitioners’ literature, cross-sectional dispersion of returns is called “variety” and is used in performance analysis with no formal link to specific volatility (see DiBartolomeo (2006)).
In the end, the bias and variance of the CSV appear to be minimum for the EW scheme, which corresponds to taking \( w_i = 1/N_t \) at each date \( t \).

For the equally-weighted scheme, we thus have:

\[
E[ CSV_{t}^{\text{EW}} ] = s_e^2(t) \left( 1 - \frac{1}{N_t} \right) \overset{N_t \to \infty}{\to} s_e^2(t) \\
\text{Var}[ CSV_{t}^{\text{EW}} ] = 2s_e^4 \left( \frac{N_t - 1}{N_t^2} \right) \overset{N_t \to \infty}{\to} 0
\]

Unlike most previous measures of average idiosyncratic variance, the CSV offers two main advantages: it can be computed directly from observed returns, with no need to estimate other parameters such as betas, and it is available at any frequency and for any universe of stocks. For any given weighting scheme (in particular EW or CW), the corresponding cross-sectional measure has the obvious advantage of being readily computable at any frequency from observed returns. This stands in contrast with the previous approaches that have used monthly measures based on time series regressions on daily returns. The second important feature of the CSV is its model-free nature, since we do not need to specify a particular factor model to compute it.

We now discuss the implications of relaxing the two simplifying assumptions of homogenous residual variances and homogenous betas.

If we first relax the homogenous residual variance assumption, we obtain that:

\[
E[ CSV_{t}^{\text{EW}} ] = \frac{1}{N_t} \sum_{i=1}^{N_t} s_e^2(t) \left( 1 - \frac{1}{N_t} \right)
\]

Hence, the assumption of homogenous residual variances comes with no loss of generality. In the general case with non-homogenous variances, the CSV simply appears to be a (biased) estimator for the average idiosyncratic variance of the stocks in the universe.

If we now relax the homogenous beta assumption, we obtain that:

\[
E[ CSV_{t}^{\text{EW}} ] = \left( \frac{1}{N_t} \sum_{i=1}^{N_t} s_e^2(t) \right) \left( 1 - \frac{1}{N_t} \right) + F_t^2 CSV_{t}^{b}
\]

where:

\[
CSV_{t}^{b} = \frac{1}{N_t} \sum_{i=1}^{N_t} b_i \left( 1 - \frac{1}{N_t} \sum_{i=1}^{N_t} b_i \right)^2
\]

Hence, the CSV appears to be a biased estimator for the average specific variance, even for an increasingly large number of stocks, when the homogenous beta assumption does not hold. Empirically, however, this bias is found to be small. In particular, Garcia, Mantilla-Garcia, and Martellini (2010) estimate that the median value for this term is a mere .348% of specific variance based on daily data for US stocks in the CRSP data base on the sample period ranging from July 1963 to December 2006.

4.2. Estimation Methodology

To construct the volatility index, we first gather information on the entire constituent universe. In particular, we require information on past returns data for purposes of filtering stocks, as well as on current returns for the construction of the current volatility index value.

As a first step, we apply filters to all available constituent stocks, with the aim of excluding certain stocks from index computation. Two specific filters are designed to remove stocks that would not add useful information to the indices. These filters remove outliers and illiquid stocks. A first filter aims at removing stocks that are outliers in terms of the returns observations.
Outlier stocks are stocks with extreme return movements within the reference period. The process uses statistical methods such as principal component analysis (such as eliminating stocks with negative weight in the first principal component) to eliminate outliers based on historical data or the constituent filter removes the stocks that are identified as outliers in terms of current daily returns. Such removal of outliers in terms of historical data or current data will achieve greater robustness of the derived risk measure. A second filter aims at removing illiquid stocks. Illiquid stocks are identified as stocks having stale prices, as indicated by zero returns over a given day, high first-order autocorrelation as well as abnormal values for other common liquidity measures including trading volume.

Once we have constructed a filtered universe of stocks, we compute current returns of each stock, as well as the average return across all stocks for the current time period. This allows us to compute deviations from the expected return for each individual stock.

For the case of the cross-sectional volatility index, we compute the following transformation of cross-sectional variance, which we have shown to be an unbiased estimator for specific volatility within the universe:

$$CVIX_t = \sqrt{\frac{1}{N_t - 1} \sum_{i=1}^{N_t} (r_{it} - \bar{r}_t)^2}$$

where $r_{it}^{EW}$ is the return on the equally-weighted portfolio at date $t$.

In fact, the actual computation of the cross-sectional volatility index is not based on straightforward standard-deviation estimates as shown in the above equation; we use instead quantile-based variance indicators meant to further enhance the robustness of the dispersion estimator by reducing its sensitivity to the presence of outliers that may remain after the filtering process has been applied. The idea behind the construction of quantile-based variance indicators is to take advantage of the elementary relationship between quantiles and the spread of a distribution to provide a more robust variance estimator. For example, for i.i.d. Gaussian returns, a robust quantile-based estimator for the standard-deviation is given by $(Q_{95\%}-Q_{5\%})/(2 \times 1.645)$, where $Q_{k\%}$ denotes the $k$th quantile of returns (see David (1970)). It is worth noting at this stage that more refined approaches to make the cross-sectional volatility index estimate less sensitive to the presence of outliers could be implemented, e.g., an approach based upon a robust regression model (see appendix 1 for more details).

5. Interpretation of the Cross-Sectional Volatility Index

In this section, we argue that the cross-sectional volatility index can be regarded as a proxy for economic uncertainty. We also show that it exhibits a high correlation with the VIX index.

5.1. Cross-sectional volatility index as a Proxy for Economic Uncertainty

As argued below, average specific volatility can be regarded as a proxy that reflects the aggregate uncertainty faced by investors at a given point in time regarding economic fundamentals. To put this analysis in the proper context, we should go back to the very nature of idiosyncratic risk. In an asset pricing model, it represents the risk that belongs proper to an individual firm, after accounting for the sources of risk that are common to all firms. In the previous sections, we have shown that the cross-sectional variance of returns provides a very good measure of this idiosyncratic risk, even if it ignores the risk exposures to the usual common risk factors such as the market return or the Fama-French factors.
To get a first sense of the relationship between the cross-sectional volatility index and economic conditions, we first plot the time-series of the cross-sectional volatility index against NBER recessions for the period 1990 to 2010 (see Garcia, Mantilla-Garcia and Martellini (2010) for a longer sample). The shaded areas in figure 1, which time stamp the NBER recession periods, indicate that the peaks in the probability of remaining in the high-mean high-variance regime coincide most of the times with the contraction periods. Therefore, the cross-sectional volatility index measure appears to be counter-cyclical, with the dispersion of returns being high and quite variable when economic growth subsides.

Figure 1: Time evolution of the cross-sectional volatility index and recession periods

To confirm the interpretation of the cross-sectional volatility index as a measure of aggregate economic uncertainty, Garcia, Mantilla-Garcia and Martellini (2010) further compare the cross-sectional volatility index series to a measure of consumption volatility used in the academic literature (see Bansal and Yaron (2004) or Tédongap (2010)). More specifically, they use the Federal Reserve Economic Data (FRED) personal consumption expenditures of non-durables and services monthly series, divided by the consumer price index and the population values to obtain a per-capita real consumption series, the volatility of which is filtered a GARCH model. While the cross-sectional volatility index series is much noisier than consumption-growth volatility, the coincident movements between the two series are found to be quite remarkable, and the correlation of the two series reaches .401. After a high volatile period just before 2000, both series show a marked downward trend after the turn of the century.

A reasonable explanation for this strong correlation is again to think about a common factor (aggregate economic uncertainty) affecting the idiosyncratic variance of each security. Aggregating over all securities will make the cross-sectional volatility index a function of economic uncertainty. The interpretation of the cross-sectional volatility index as a proxy for economic uncertainty may suggest that an investable form of the cross-sectional volatility index may serve as a hedge against economic slowdown uncertainty.

5.2. Comparison of VIX and cross-sectional volatility index

As argued above, while representing a priori two different underlying risk measures, systematic and average specific volatility indicators should be highly correlated, since they both reflect the aggregate uncertainty faced by investors at a given point in time regarding economic fundamentals.

In what follows, we confirm this intuition and find high correlation between the VIX index, an imperfect measure of systematic risk based on option prices, and the cross-sectional volatility index, a model-free efficient and unbiased proxy for specific risk.
Figure 2 shows the time-series of the quantile-based cross-sectional volatility index, in comparison with the VIX index, based on daily stock return data for the S&P500 universe on the sample period ranging from January 1990 to November 2010. We find a high correlation (.70) on the sample period that confirms that option-implied volatility is closely related to its average idiosyncratic volatility counterpart. We have also noted that the high correlation is robust to changes in market conditions, with a conditional correlation that tends to be higher in down markets. For example, the correlation between VIX and the cross-sectional volatility index is estimated to be .73, slightly higher than the unconditional estimate of .70, when daily market returns are one standard deviation below the mean.

Figure 2: Time evolution of the cross-sectional volatility index and the VIX index

Quantile-based cross-sectional volatility index performance, compared with the VIX index, based on daily stock return data for the S&P500 universe on the sample period ranging from January 1990 to November 2010.

In order to provide further robustness checks for the relationship between VIX and the cross-sectional volatility index, and check whether our results remained consistent over time, we have generated a ten-year rolling window correlation estimate for the two series on the period ranging from January 1990 to November 2010 (see figure 3). These results suggest that the correlation level is systematically high, whatever the time period under consideration. They also suggest evidence of an upward-sloping trend, with a correlation level that has increased to .77 for the most recent ten-year period (2000-2010).

Figure 3: Correlation of the cross-sectional volatility index and the VIX index across Time

Quantile-based cross-sectional volatility index correlation with the VIX index, based on ten-year daily stock return rolling window data on the sample period ranging from January 1990 to November 2010.

---

3 - This time-series is obtained from raw data, without any filtering of outlier data. A smoother time-series is obtained when the filters are applied.
As a final robustness check, we have also tested whether the correlation between implied volatility and the corresponding cross-sectional volatility index is also high in other regions for which a VIX-type index exists. The results are presented in Table 4 below (see also appendix 2 for more details). The correlation is uniformly high across all countries, which suggests that the cross-sectional volatility index can be regarded as a good proxy for the VIX index in regions/sectors where a VIX-type index cannot be computed because of unavailability of a sufficiently liquid option market. It is only in the case of the KOSPI 200 volatility index and the Mexico volatility index that correlation values are somewhat lower. This can probably be explained by the fact that these are the two countries with some of the shortest data history as well (South Korea data is available only from 2003 on and Mexico data is available only from 2004 on).

Table 4: Volatility Indices and their Correlation with Corresponding Cross-Sectional Volatility Index Series.

<table>
<thead>
<tr>
<th>Index</th>
<th>Ticker</th>
<th>Correlation</th>
<th>Sample period</th>
</tr>
</thead>
<tbody>
<tr>
<td>CBOE Volatility Index®</td>
<td>VIX</td>
<td>0.6913</td>
<td>01.1990 – 12.2010</td>
</tr>
<tr>
<td>AEX Volatility</td>
<td>VAEX</td>
<td>0.6192</td>
<td>01.2000 – 12.2010</td>
</tr>
<tr>
<td>BEL 20 Volatility</td>
<td>VBEL</td>
<td>0.6784</td>
<td>01.2000 – 12.2010</td>
</tr>
<tr>
<td>CAC 40 Volatility</td>
<td>VCAC</td>
<td>0.6555</td>
<td>01.2000 – 12.2010</td>
</tr>
<tr>
<td>FTSE 100 Volatility</td>
<td>VFTSE</td>
<td>0.7235</td>
<td>01.2000 – 12.2010</td>
</tr>
<tr>
<td>DAX Volatility</td>
<td>VDAX-NEW</td>
<td>0.6864</td>
<td>01.1992 – 12.2010</td>
</tr>
<tr>
<td>SMI Volatility</td>
<td>VSMI</td>
<td>0.6251</td>
<td>06.1999 – 12.2010</td>
</tr>
<tr>
<td>EURO STOXX 50 Volatility</td>
<td>VSTOXX</td>
<td>0.6783</td>
<td>01.1999 – 12.2010</td>
</tr>
<tr>
<td>NIKKEI Volatility Index</td>
<td>VNKY</td>
<td>0.7032</td>
<td>01.2001 – 12.2010</td>
</tr>
<tr>
<td>India NSE VIX</td>
<td>INVIXN</td>
<td>0.6709</td>
<td>11.2007 – 12.2010</td>
</tr>
<tr>
<td>KOSPI 200 Volatility Index</td>
<td>VKOSPI</td>
<td>0.5863</td>
<td>01.2003 – 12.2010</td>
</tr>
<tr>
<td>Mexico Volatility Index</td>
<td>VIMEX</td>
<td>0.5727</td>
<td>03.2004 – 12.2010</td>
</tr>
</tbody>
</table>

In summary, we find high correlation between the VIX index and the corresponding cross-sectional volatility index series. This is confirmed in all regions for which a VIX-type index exists, suggesting that our measure for volatility is intimately related to option-implied measures where and when such measures are available. We also find the high correlation between the two volatility measures to be robust with respect to time and changes in market conditions.

6. Conclusion
We introduce a new form of volatility index, the cross-sectional volatility index. Through formal central limit arguments, we show that the cross-sectional dispersion of stock returns can be regarded as an efficient estimator for the average idiosyncratic volatility of stocks within the universe under consideration. Among the key advantages of the cross-sectional volatility index measure over currently available measures are its observability at any frequency, its model-free nature, and its availability for every region, sector, and style of the world equity markets, without the need to resort to any auxiliary option market. We also provide some interpretation of the cross-sectional volatility index as a proxy for aggregate economic uncertainty, which suggests that the cross-sectional volatility index should be closely related to option-based implied volatility measures. We confirm this intuition by reporting high correlation between the VIX index and the corresponding cross-sectional volatility index based on the S&P 500 universe. We also find the high correlation between the two volatility measures to be robust with respect to changes in sample period, changes in market conditions, and changes in the region under consideration. Overall, these results suggest that the cross-sectional volatility index is intimately related to other volatility measures where and when such measures are available, and that it can be used as a reliable proxy for volatility when such measures are not available.
7. References


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Appendix 1 – Using robust regressions to generate a cross-sectional volatility index estimates
Broadly speaking this approach consists of regressing cross-sectional returns (after filters have been applied to remove obvious outliers) against a vector of ones using a robust regression. The weights obtained indicate which observations the robust method identifies as outliers, to which small weight will be applied.

More specifically, we start with the model $r_{it} = \beta_{i} F_{t} + \varepsilon_{it}$, where $F_{t}$ is the factor excess return at time $t$, $\beta_{i}$ is the beta of stock $i$ at time $t$, and $\varepsilon_{it}$ is the residual or specific return on stock $i$ at date $t$, with $E(\varepsilon_{it})=0$ and $\text{cov}(F_{t},\varepsilon_{it})=0$. We assume that the factor model under consideration is a strict factor model, that is $\text{cov}(\varepsilon_{it},\varepsilon_{jt})=0$ for $i \neq j$, and also

• Homogenous beta assumption: $\beta_{it} = \beta_{i}$ for all $i$;
• Homogenous residual variance assumption: $E(\varepsilon_{it}^2)=\sigma_{\varepsilon}^2(t)$ for all $i$.

which implies the following model with a simplified notation,

$$r_{it} = a_{t} + \varepsilon_{it}$$

where $a_{t} = \beta_{t} F_{t}$. With $t$ fixed, we can argue that the distribution of $\varepsilon_{it}$ is contaminated with outliers and we can estimate the intercept through a robust method. Following the M-estimation technique (see Rousseeuw and Leroy (1987)), the general form of the estimator of a general linear model is :

$$b = [X'WX]^{-1}X'Wy$$

where $X$ is a matrix denoting the data of the explanatory variables, $y$ is a vector denoting the data of the dependent variable, and $W$ is a diagonal matrix of weights produced by the fitting method. The solution method is iterative and $W$ depends on the data; it is not known in advance. In our case, $X = e$, where $e$ is a vector of ones, i.e.,

$$a = [e'Ve]^{-1}e'Wy = e'Wy$$

since the weights sum up to 1. Rewriting the equation without the matrix notation leads to,

$$a = \sum_{i} w_{i} r_{it}$$

The same weights $w$ can be used in the cross-sectional volatility index estimation since they are reported by the fitting routine. Under the conditions in which the robust estimator converges to the OLS method, the weights $w$ converge to $1/N$. 

Since the weights $w$ resulting from this estimation depend on the sample and are, therefore, stochastic, the convergence result in the paper will hold with some additional bias. The exact bias cannot be calculated explicitly since it depends on the distribution of the returns, but it can be numerically calibrated to the Gaussian case, just as with the quantile based estimator for standard deviation, since the limit relationship implies that for a large $N$, the distribution of the (properly normalized) cross-sectional volatility index will be approximately Gaussian.

On the other hand, the convergence property should still hold since by construction the weights from the robust method satisfy the inequality

$$\max_i (w_i) \leq \frac{B}{N}, \text{ with } B > 1,$$

almost surely. Therefore,

$$\text{CSV}_{w_i} \leq \sum_{i=1}^{N_t} w_i (r_i) \leq \frac{B}{N} \sum_{i=1}^{N_t} (r_i)^2 \xrightarrow{N_t \to \infty} C > 0,$$

i.e., at the limit, cross-sectional volatility index converges to a positive constant.

**Appendix 2 - Comparison of Option-Based Volatility Indices and Cross-Sectional Volatility Index Indices across Countries**
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