Risk Allocation, Factor Investing and Smart Beta: Reconciling Innovations in Equity Portfolio Construction

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About the Authors
Executive Summary
Introduction: From Cap-Weighted Indices to Smart Factor Indices

This publication argues that current smart beta investment approaches only provide a partial answer to the main shortcomings of capitalisation-weighted (cap-weighted) indices, and develops a new approach to equity investing referred to as smart factor investing. It provides an assessment of the benefits of simultaneously addressing the two main shortcomings of cap-weighted indices, namely their undesirable factor exposures and their heavy concentration, by constructing factor indices that explicitly seek exposures to rewarded risk factors while diversifying away unrewarded risks. The results we obtain suggest that such smart factor indices lead to considerable improvements in risk-adjusted performance. For long-term US data, smart factor indices for a range of different factor tilts roughly double the Sharpe ratio of the broad cap-weighted index. Outperformance of such indices persists at levels ranging from 2.92% to 4.46%, even when assuming unrealistically high transaction costs. Moreover, by providing explicit tilts to consensual factors, such indices improve upon many current smart beta offerings where, more often than not, factor tilts result as unintended consequences of ad hoc methodologies. In fact, this publication shows that by using consensual results from asset pricing theory concerning both the existence of factor premia and the importance of diversification, it is possible to go beyond existing smart beta approaches which provide partial solutions by only addressing one of these issues.

Designing Efficient and Investable Proxies for Risk Premia

In this study we focus on four well-known rewarded factors – the Size and Value factors (Fama and French (1993)\(^1\)), the Momentum factor (Carhart (1997)\(^2\)) and the Low Volatility factor (Ang et al. (2006, 2009)\(^3\)). For each rewarded factor, we introduce a corresponding smart factor index, which can be regarded as an efficient investable proxy for a given risk premium. In a nutshell, a risk premium can be thought of as a combination of a risk (exposure) and a premium (to be earned from the risk exposure). Smart factor indices have been precisely engineered to achieve a pronounced factor tilt emanating from the stock selection procedure (relevant risk exposure), as well as high Sharpe ratio emanating from the efficient diversification of unrewarded risks related to individual stocks (fair reward for the risk exposure). The access to the fair reward for the given risk exposure is obtained through a well-diversified, also known as smart-weighted, portfolio, as opposed to a concentrated cap-weighted portfolio, of the selected stocks so as to ensure that the largest possible fraction of individual stocks’ unrewarded risks is eliminated.

The results in Exhibit A confirm that the combination of relevant security selection and appropriate weighting schemes in a two-step process leads to substantial improvements in risk-adjusted performance with respect to the use of a standard cap-weighted index, which typically implies an inefficient set of factor exposures and an excess of unrewarded risk.

On the one hand, starting with a focus on the systematic risk exposure, we find...
that a higher Sharpe ratio can be achieved with the same weighting scheme, here a cap-weighting scheme, for stocks selected on the basis of their loadings on the value, size, momentum and low volatility factors, compared to the case where the full universe is held in the form of a cap-weighted portfolio.

The results we obtain, reported in Exhibit A, show that while the Sharpe ratio of the cap-weighted index is 0.24 on the sample period, it reaches values as high as 0.39 for a mid cap stock selection, 0.30 for a high momentum stock selection, 0.29 for a low volatility stock selection or 0.35 for a value stock selection. These results suggest that a systematic attempt to harvest equity risk premia above and beyond broad market exposure leads to additional risk-adjusted performance. It should be noted at this stage that substantially higher levels of max Drawdown are incurred for the mid cap and value selections, confirming that the reward harvested through the factor exposure is a compensation for a corresponding increase in risk. In contrast, we note that high momentum and low volatility selections lead to lower levels of max Drawdown compared to the no selection case, suggesting that the excess performance earned on these two factors has at best a behavioural explanation, and is not necessarily related to an increased riskiness.

On the other hand, shifting to the management of specific risk exposures, we find that even higher levels of Sharpe ratio can be achieved for each selected factor exposure through the use of a well-diversified weighting scheme, which we take to be an equally-weighted combination of 5 popular smart weighting schemes. Thus, the Sharpe ratio of the so-called diversified multi-strategy combination reaches 0.52 for mid cap stocks, 0.48 for high momentum stocks, 0.50 for low volatility stocks, 0.35 for value stocks, 0.54 for low volatility stocks, 0.54 for low volatility stocks.
volatility stocks and 0.54 for value stocks. These results suggest that multi-strategy factor-tilted indices obtain the desired factor tilts without undue concentration, which provides an explanation for their superior risk-adjusted performance with respect to the cap-weighted combination of the same selection of stocks.

Overall, it appears that the combined effects of a rewarded factor exposure ensured by a dedicated proper security selection process and an efficient harvesting of the associated premium through improved portfolio diversification leads to a Sharpe ratio improvement of around 100% compared to the broad cap-weighted index.

Risk Allocation with Smart Factor Indices

Once a series of smart factor indices have been developed for various regions of the equity universe, they can be used as attractive building blocks in the design of an efficient allocation to these multiple risk premia.

In an attempt to identify, and analyse the benefits of, the possible approaches to efficient risk allocation across the various smart factor indices, we identify four main dimensions that can be taken in consideration when designing a sophisticated allocation methodology (see Exhibit B).

The first, and arguably most important, dimension relates to whether risk is defined by the investor from an absolute perspective in the absence of a benchmark, or whether it is instead defined in relative terms with respect to an existing benchmark, which is more often than not a cap-weighted index. In the former situation, one would use volatility as a relevant risk measure, while tracking error with respect to the cap-weighted index would instead be used in the latter case.

The second dimension concerns whether one would like to incorporate views regarding factor returns in the optimisation process. While additional benefits can be obtained from the introduction of views on factor returns at various points of the business cycle, we focus in what follows only on approaches that are solely based on risk parameters, which are notoriously easier to estimate with a sufficient degree of robustness and accuracy (Merton (1980)). The third dimension is related to the objective of the allocation procedure. Indeed, there are several possible targets for the design of a well-diversified portfolio of factor exposure, depending upon whether

Exhibit B – The Various Dimensions of Allocation Methodologies across Assets or Risk Factors.
one would like to use naive approaches (equal dollar allocation or equal risk allocation) or scientific approaches based on minimising portfolio risk (volatility in the absolute return context or tracking error in the relative return context). The fourth and last dimension related to the presence of various forms of constraints such as minimum/maximum weight constraints, turnover constraints, or factor exposure constraints, which are obviously highly relevant in the context of risk factor allocation.

To illustrate the benefits of an efficient allocation to smart factor indices, we consider a second dataset over the 10-year period from 31-Dec-2003 to 31-Dec 2013 using five sub-regions of the global developed universe, namely, US, UK, Dev. Europe Ex UK, Japan, Dev. Asia Pacific Ex Japan. Using the four smart multi-strategy indices as proxies for the value, size, momentum and volatility rewarded tilts in each region, we obtain a total of (5x4) 20 constituents.

Absolute Return Perspective
We start from the absolute return perspective and consider in Exhibit C five allocation strategies to the 20 aforementioned smart factor indices – an equal dollar contribution portfolio (denoted by Multi Beta EW Allocation), an equal risk contribution portfolio (denoted by Multi Beta ERC Allocation), and then a global minimum variance portfolio (denoted by Multi Beta GMV Allocation). Given that these allocation strategies lead in general to concentrated factor exposures (for example the minimum variance portfolio heavily loads on the low volatility factor indices in each region), we also introduce factor risk parity constraints – that is we restrict our analysis to portfolios such that each one of the four factors has the same contribution to the portfolio volatility. More precisely we consider a global minimum variance portfolio subject to factor risk parity constraints (denoted by Multi Beta GMV - Fact. Allocation), as well as a maximum deconcentration portfolio subject to factor risk parity constraints (denoted by Multi Beta MDecon - Fact. Allocation), a portfolio which can be regarded as the closest approximation to an equally-weighted portfolio that satisfies the factor risk parity constraints.5

We note that the GMV allocation process leads to the lowest volatility, as expected. When analysing the performances in terms of bull versus bear market regimes (defined as positive versus negative returns for the cap-weighted index), we observe that the addition of risk parity constraints to the GMV allocation tends to stabilise the returns across market conditions. For example, in the absence of a factor risk parity constraint, the GMV allocation leads to a massive outperformance of 11.94% with respect to the cap-weighted index in bear markets, which is due to the almost exclusive domination of the low volatility factor, with a defensive bias that proves extremely useful in such market conditions. On the other hand, the relative return in bull markets is negative at -3.90% due to the performance drag associated with exclusively holding defensive equity exposure in bull market conditions. In this context, one key advantage of the introduction of factor risk parity constraints is that it leads to a much more balanced return profile across market conditions with positive outperformance in both

5 - So as to avoid introducing overly strong biases in country exposures, we also introduce a set of constraints dedicated to ensure that each one of the five regions is not too strongly under- or over-represented with respect to its market capitalisation in the cap-weighted global developed index.
We also find that the introduction of factor risk parity constraints has led to a substantial improvement in information ratios with an information ratio above 1 for the Max-Deconcentration allocation under risk parity constraints. Interestingly we note that the introduction of factor risk parity constraints leads to 100% out-performance probabilities over a three-year horizon. Overall, all tested strategies lead to extremely substantial levels of outperformance with respect to the cap-weighted index, with excess returns ranging between 276 and 356 basis points per annum.

Relative Return Perspective

It is often the case that investors maintain the cap-weighted index as a benchmark, which has the merit of macro-consistency and is well-understood by all stakeholders.

In this context, a multi-smart beta solution can be regarded as a reliable cost-efficient substitute to expensive active managers, and the most relevant perspective is not an absolute return perspective, but a relative return perspective, with respect to the cap-weighted index.

In what follows, we focus on two approaches, a naive diversification approach leading to a relative equal risk allocation (R-ERC) portfolio, which focuses on equalising the contribution of the smart factor-tilted indices to the portfolio tracking error, and a scientific diversification approach leading to a relative global minimum variance (R-GMV) portfolio, also known as minimum tracking error portfolio, which focuses on minimising the variance of the portfolio relative returns with respect to the cap-weighted index.

From the results reported in Exhibit D, we note that the focus on relative return leads to lower tracking error levels compared to bear and bull markets (at 2.66% and 3.18% respectively).
the portfolios that had an absolute return focus. For example, the ex-post tracking error is around 2.50% for these two portfolios (2.43% for the relative minimum variance portfolio and 2.56% for the relative equal risk contribution portfolio). Such low tracking error levels, associated with substantial outperformance (more than 300 basis points per annum for the R-ERC portfolio), eventually leads to exceedingly high information ratios. In particular, the relative ERC has an information ratio of 1.22, which is the highest level among all portfolio strategies tested so far, with an outperformance probability of 100% over any given three-year investment horizon during the same period.

**Conclusion: From Cap-Weighted Indices to Smart Factor Indices**

We find that well-rewarded factor-tilted indices constitute attractive building blocks for the design of an improved equity portfolio. First-generation smart beta investment approaches only provide a partial answer to the main shortcomings of cap-weighted indices. Multi-Strategy factor indices, which diversify away unrewarded risks and seek exposure to rewarded risk factors, address the two main problems of cap-weighted indices (their undesirable factor exposures and their heavy concentration) simultaneously.

The results suggest that such Multi-Strategy factor indices lead to considerable improvements in risk-adjusted performance. For long-term US data, smart factor indices for a range of different factor tilts roughly double the Sharpe ratio of the broad cap-weighted index. Moreover, outperformance of such indices persists at levels ranging from 2.92% to 4.46%, even when assuming unrealistically high transaction costs. The outperformance of Multi-Strategy factor indices over cap-weighted factor indices is observed for

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**Executive Summary**

*Exhibit D – Relative ERC and GMV Allocation across Smart Factor Indices (Developed Universe). The table compares performance and risk of Scientific Beta Diversified Multi-Strategy indices converted in US dollars. We look at relative ERC and relative GMV allocations invested in the 20 Diversified Multi-Strategy indices with stock selection based on mid cap, momentum, low volatility, and value in the five sub-regions – US, UK, Dev. Europe Ex UK, Japan and Dev. Asia Pacific Ex Japan. The period goes from 31-December-2003 to 31-December-2013.*

<table>
<thead>
<tr>
<th>Developed (2004-2013)</th>
<th>CW (All Stocks)</th>
<th>Diversified Multi-Strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Multi Beta Relative ERC Allocation</td>
<td>Multi Beta Relative GMV Allocation</td>
</tr>
<tr>
<td>Ann. Returns</td>
<td>7.80%</td>
<td>10.92%</td>
</tr>
<tr>
<td>Ann. Volatility</td>
<td>17.09%</td>
<td>16.10%</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
<td>0.36</td>
<td>0.58</td>
</tr>
<tr>
<td>Max Drawdown</td>
<td>57.13%</td>
<td>54.14%</td>
</tr>
<tr>
<td>Excess Returns</td>
<td>-</td>
<td>3.12%</td>
</tr>
<tr>
<td>Tracking Error (TE)</td>
<td>-</td>
<td>2.56%</td>
</tr>
<tr>
<td>95% TE</td>
<td>-</td>
<td>4.70%</td>
</tr>
<tr>
<td>Information Ratio</td>
<td>-</td>
<td>1.22</td>
</tr>
<tr>
<td>Outperf Prob (3Y)</td>
<td>-</td>
<td>100.00%</td>
</tr>
<tr>
<td>Max Rel. Drawdown</td>
<td>-</td>
<td>5.10%</td>
</tr>
<tr>
<td>Ann. Returns Bull</td>
<td>-</td>
<td>31.38%</td>
</tr>
<tr>
<td>Ann. Returns Bear</td>
<td>-</td>
<td>-25.25%</td>
</tr>
</tbody>
</table>
other developed stock markets as well. By providing explicit tilts to consensual factors, such indices improve upon many current smart beta offerings where, more often than not, factor tilts result as unintended consequences of ad hoc methodologies.

Moreover, additional value can be added at the allocation stage, where the investor can control for the dollar and risk contributions of various constituents or factors to the absolute (volatility) or relative risk (tracking error) of the portfolio. As a result, extremely substantial levels of risk-adjusted outperformance (information ratios) can be achieved even on the absence of views on factor returns. The portfolio strategies we have presented in this publication can be regarded as robust attempts at generating an efficient strategic factor allocation benchmark in the equity space. Obviously, active portfolio managers may generate additional value on top of this efficient benchmark by incorporating forecasts of factor returns at various points of the business cycle in the context of tactical factor allocation decisions.
Introduction
Introduction

Alternative equity indices or smart beta strategies are seen to provide tremendous growth potential. A recent survey (the EDHEC European ETF Survey 2013 by Ducoulombier et al. (2014)) reveals that while only 30% of investment professionals already use products tracking smart beta indices, more than one third of respondents are considering investing in such products in the near future. This publication argues that current smart beta investment approaches only provide a partial answer to the main shortcomings of capitalisation-weighted (cap-weighted) indices, and develops a new approach to equity investing referred to as smart factor investing. It then provides an assessment of the benefits of addressing the two main problems of cap-weighted indices (their undesirable factor exposures and their heavy concentration) simultaneously by constructing factor indices that explicitly seek exposures to rewarded risk factors, while diversifying away unrewarded risks. The results suggest that such smart factor indices lead to considerable improvements in risk-adjusted performance. For long-term US data, smart factor indices for a range of different factor tilts roughly double the Sharpe ratio of the broad cap-weighted index. Outperformance of these indices ranges from 2.92% to 4.46% per annum on this long period and persists even when assuming unrealistically high transaction costs. Moreover, by providing explicit tilts to consensual factors, such indices improve upon many current smart beta offerings where, more often than not, factor tilts result as unintended consequences of ad hoc methodologies. In fact, this publication shows that by using consensual results from asset pricing theory concerning both the existence of factor premia and the importance of diversification, it is possible to go beyond existing smart beta approaches which provide partial solutions by only addressing one of these issues.

Asset pricing theory in fact suggests that there are two main challenges involved in a sound approach to equity investing. The first challenge is the efficient diversification of unrewarded risks, where “diversification” means “reduction” or “cancellation” (as in “diversify away”). Indeed, unrewarded risks are by definition not attractive for investors who are inherently risk-averse and therefore only willing to take risks if there is an associated reward to be expected in exchange for such risk taking, as shown by Harry Markowitz (1952) in his seminal work on portfolio diversification. The second challenge is the efficient diversification of rewarded risks. Here the goal is not to diversify away rewarded risk exposures so as to eventually eliminate or at least minimise them, since this would imply giving up on the risk premia; the goal is instead to efficiently allocate to rewarded risk factors so as to achieve the highest reward per unit of risk. In William Sharpe’s (1964) CAPM, there is a single rewarded risk factor so the second challenge is non-existent, and the only focus should be on holding a well-diversified proxy for the market portfolio. In a multi-factor world, where the equity risk premium is multi-dimensional (including not only market risk, but also size, book-to-market (B/M) ratio, momentum, volatility, etc.), an important component of an investor’s equity investment process is the determination of the appropriate (e.g. Sharpe ratio maximising) allocation to these rewarded risk exposures.
This analysis of the dual challenges to rational equity investing is enlightening with respect to a proper understanding of the intrinsic shortcomings of cap-weighted (CW) indices that are typically used as default investment benchmarks by asset owners and asset managers. On the one hand, CW indices are ill-suited investment benchmarks because they tend to be concentrated portfolios that contain an excessive amount of unrewarded risk. On the other hand, CW indices implicitly embed a bundle of factor exposures that are highly unlikely to be optimal for any investor, if only because they have not been explicitly controlled for. For example, CW indices show by construction a large cap bias and a growth bias, while the academic literature has instead shown that small cap and value where the positively rewarded risk exposures.

This analysis also allows light to be shed on the benefits and shortcomings of existing alternatives to CW indices. Broadly speaking, there have been two main innovations in recent years. On the one hand, a number of index providers have launched so-called smart indices or smart beta indices, which focus on addressing the first shortcoming of CW indices, namely their excessive concentration that leads to an excessive presence of unrewarded risk. Such smart beta indices include various approaches that are based either on scientific diversification (e.g. indices aiming to implement a minimum variance or MSR allocation to selected stocks subject to a number of constraints either on weights or on parameter estimates that are meant to improve the robustness of the portfolio construction methodology) or naive diversification (equal-dollar contribution or equal risk contribution indices). One problem with these smart beta indices, however, is that they fail to address the second problem, namely the explicit control of rewarded risk exposures. Hence, by switching from a CW index to an equally-weighted (EW) or global minimum variance (GMV) or GMV index for example, the investor is switching from one arbitrary bundle of factor exposures to another arbitrary bundle of factor exposures, which may or may not be consistent with the investor’s needs and beliefs. On the other hand, index providers have also launched so-called factor indices, which focus on addressing the second shortcoming of CW indices, namely their lack of controlled factor exposure. Such factor indices are meant to be investable long-only or long-short proxies for some of the rewarded factors that have been analysed in academic literature, such as the value factor, the size factor, the momentum factor or the low volatility factor. One problem with these factor indices, however, is that they fail to address the first problem, namely the excessive concentration problem leading to the presence of unrewarded risk. This is because the weighting scheme used in the design of factor indices is either CW (leading to an excessive degree of concentration) or factor exposure maximising (also leading to a lack of diversification).

In a nutshell, CW indices suffer from two main problems, namely the presence of excessive concentration and the presence of an underlying arbitrary set of factor exposures, and existing alternatives (namely smart indices or factor indices) are reasonably successful attempts...
at addressing one of these problems, but they do so while leaving the other dimension unattended. In the end, risk factors are like vectors; they are defined through the direction they point to, but also their magnitude. Having access to a good proxy for a factor is hardly relevant if the investable proxy only gives access to a fraction of the fair reward per unit of risk to be expected from the factor exposure because of the presence of unrewarded risk due to excessive concentration. The first contribution of this publication is to demonstrate that it is in fact feasible to address two problems simultaneously through the use of smart factor indices, which are smart (meaning well-diversified) indices with selected factor exposures that naturally combine the benefits of smart indices and the benefits of factor indices. In brief, smart factor indices are meant to be the outcome of a process carefully distinguishing the security selection stage from the portfolio construction process.6

The security selection stage is meant to ensure that the right factor-tilt will be associated to each index. For example, one would select a set of value stocks to construct a proxy for a value factor or a set of low volatility stocks to construct a proxy for the low volatility factor. On the other hand, the portfolio construction phase is meant to seek to diversify away unrewarded risk as much as possible by using some naive or scientific approach to diversification. As such the factor index is made "smart", that is better diversified, and the investor can hope to gain a larger fraction of the reward (Sharpe ratio) associated with these factors. The second contribution of this publication is to introduce a formal framework that can be used by investors to allocate to the various smart factor indices, once they have been carefully constructed. This portfolio construction process distinguishes from the unconditional approach, where the investor seeks the optimal exposure to risk factors that are rewarded in the long term by utilising a sophisticated risk allocation framework. A sound approach to smart factor index allocation requires the proper execution of three different steps:

- Choice of factors that are rewarded in the long term;
- Designing factor-tilted portfolios that capture the fair risk-adjusted reward associated with exposure to the factor;
- Choice of a methodology for deriving the optimal multi-factor exposures.

This paper discusses investment choices at each of these steps different steps in detail, and provides an analysis of readily implementable investment solutions that draw on allocations across publicly available factor indices. The remainder of the paper is organised as follows. In the next section, we describe the selection of appropriate factors. Then we describe the design of well-diversified factor indices (referred to as smart factor indices) and compare them with the conventional approach to factor indices. The third section illustrates suitable allocation decisions across smart factor indices. The last section provides conclusions.
1. Selecting Desired Factor Exposures
1. Selecting Desired Factor Exposures

In this section, we review the empirical asset pricing literature to identify the factors that are most likely to bear a long-term reward. Both equilibrium models such as Merton's (1973) inter-temporal capital asset pricing model and no arbitrage models such as Ross's (1976) Arbitrage Pricing Theory allow for the existence of multiple priced risk factors. The economic intuition for the existence of a reward for a given risk factor is that exposure to such a factor is undesirable for the average investor because it leads to losses in bad times when marginal utility is high (Cochrane (2001)). While asset pricing theory provides a sound rationale for the existence of multiple factors, theory provides little guidance on which factors should be expected to be rewarded.

The first order necessary condition for a factor to be deemed important is the existence of empirical research which shows that the identified factor has a significant impact on the cross section of stock returns in US and international equity markets. Several systematically rewarded risk factors have been documented in literature; Harvey et al. (2013) document a total of 314 of such factors. The practice of identifying empirical factors is referred to as 'factor fishing'. Therefore, a key requirement of investors to accept factors as relevant in their investment process is, however, that there is a clear economic intuition as to why the exposure to this factor constitutes a systematic risk that requires a reward and is likely to continue producing a positive risk premium.7

In this study we focus on four well known rewarded factors – Size, Value, Momentum and Low Volatility. Fama and French have identified that value (book-to-market) and size (market cap) explain average asset returns, as a complement to the market beta (Fama and French (1993)). Carhart (1997) empirically proved the existence of another priced factor – the momentum factor. The low volatility factor, which qualifies as an anomaly rather than a risk factor, is the result of the famous 'volatility puzzle,' which states that low-volatility stocks tend to outperform high-volatility stocks in the long run (Ang et al. (2006)).

Empirical Illustration

Exhibit 1.1 shows the returns of signal weighted quintile portfolios that represent portfolios with varying degree of exposure to each factor. Signal weighting is done by weighting the stocks in proportion to their rank by relevant sorting characteristic following Asness et al. (2013). For example,

<table>
<thead>
<tr>
<th>Quintile</th>
<th>Market Cap</th>
<th>B/M Ratio</th>
<th>Momentum</th>
<th>Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>9.74%</td>
<td>19.67%</td>
<td>15.19%</td>
<td>10.21%</td>
</tr>
<tr>
<td>Quintile 2</td>
<td>10.92%</td>
<td>13.78%</td>
<td>13.43%</td>
<td>12.31%</td>
</tr>
<tr>
<td>Quintile 3</td>
<td>13.23%</td>
<td>12.07%</td>
<td>14.01%</td>
<td>12.17%</td>
</tr>
<tr>
<td>Quintile 4</td>
<td>12.14%</td>
<td>9.99%</td>
<td>11.99%</td>
<td>12.97%</td>
</tr>
<tr>
<td>Low</td>
<td>14.71%</td>
<td>8.05%</td>
<td>9.64%</td>
<td>12.44%</td>
</tr>
<tr>
<td>High-Low</td>
<td>-4.97%</td>
<td>11.62%</td>
<td>5.56%</td>
<td>-2.23%</td>
</tr>
</tbody>
</table>

Exhibit 1.1: Performance of quintile portfolios sorted by factors and weighted by rank – The exhibit shows mean annualised returns of quintile portfolios. For each factor, the quintiles are constructed on related stock characteristics – market cap for size, B/M ratio for value, past 1 year minus 1 month returns for momentum, and past 2-year volatility for low volatility. Stocks in each quintile are rank weighted. All portfolios are rebalanced quarterly and the analysis is based on daily total returns from 31/12/1972 to 31/12/2012 (40 years).
in any value quintile consisting of 100 stocks, the stock with highest B/M ratio will have 100 times more weight than the stock with lowest B/M ratio. Same rank based weighting is followed in all quintiles for all factors. The difference between Value and Growth quintiles is 11.62% and that between Mid Cap and Large Cap quintiles is 4.97%.

The debate about the existence of positive premia for these factors is far from closed. While positive premia for these factors are documented in an extensive literature, some authors question the robustness or the persistence of the reward associated with these factors. In fact, one can argue that empirical evidence will not be sufficient to draw a clear conclusion as to which set of factors are acceptable for a given investor. Empirical results always carry a risk of data-mining, i.e. strong and statistically significant factor premia may be a result of many researchers searching through the same dataset to find publishable results. Therefore, the choice of relevant factors should consider the economic rationale behind the reward for a given factor. The following subsection explains why investors should expect a reward for the four main risk factors discussed in this paper.

Moreover, simple, straightforward factor definitions may be useful to avoid the risk of data-mining of complex and unproven factor definitions.

1. Selecting Desired Factor Exposures

Economic Rationale

Given the wide fluctuation in equity returns, the equity risk premium can be statistically indistinguishable from zero even for relatively long sample periods. However, one may reasonably expect that stocks have higher rewards than bonds because investors are reluctant to hold too much equity due to its risks. For other equity risk factors, such as value, momentum, low risk and size, similar explanations that interpret the factor premia as compensation for risk have been put forth in the literature.

It is worth noting that the existence of the factor premia could also be explained by investors making systematic errors due to behavioural biases such as over-reaction or under-reaction to news on a stock. However, whether such behavioural biases can persistently affect asset prices in the presence of some smart investors who do not suffer from these biases is a point of contention. In fact, even if the average investor makes systematic errors due to behavioural "biases", it could still be possible that some rational investors who are not subject to such biases exploit any small opportunity resulting from the irrationality of the average investor. The trading activity of such smart investors may then make the return opportunities disappear. Therefore, behavioural explanations of persistent factor premia often introduce so-called "limits to arbitrage," which prevent smart investors from fully exploiting the opportunities arising from the irrational behaviour of other investors. The most commonly-mentioned limits to arbitrage are short-sales constraints and funding-liquidity constraints.

The table below summarises the main economic explanations for common factor premia.
Value
Zhang (2005) provides a rationale for the value premium based on costly reversibility of investments. The stock price of value firms is mainly made up of tangible assets which are hard to reduce while growth firms' stock price is mainly driven by growth options. Therefore value firms are much more affected by bad times. Choi (2013) shows that value firms have increasing betas in down markets (due to rising asset betas and rising leverage) while growth firms have more stable betas. The value premium can thus be interpreted as compensation for the risk of suffering from losses in bad times. In an influential paper, Lakonishok, Shleifer and Vishny (1994) argue that “value strategies exploit the suboptimal behaviour of the typical investor”. Their explanation of the value premium focuses on the psychological tendency of investors to extrapolate recent developments into the future and to ignore evidence that is contrary to the extrapolation. Glamour firms with high recent growth thus tend to obtain valuations that correspond to overly optimistic forecasts while distressed firms obtain stock market valuations which are overly pessimistic.

Momentum
Momentum stocks are exposed to macroeconomic risk. In particular, Liu and Zhang (2008) provide empirical evidence that past winners have temporarily higher loadings on the growth rate of industrial production. This higher sensitivity of firms with higher expected growth rates is a natural result of firm valuation and is similar to the higher interest rate sensitivity (duration) of bonds at high levels of interest rate (see Johnson (2002)). Low momentum stocks on the other hand have low expected growth and are less sensitive to changes in expected growth.

Behavioural explanations for momentum profits focus on the short-term over-reaction of investors. Daniel et al. (1998) show that two cognitive biases, overconfidence and self-attribution, can generate momentum effects. In particular, they show that investors will attribute the recent performance of the winning stocks they have selected to their stock picking skill and thus further bid up the prices for these stocks, thus generating a momentum effect in the short term, with stock prices only reverting to their fundamental values at longer horizons.

1. Selecting Desired Factor Exposures

<table>
<thead>
<tr>
<th>Risk-Based Explanation</th>
<th>Behavioural Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Value</strong></td>
<td></td>
</tr>
<tr>
<td>Costly reversibility of assets in place leads to high sensitivity to economic shocks in bad times</td>
<td>Overreaction to bad news and extrapolation of the recent past leads to subsequent return reversal</td>
</tr>
<tr>
<td><strong>Momentum</strong></td>
<td></td>
</tr>
<tr>
<td>High expected growth firms are more sensitive to shocks to expected growth</td>
<td>Investor overconfidence and self-attribution bias leads to returns continuation in the short term</td>
</tr>
<tr>
<td><strong>Low Risk</strong></td>
<td></td>
</tr>
<tr>
<td>Liquidity-constrained investors hold leveraged positions in low-risk assets which they may have to sell in bad times when liquidity constraints become binding</td>
<td>Disagreement of investors about high-risk stocks leads to overpricing in the presence of short sales constraints</td>
</tr>
<tr>
<td><strong>Size</strong></td>
<td></td>
</tr>
<tr>
<td>Low profitability leads to high distress risk and downside risk. Low liquidity and high cost of investment needs to be compensated by higher returns</td>
<td>N.A.</td>
</tr>
</tbody>
</table>

Low Risk
Frazzini and Pedersen (2014) provide a model in which liquidity-constrained investors are able to invest in leveraged positions of low-beta assets, but are forced
1. Selecting Desired Factor Exposures

to liquidate these assets in bad times when their liquidity constraints mean they can no longer sustain the leverage. Thus low-risk assets are exposed to a risk of liquidity shocks and investors are compensated for this risk when holding low-beta assets. High-beta assets, on the other hand, expose investors to less liquidity risk and rational investors may thus require less expected return from these stocks than what would be in line with their higher market beta.

Behavioural explanations for the low-risk premium argue that high-risk stocks tend to have low returns because irrational investors bid up prices beyond their rational value. For example, Hong and Sraer (2012) show that when there is disagreement among investors on the future cash flow of firms, short sales constraints will lead to overpricing of stocks where investor disagreement is high. As disagreement increases with a stock’s beta, high-beta stocks are more likely to be overpriced.

Size
Small stocks tend to have lower profitability (in terms of return on equity) and greater uncertainty of earnings (see Fama and French (1995)), even when adjusting for book-to-market effects. Therefore, such stocks are more sensitive to economic shocks, such as recessions. It has also been argued that stocks of small firms are less liquid and expected returns of smaller firms have to be large in order to compensate for their low liquidity (Amihud and Mendelson (1986)). It has also been argued that smaller stocks have higher downside risk (Chan, Chen and Hsieh (1985)).
1. Selecting Desired Factor Exposures
2. Designing Efficient and Investable Proxies for Risk Premia
2. Designing Efficient and Investable Proxies for Risk Premia

2.1. Conventional Approaches to Factor Indices

Factor indices fall into two major categories. The first involves selecting stocks that are most exposed to the desired risk factor and the application of a cap-weighting scheme to this selection. While this approach responds to one limitation of cap-weighted indices, namely the choice of exposure to a good factor, the problem of poor diversification arising from high concentration in a small number of stocks remains unanswered. The second method involves maximising the exposure to a factor, either by weighting the whole of the universe on the basis of the exposure to this factor (score/rank weighting), or by selecting and weighting by the exposure score of the stock to that factor. Here again, the maximisation of the factor exposure does not guarantee that the indices are well-diversified.

To overcome these difficulties, index providers that generally offer factor indices on the basis of the first two approaches have recently sought to take advantage of the development of smart beta indices to offer investors a new framework for factor investing (Bender et al. (2013)). In fact, index providers have recognised that the traditional factor indices they previously offered are not good investable proxies of the relevant risk factors due to their poor diversification, and that the smart beta indices aiming at improved diversification have implicit risk exposures. As a result, providers are proposing to select and combine indices according to their implicit factor exposures. For example, one could seek exposure to the value factor through a fundamental-weighted index. This, however, will not produce a well-diversified index, simply because the integration of the attributes characterising the value exposure into the weighting does not take the correlations between these stocks into account.

Exhibit 2.1 shows that although Fundamentals weighted strategy outperforms the broad cap-weighted index and delivers a Sharpe ratio of 0.34, the Value Multi-Strategy Index exhibits even better Sharpe ratio (of 0.54) as it takes into account the correlation between the value stocks. This is confirmed by a lower Goetzmann-Li-Rouwenhorst (GLR) measure (19.51%) than the Fundamentals based strategy. The index also remains de-concentrated with high Effective number of stocks (ENS).

Moreover, the value tilt is an implicit result of the weighting methodology and it is questionable whether an investor seeking a value tilt would wish to hold any weight in growth stocks which will be present in a fundamentally-weighted index. Exhibit 2.2 shows that the fundamentally-weighted strategy holds, albeit in relatively lower amount, growth stocks, which is not in line with the objective of a value exposure seeking investor. On the other hand, the Value Multi-Strategy index, by construction, invests a larger percentage of the portfolio in value stocks.

Similarly, seeking exposure to the size factor through equal weighting of a broad universe is certainly less effective than selecting the smallest size stocks in the universe and then diversifying them, including with an equal-weighted weighting scheme. Furthermore, a minimum volatility portfolio on a broad universe does not guarantee either the highest exposure to low volatility stocks
2. Designing Efficient and Investable Proxies for Risk Premia

Exhibit 2.1: Performance of the USA Fundamentals Based Strategy and the USA Value Multi-Strategy Index – The exhibit shows the absolute performance, relative performance, and diversification indicators for the Fundamentals Based Index and the USA Value Multi-Strategy Index. The Fundamentals Based Strategy is constructed by selecting the largest 500 stocks in the USA universe by the average of their book value, trailing five-year dividend, trailing five-year cash flow and trailing five-year sales and then weighting the selected stocks by the same measure of size. The GLR measure is defined as the ratio of the portfolio variance to the weighted variance of its constituents. The effective number of stocks (ENS) is defined as the reciprocal of the Herfindahl Index, which in turn is defined as the sum of squared weights of portfolio constituents. The complete stock universe consists of 500 largest stocks in USA. The benchmark is the cap-weighted portfolio of the full universe. The yield on secondary market US Treasury Bills (3M) is the risk-free rate. The return based analysis is based on daily total returns from 31/12/1972 to 31/12/2012 (40 years). All weight based statistics are average values across 160 quarters (40 years) from 31/12/1972 to 31/12/2012.

<table>
<thead>
<tr>
<th></th>
<th>Broad CW</th>
<th>Fundamentals Based Strategy</th>
<th>Value Diversified Multi Strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ann. Returns</td>
<td>9.74%</td>
<td>11.20%</td>
<td>14.44%</td>
</tr>
<tr>
<td>Ann. Volatility</td>
<td>17.47%</td>
<td>16.94%</td>
<td>16.55%</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
<td>0.24</td>
<td>0.34</td>
<td>0.54</td>
</tr>
<tr>
<td>GLR</td>
<td>26.51%</td>
<td>25.82%</td>
<td>19.51%</td>
</tr>
<tr>
<td>Effective Number of Stocks</td>
<td>113</td>
<td>108</td>
<td>190</td>
</tr>
<tr>
<td>Ann. Alpha</td>
<td>0.0%</td>
<td>-0.08%</td>
<td>2.26%</td>
</tr>
<tr>
<td>Market Beta</td>
<td>1.00</td>
<td>0.98</td>
<td>0.91</td>
</tr>
<tr>
<td>Small Minus Big (SMB)</td>
<td>-</td>
<td>0.00</td>
<td>0.16</td>
</tr>
<tr>
<td>High Minus Low (HML)</td>
<td>-</td>
<td>0.34</td>
<td>0.31</td>
</tr>
<tr>
<td>Momentum (MOM) Beta</td>
<td>-</td>
<td>-0.04</td>
<td>0.03</td>
</tr>
<tr>
<td>R-squared</td>
<td>100%</td>
<td>98.0%</td>
<td>95.0%</td>
</tr>
</tbody>
</table>

Exhibit 2.2: Distribution across Value Quintiles of the USA Fundamentals Based Index and the USA Value Multi-Strategy Index – The exhibit shows the distribution of portfolio weights across value quintiles for the Fundamentals Based Index and the USA Value Multi-Strategy Index. The analysis based statistics are average values across 160 quarters (40 years) from 31/12/1972 to 31/12/2012.
2. Designing Efficient and Investable Proxies for Risk Premia

2.2. Smart Beta 2.0 Approach to Smart Factor Indices

An important challenge in factor index construction is to design well-diversified factor indices that capture rewarded risks while avoiding unrewarded risks. The Smart Beta 2.0 approach allows investors to explore different Smart Beta index construction methods in order to construct a benchmark that corresponds to their own choice of factor tilt and diversification method. It allows investors to manage the exposure to systematic risk factors and diminish the exposure to unrewarded strategy specific risks (see Amenc, Goltz and Lodh (2012), and Amenc and Goltz (2013)).

Stock selection, the first step in Smart Beta 2.0, allows investors to choose the right (rewarded) risk factors to which they want to be exposed. When it is performed upon a particular stock-based characteristic linked to stocks’ specific exposure to a common factor, such as size, stock selection allows this specific factor exposure to be shifted, regardless of the weights that will be applied to portfolio individual components. A well-diversified weighting scheme allows for the reduction of unrewarded or specific risks. Stock specific risk (such as management decisions, product success, etc.) is reduced through the use of a suitable diversification strategy. However, due to imperfections in the model there remain residual exposures to unrewarded strategy specific risks. For example, Minimum Volatility portfolios are often exposed to significant sector biases. Similarly, in spite of all the attention paid to the quality of model selection and the implementation methods for these models, the specific operational risk remains present to certain extent. For example, robustness of Maximum Sharpe Ratio scheme depends on a good estimation of the covariance matrix and expected returns. The parameter estimation errors of optimised portfolio strategies are not perfectly correlated and therefore have a potential to be diversified away (Kan and Zhou (2007), Amenc et al. (2012)). A Diversified Multi-Strategy approach,\(^{11}\) which combines the 5 different weighting schemes in equal proportions, enables the non-rewarded risks associated with each of the weighting schemes to be diversified away.

The Smart Beta 2.0 framework thus allows the full benefits of smart beta to be harnessed, where the stock selection defines exposure to the right (rewarded) risk factors and the smart weighting scheme allows unrewarded risks to be reduced.

We now turn to an empirical analysis on US long-term data of a set of Multi-Strategy factor indices constructed for the four main factors introduced above.\(^{12}\) All indices are designed within a Smart Beta 2.0 framework.

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\(^{11}\) - The Diversified Multi-Strategy weighting is an equal weighted combination of the following five weighting schemes: Maximum Deconcentration, Diversified Risk Weighted, Maximum Decorrelation, Efficient Minimum Volatility and Efficient Maximum Sharpe Ratio.

\(^{12}\) - The following selection rules are applied to select stocks for each tilt: Mid Cap: bottom 50% free float adjusted market cap stocks. Value: top 50% stocks by book-to-market (B/M) ratio defined as the ratio of available book value of shareholders’ equity to company market cap; High Momentum: top 50% stocks by returns over the past 52 weeks excluding the last 4 weeks; Low Volatility: bottom 50% stocks by the standard deviation of weekly stock returns over the past 104 weeks. This score based selection is done twice a year (June and December) for Momentum and once a year (June) for the other three factors. Multi-Strategy factor indices are constructed by applying the Diversified Multi-Strategy weighting scheme and CW factor indices are constructed by applying float adjusted market cap weighting on each stock selection.

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Exhibit 2.3: Smart Beta 2.0 framework – The exhibit shows the two key index construction steps which allow factor indices to be designed within a Smart Beta 2.0 framework.
2. Designing Efficient and Investable Proxies for Risk Premia

rebalanced quarterly and dividends are reinvested in the index. The analytics on US indices in subsequent sections use 40 years daily total returns. Stock-level data for portfolio construction and portfolio valuation is obtained from CRSP. We first assess the achievement of the desired factor tilts and then assess risk-adjusted performance.

2.3. Achieving the Objective of Factor Exposure

First we examine how well these Multi-Strategy factor indices fulfill their first objective (i.e. to provide exposure to the desired risk factor). Exhibit 2.4 shows the Carhart 4-factor regression statistics for the Multi-Strategy factor indices and for the cap-weighted (poorly diversified) factor indices. The Mid Cap Multi-Strategy index has a size beta of 0.32, the High Momentum Multi-Strategy index has a momentum beta of 0.17, and the value tilted index has a momentum beta of 0.17, and the value tilted index has a value beta of 0.31. Similarly, the low volatility smart factor index has a low market beta (0.78), as low market beta stocks are usually also low volatility stocks. Cap weighted indices, by construction, load heavily on large cap stocks. Therefore any alternative to cap weighting, especially diversification-based weighting schemes which aim to be more deconcentrated, will induce the exposure to the small cap factor. As a result, smart factor indices have small size exposure as well. However, it is important to note that the magnitude of the small size beta is largest for the smart factor index that is explicitly exposed to small size (i.e. the Mid Cap Diversified Multi-Strategy index (0.32)), while the average small size beta for the other three smart factor indices is 0.12. Similarly the Momentum Diversified Multi-Strategy index has a momentum beta of 0.17 compared to 0.01 on average for the other smart factor indices; and the Value Diversified Multi-Strategy index has a value beta of 0.31 compared to 0.13 on average for the other smart factor indices.

Exhibit 2.4: Exposure of USA Cap Weighted Factor Indices and USA Multi-Strategy Factor Indices to Equity Risk Factors – The exhibit shows 4-factor regression analysis indicators for Cap Weighted Factor Indices and Multi-Strategy Factor Indices for four factor tilts – Mid Cap, High Momentum, Low Volatility, and Value. The Market factor is constructed on the basis of the daily returns of a cap-weighted index of all stocks net of the risk-free rate. The Small size factor is long the CW portfolio of market cap deciles 6 to 8 (NYSE, Nasdaq, AMEX) and short the CW portfolio of the largest 30% of stocks. The Value factor is long the CW portfolio of the highest 30% and short the CW portfolio of the lowest 30% of stocks ranked according to B/M ratio. The Momentum factor is long the CW portfolio of the highest 30% and short the CW portfolio of the lowest 30% stocks ranked by past returns over 52 weeks (excluding the most recent 4 weeks). The regression coefficients (betas and alphas) which are statistically significant at the 95% level are highlighted in bold. The complete stock universe consists of the 500 largest stocks in the USA. The yield on secondary market US Treasury Bills (3M) is the risk-free rate. All statistics are annualised. The analysis is based on daily total returns from 3/12/1972 to 3/12/2012 (40 years).

<table>
<thead>
<tr>
<th></th>
<th>Mid Cap</th>
<th>High Momentum</th>
<th>Low Volatility</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CW</td>
<td>Diversified</td>
<td>CW</td>
<td>Diversified</td>
</tr>
<tr>
<td></td>
<td>Multi</td>
<td>Multi</td>
<td></td>
<td>Multi</td>
</tr>
<tr>
<td></td>
<td>Strategy</td>
<td>Strategy</td>
<td></td>
<td>Strategy</td>
</tr>
<tr>
<td>Ann. Alpha</td>
<td>0.88%</td>
<td>2.59%</td>
<td>0.07%</td>
<td>1.73%</td>
</tr>
<tr>
<td>Market Beta</td>
<td>1.01</td>
<td>0.93</td>
<td>1.01</td>
<td>0.94</td>
</tr>
<tr>
<td>Small Size Beta</td>
<td>0.31</td>
<td>0.32</td>
<td>0.01</td>
<td>0.16</td>
</tr>
<tr>
<td>Value Beta</td>
<td>0.15</td>
<td>0.16</td>
<td>0.00</td>
<td>0.09</td>
</tr>
<tr>
<td>Momentum Beta</td>
<td>0.02</td>
<td>0.00</td>
<td>0.20</td>
<td>0.17</td>
</tr>
<tr>
<td>R-squared</td>
<td>94.3%</td>
<td>92.0%</td>
<td>98.6%</td>
<td>95.5%</td>
</tr>
</tbody>
</table>
2. Designing Efficient and Investable Proxies for Risk Premia

The exercise shows that the simple stock selection process (prior to any optimisation) results in portfolios which have the desired exposure ex-post. If one wants to have a strong factor tilt, using stock selection is the most transparent and simple way to achieve it. In other words, a careful distinction between security selection and weighting scheme allows investors to turn risk into "a choice rather than a fate", to paraphrase an insightful comment by the late Peter Bernstein (1996).

2.4. Avoiding Non Rewarded Risks through Diversification

Exhibit 2.5 presents the performance summary of the four Multi-Strategy factor indices and their Cap-Weighted counterparts. As broad cap-weighted indices remain the widely accepted reference, we use the broad cap-weighted index of the 500 largest stocks as the benchmark. All factor-tilted portfolios, irrespective of the weighting scheme used, outperform the broad cap-weighted index. It verifies that the four chosen risk factors do earn, on average, a positive risk premium in the long run.

For each factor tilt, the Multi-Strategy factor index earns higher returns than the CW factor index for the same tilt. Value and Mid Cap have been the most rewarding factors in the last 40 years in the US market. The Mid Cap and Value smart factor indices earn a premium of 4.45% and 4.70% annual respectively. Low Volatility and High Momentum are comparatively less rewarded; however, their smart factor indices have earned 2.90% and 3.56% excess returns respectively. If one looks at the risk-adjusted performance, Multi-Strategy factor indices consistently post superior Sharpe ratio than CW factor indices. The historical Daily 5% Value-at-Risk and Maximum Drawdown of Multi-Strategy factor indices and CW factor indices are similar. The maximum Drawdown of Multi-Strategy Factor indices is not different from that of CW factor indices. It shows that the increase in performance and the reduction in portfolio risk do not come at the cost of extreme risk.

In addition to the non-parametric methods to compute extreme risk (historical VaR), we also report VaR based on a semi-parametric approach based on Extreme Value Theory (EVT). This approach avoids the disadvantages of non-parametric methods such as the limited number of data points for extreme events and the fact that the information on skewness, kurtosis, and higher-order moments contained in the data points from the body of the distribution becomes less relevant for the lower quintiles in the left tail. At the same time, unlike full parametric approach, it is not exposed to model risk. The results show that Multi-Strategy indices are way superior to the CW factor indices in terms of both EVT 1%VaR (extreme risk) and Return to EVT 1%VaR ratio (extreme risk-adjusted returns).

Both Multi-Strategy and Cap Weighted factor indices are exposed to systematic risk factors which are quite different from those of the broad CW index. Reward to these risk factors varies over time and they experience periods of underperformance relative to the broad market. Consequently all factor indices are exposed to relative risk i.e. risk of underperforming the broad CW benchmark in the short term which is shown by the ‘maximum relative Drawdown’ numbers.
Each factor index, be it a CW factor index or a Multi-Strategy factor index, undergoes periods of relative Drawdown which can be triggered by events specific to stock markets. The occurrence of these Drawdowns does not mean that the factor premium is not robust across different economic conditions. An analysis conditional on business market cycles, presented in the following sections, shows that all Multi-Strategy factor indices earn positive premia in both the contraction and expansion phases of the USA business cycles.

Also, one must not forget that Multi-Strategy factor indices have a limited set of securities to diversify across as they are constructed on 50% of the stock universe. This induces considerable tracking error relative to the broad CW index. However, this tracking error is not a drawback if the associated outperformance is high enough, which is the case with Multi-Strategy factor indices, suggesting that they harvest the relevant factor premia in an efficient way. In fact, the results show that the information ratios of Multi-Strategy factor indices range from 0.47 for Low Volatility to 0.81 for Value.  

This outperformance of smart factor indices over traditional factor indices is not surprising. In fact, a lack of diversification has been identified as a major drawback of CW indices. When it comes to factor-tilted indices, Multi-Strategy factor indices show considerable improvement both over the broad cap-weighted index and over the CW factor indices. We report the effective number of stocks (ENS) which can be used as a measure of deconcentration. Given a fixed number of constituent stocks, an index with balanced weights will have a high ENS.

Going a step further and taking correlations into account, we also report the ratio of portfolio variance to the weighted variance of its constituents (based on the GLR ratio) as a measure of diversification. A weighting scheme which exploits correlations to bring down portfolio volatility will have a low GLR ratio.

The results in Exhibit 2.5 show that Multi-Strategy factor indices are in fact better diversified as they have considerably higher ENS and lower GLR ratio than their CW counterparts. On the other hand, CW factor indices display high GLR ratios and - with the exception of Mid Cap factor - a low effective number of stocks, suggesting that while they may improve the exposure to rewarded risk factors compared to the broad cap-weighted index, they actually aggravate the concentration problem. In contrast, multi-strategy factor-tilted indices obtain the desired factor tilts without undue concentration, which provides an explanation for their superior risk-adjusted performance.

2.5. Robustness of Performance Benefits

2.5.1. Probability of Outperformance

Of particular interest is the information on the probability of outperformance which is defined as the historical probability of outperforming the cap-weighted reference index over a given investment horizon. This measure is reported for investment horizons of 5 years by using a rolling window analysis with one-week step size. We compute the frequency of obtaining positive excess returns if one invests in the strategy for a period of three or five years.
and is computed using a rolling window analysis with 1 week step size. It is an intuitive measure to show how often the strategy has managed to outperform the cap-weighted reference index in the past irrespective of the entry point.

Exhibit 2.6 shows that all Multi-Strategy factor indices have better chances of outperforming the CW benchmark relative to their respective CW factor indices. For a 5-year horizon, the Low Volatility CW index beats the benchmark with a 54.27% probability while the Low Volatility Multi-Strategy index beats it with 84.96% probability. It shows that the long-term outperformance of Multi-Strategy factor indices (over CW factor indices) is well spread out over time and therefore is consistent. The outperformance cannot be explained merely by some specific event or some specific period, but by diversifying away the bulk of the idiosyncratic risk at the constituent level as well as the weighting-scheme specific risk.

2.5.2. Transaction Costs, Liquidity and Capacity

Smart beta strategies, in their unaltered form often incur large turnover and are exposed to liquidity risk – the risk of investing substantial amount in illiquid

<table>
<thead>
<tr>
<th>Broad</th>
<th>Mid Cap</th>
<th>High Momentum</th>
<th>Low Volatility</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>CW</td>
<td>CW</td>
<td>CW</td>
<td>CW</td>
<td>CW</td>
</tr>
<tr>
<td></td>
<td>Diversified Multi Strategy</td>
<td>Diversified Multi Strategy</td>
<td>Diversified Multi Strategy</td>
<td>Diversified Multi Strategy</td>
</tr>
<tr>
<td>Ann. Returns</td>
<td>9.74%</td>
<td>12.54%</td>
<td>14.19%</td>
<td>10.85%</td>
</tr>
<tr>
<td>Ann. Volatility</td>
<td>17.47%</td>
<td>17.83%</td>
<td>16.73%</td>
<td>17.60%</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
<td>0.24</td>
<td>0.39</td>
<td>0.52</td>
<td>0.30</td>
</tr>
<tr>
<td>Historical Daily 5% VaR</td>
<td>1.59%</td>
<td>1.60%</td>
<td>1.64%</td>
<td>1.50%</td>
</tr>
<tr>
<td>Max Drawdown</td>
<td>54.53%</td>
<td>60.13%</td>
<td>58.11%</td>
<td>49.91%</td>
</tr>
<tr>
<td>EVT 1% VaR</td>
<td>2.39%</td>
<td>2.28%</td>
<td>2.11%</td>
<td>2.16%</td>
</tr>
<tr>
<td>Return to EVT 1% VaR</td>
<td>0.11</td>
<td>0.19</td>
<td>0.26</td>
<td>0.14</td>
</tr>
<tr>
<td>Monthly EVT 1% VaR</td>
<td>11.05%</td>
<td>11.46%</td>
<td>10.68%</td>
<td>11.19%</td>
</tr>
<tr>
<td>Ann. Excess Returns</td>
<td>-</td>
<td>2.80%</td>
<td>4.45%</td>
<td>1.10%</td>
</tr>
<tr>
<td>Ann. Tracking Error</td>
<td>-</td>
<td>5.99%</td>
<td>6.89%</td>
<td>3.50%</td>
</tr>
<tr>
<td>95% Tracking Error</td>
<td>-</td>
<td>9.39%</td>
<td>11.56%</td>
<td>8.64%</td>
</tr>
<tr>
<td>Information Ratio</td>
<td>-</td>
<td>0.47</td>
<td>0.66</td>
<td>0.32</td>
</tr>
<tr>
<td>Max Relative Drawdown</td>
<td>-</td>
<td>35.94%</td>
<td>42.06%</td>
<td>14.44%</td>
</tr>
<tr>
<td>GLR</td>
<td>26.51%</td>
<td>19.12%</td>
<td>16.72%</td>
<td>28.52%</td>
</tr>
<tr>
<td>Effective Number of Stocks</td>
<td>113</td>
<td>181</td>
<td>191</td>
<td>65</td>
</tr>
</tbody>
</table>
2. Designing Efficient and Investable Proxies for Risk Premia

Exhibit 2.6: Probability of Outperformance of USA Cap Weighted Factor Indices and USA Multi-Strategy Factor Indices – The exhibit shows the outperformance probability for Cap Weighted Factor Indices and Multi-Strategy Factor Indices for four factor tilts – Mid Cap, High Momentum, Low Volatility, and Value. The probability of outperformance is the historical empirical probability of outperforming the benchmark over a typical investment horizon of 3 or 5 years irrespective of the entry point in time. The analysis is based on daily total returns from 31/12/1972 to 31/12/2012 (40 years).

<table>
<thead>
<tr>
<th>Outperformance Probability</th>
<th>Broad CW</th>
<th>Mid Cap</th>
<th>High Momentum</th>
<th>Low Volatility</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3Y)</td>
<td>-</td>
<td>70.08%</td>
<td>74.12%</td>
<td>78.21%</td>
<td>64.42%</td>
</tr>
<tr>
<td>(5Y)</td>
<td>-</td>
<td>75.33%</td>
<td>78.88%</td>
<td>86.76%</td>
<td>91.25%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>54.27%</td>
<td>84.96%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>72.05%</td>
<td>88.35%</td>
</tr>
</tbody>
</table>

stocks. Both these limitations could result in high transaction costs and other operational hurdles such as large trading lags in the implementation of the strategy. The multi-strategy index performance reported here relates to portfolios which have been subjected to turnover control and capacity adjustments which ensure easy implementation of these strategies.

Exhibit 2.7 compares the turnover incurred by rank weighted (and hence concentrated) factor indices constructed using 500, 250, 100, and 50 stocks with the turnover of Multi-Strategy factor indices (on 250 stocks). It is clear that as one concentrates in a lesser number of stocks in an attempt to select the most value (or least volatile stocks), the associated turnover increases to very high levels. In a nutshell the concentrated factor indices not only suffer from poor diversification, but also exhibit quite high levels of turnover. Interestingly, on the other hand, Multi-Strategy indices on 250 stocks result in lower turnover than that incurred by the rank weighting of 500 stocks.

Indeed, with the exception of the momentum tilt, all smart factor indices have one-way annual turnover in the range of 22%-25%, which is well below the threshold of 30%.

Since Multi-Strategy factor indices aim to replace active style investing, the impact of turnover on the performance, and not the absolute turnover, is the matter of concern. A transaction cost of 20 bps per 100% one-way turnover represents the worst case observed historically and 100 bps represents an 80% reduction in market liquidity. The excess returns net of unrealistically high transaction costs, even for high momentum indices, remain quite significantly high.

Another wide-spread criticism of smart beta strategies is their limited capacity compared to the CW benchmark which by definition invests very small amounts in smaller and less liquid stocks. Exhibit 2.8 shows that the weighted average market capitalisation of factor indices ranges from $2.73 billion for the Mid Cap Multi-Strategy index to $13.67 billion for the Low Volatility Multi-Strategy index compared to $44.9 billion for the broad CW index. Another way to assess the impact of holding less liquid securities is to have an estimation of trading days required to enter (or exit) the investment. ‘Days to Trade’ is average number of days required to trade the total stock position in the portfolio of $1 billion, assuming that 100% of the ‘Average Daily Traded Volume (ADTV)’ can be traded every day.
We report the 95% percentile of this statistic across all stocks and across all rebalancing dates to get an estimate of extremely difficult trades. The results show that all Multi-Strategy factor indices have extreme trades which can be implemented within about 1/4th of a trading day.

Recent surveys show that smart beta is perceived as an alternative to active management and/or as a complement to the existing cap-weighted indices rather than as a replacement of the cap-weighted benchmark. Therefore the assessment of the capacity effect (liquidity) and turnover of smart beta indices, including smart factor indices, must be done in a way similar to that of active managers. In that regard, both the turnover and weighted average market cap of Multi-Strategy factor indices stay at quite manageable levels. Furthermore, the systematic rules-based methodology of Multi-Strategy factor indices guarantee increased transparency that active managers cannot provide.

We show above that Multi-Strategy factor indices in the USA universe, which are based on the 500 largest stocks, do not show any significant illiquidity that could hinder smooth implementation of the strategy. However, it is interesting to assess whether liquidity can be further improved. We thus construct high liquidity versions of the same portfolios by selecting the top 60% of stocks by liquidity among the stocks included in the factor-tilted portfolios. Exhibit 2.9 displays performance and risk characteristics of the resulting High Liquidity Multi-Strategy factor indices. As expected, weighted average market cap and ‘Days to Trade’ numbers show significant improvement. Furthermore, the indices maintain most of the outperformance of the original portfolios even though outperformance is reduced by a few basis points which can be explained by a potential illiquidity premium (Xiong et al. (2012)).
2. Designing Efficient and Investable Proxies for Risk Premia

Exhibit 2.8: Implementation Costs of USA Multi-Strategy Factor Indices – The exhibit shows weighted average market cap, turnover, and outperformance net of transaction costs of Multi-Strategy Factor Indices for four factor tilts – Mid Cap, High Momentum, Low Volatility, and Value. The complete stock universe consists of the 500 largest stocks in the USA. All statistics are average values across 160 quarters (40 years) from 31/12/1972 to 31/12/2012.

<table>
<thead>
<tr>
<th></th>
<th>USA Broad CW</th>
<th>USA Diversified Multi-Strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ann. One-Way Turnover</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Relative Returns</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Relative Returns net of 20 bps transaction costs</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Relative Returns net of 100 bps transaction costs</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Weighted Avg Mkt Cap ($m)</td>
<td>44 959</td>
</tr>
<tr>
<td></td>
<td>Days to Trade $1 bn Investment (95% quintile)</td>
<td>0.03</td>
</tr>
</tbody>
</table>

Exhibit 2.9: Implementation Costs of USA High Liquidity Multi-Strategy Factor Indices – The exhibit shows weighted average market cap, turnover, and outperformance net of transaction costs of High Liquidity Multi-Strategy Factor Indices for four factor tilts – Mid Cap, High Momentum, Low Volatility, and Value. The complete stock universe consists of the 500 largest stocks in the USA. All statistics are average values across 160 quarters (40 years) from 31/12/1972 to 31/12/2012.

<table>
<thead>
<tr>
<th></th>
<th>USA Broad CW</th>
<th>USA High Liquidity Diversified Multi-Strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ann. One-Way Turnover</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Relative Returns</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Relative Returns net of 20 bps transaction costs</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Relative Returns net of 100 bps transaction costs</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Weighted Avg Mkt Cap ($m)</td>
<td>44 959</td>
</tr>
<tr>
<td></td>
<td>Days to Trade $1 bn Investment (95% quintile)</td>
<td>0.03</td>
</tr>
</tbody>
</table>

2.5.3. Sensitivity of Performance to Economic Cycles and Market Conditions

As discussed before, the rewarded factors yield premiums in the long term in exchange of risks that can lead to considerable underperformance or relative drawdowns in smaller periods. Therefore it is important to analyse the time varying performance of Multi-Strategy factor indices in an attempt to identify and characterise the nature of the risk premiums. One approach is to use the NBER definition of business cycle to breakdown the analysis period into alternating sub-periods of ‘contraction’ and ‘expansion’ phases.24 In addition to economic cycles, equity market conditions such as bullish or bearish markets may have a considerable impact on how different portfolio strategies perform. For example, Amenc et al. (2012) show considerable variation in the performance of some popular smart beta strategies in different sub-periods, revealing the pitfalls of aggregate performance analysis based on long periods. Moreover, separating bull and bear market periods to evaluate performance has been proposed by various authors such as Levy (1974), Turner, Starz and Nelson (1989) and Faber (2007). Ferson and Qian (2004) note that an unconditional evaluation made for example during bearish
markets will not be a meaningful estimation of forward performance if the next period were to be bullish. It is therefore important to assess the robustness of performance with respect to such conditions.

Exhibit 2.10 shows annualised excess returns of the four Multi-Strategy factor indices over broad CW index in different business cycles and different equity market conditions. Exhibit 2.10 shows that the performance of Multi-Strategy factor indices depends on market conditions. For example, the Mid Cap Multi-Strategy index post much higher outperformance in bull markets (+5.37%) than in bear markets (+3.02%). The converse is true for the Low Volatility Multi-Strategy index which underperforms by 0.81% in bull markets and outperforms by 7.33% in bear markets. Similarly, the Mid Cap Multi-Strategy index has outperformed by a larger margin in expansion phases while the Low Volatility Multi-Strategy index was favoured by contraction phases. This difference in sensitivities to market conditions suggests room for improvement through allocating across multiple Multi-Strategy factor indices, an issue we turn to in the next subsection.

2.5.4. Performance of Regional Indices
Having shown the robustness of Multi-Strategy factor indices using US long-term track records, we test the consistency of their performance across different developed stock markets. Due to limited availability of reliable data for non-US markets the time period of analysis is 31 December 2003 to 31 December 2013 (10 years). The Multi-Strategy factor indices and CW indices are governed by the same methodology as described for the USA data.
and the only difference across regions is the number of stocks. Stock universe sizes for developed regions are: 500 (USA), 300 (Eurozone), 100 (UK), 500 (Japan), and 400 (Asia Pacific ex Japan).

Exhibit 2.11 shows that all Multi-Strategy factor indices exhibit superior Sharpe ratios relative to the broad CW index and their respective CW factor indices. Information ratios of the four Multi-Strategy factor indices are usually higher than those of CW factor indices and often reach impressive levels such as 0.84 for USA Value and 0.69 for UK Momentum.

Since the analysis period is very short, certain CW factor indices in certain regions do not necessarily outperform the broad CW index despite being tilted towards the long-term rewarded factors – the problem of sample time dependency. For example, Japan High Momentum CW and UK Value CW indices have excess returns of -0.45% and -2.27% respectively in the 10-year period. The benefit from using a well-diversified weighting scheme is more visible in these cases as their corresponding Multi-Strategy factor indices outperform by 1.22% and 1.77%.
2. Designing Efficient and Investable Proxies for Risk Premia

Exhibit 2.11: Performance of Multi-Strategy Factor Indices in developed markets – The exhibit shows the absolute and relative performance of Multi-Strategy Factor Indices in 5 developed regions for four factor tilts – Mid Cap, High Momentum, Low Volatility, and Value. Developed universes and their respective stock universe sizes are: USA (500), Eurozone (300), UK (100), Japan (500), and Developed Asia Pacific ex-Japan (400). The benchmark is the Cap-Weighted index on the full universe for each region. The risk-free rates used for these regions are Secondary Market US T-bill (3M), Euribor (3M), UK T-bill (3M), Japan Gensaki T-bill (1M) and Secondary Market US T-bill (3M) respectively. All statistics are annualised. The analysis is based on daily total returns from 31/12/2003 to 31/12/2013 (10 years).

<table>
<thead>
<tr>
<th></th>
<th>Broad CW</th>
<th>Mid Cap</th>
<th>High Momentum</th>
<th>Low Volatility</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CW</td>
<td>Diversified Strategy</td>
<td>CW</td>
<td>Diversified Strategy</td>
<td>CW</td>
</tr>
<tr>
<td><strong>USA</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ann. Returns</td>
<td>7.68%</td>
<td>10.41%</td>
<td>10.80%</td>
<td>8.64%</td>
<td>9.40%</td>
</tr>
<tr>
<td>Ann. Volatility</td>
<td>20.23%</td>
<td>22.33%</td>
<td>20.29%</td>
<td>20.38%</td>
<td>20.07%</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
<td>0.30</td>
<td>0.40</td>
<td>0.45</td>
<td>0.35</td>
<td>0.39</td>
</tr>
<tr>
<td>Ann. Relative Returns</td>
<td>-</td>
<td>2.73%</td>
<td>3.12%</td>
<td>0.96%</td>
<td>1.72%</td>
</tr>
<tr>
<td>Ann. Tracking Error</td>
<td>-</td>
<td>5.07%</td>
<td>4.23%</td>
<td>4.29%</td>
<td>5.07%</td>
</tr>
<tr>
<td>Information Ratio</td>
<td>-</td>
<td>0.54</td>
<td>0.74</td>
<td>0.22</td>
<td>0.34</td>
</tr>
<tr>
<td><strong>Eurozone</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ann. Returns</td>
<td>6.35%</td>
<td>7.99%</td>
<td>8.41%</td>
<td>9.09%</td>
<td>10.60%</td>
</tr>
<tr>
<td>Ann. Volatility</td>
<td>20.58%</td>
<td>18.63%</td>
<td>16.69%</td>
<td>19.74%</td>
<td>16.66%</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
<td>0.21</td>
<td>0.32</td>
<td>0.38</td>
<td>0.35</td>
<td>0.51</td>
</tr>
<tr>
<td>Ann. Relative Returns</td>
<td>-</td>
<td>1.64%</td>
<td>2.05%</td>
<td>2.73%</td>
<td>4.25%</td>
</tr>
<tr>
<td>Ann. Tracking Error</td>
<td>-</td>
<td>6.28%</td>
<td>7.07%</td>
<td>4.82%</td>
<td>7.05%</td>
</tr>
<tr>
<td>Information Ratio</td>
<td>-</td>
<td>0.26</td>
<td>0.29</td>
<td>0.57</td>
<td>0.60</td>
</tr>
<tr>
<td><strong>UK</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ann. Returns</td>
<td>8.32%</td>
<td>11.76%</td>
<td>11.10%</td>
<td>9.46%</td>
<td>12.71%</td>
</tr>
<tr>
<td>Ann. Volatility</td>
<td>19.18%</td>
<td>19.67%</td>
<td>17.95%</td>
<td>20.57%</td>
<td>17.99%</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
<td>0.30</td>
<td>0.46</td>
<td>0.47</td>
<td>0.33</td>
<td>0.56</td>
</tr>
<tr>
<td>Ann. Relative Returns</td>
<td>-</td>
<td>3.44%</td>
<td>2.78%</td>
<td>1.14%</td>
<td>4.39%</td>
</tr>
<tr>
<td>Ann. Tracking Error</td>
<td>-</td>
<td>7.17%</td>
<td>7.29%</td>
<td>5.95%</td>
<td>6.37%</td>
</tr>
<tr>
<td>Information Ratio</td>
<td>-</td>
<td>0.48</td>
<td>0.38</td>
<td>0.19</td>
<td>0.69</td>
</tr>
<tr>
<td><strong>Japan</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ann. Returns</td>
<td>4.09%</td>
<td>4.97%</td>
<td>5.72%</td>
<td>3.64%</td>
<td>5.31%</td>
</tr>
<tr>
<td>Ann. Volatility</td>
<td>22.62%</td>
<td>21.21%</td>
<td>19.26%</td>
<td>22.39%</td>
<td>19.95%</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
<td>0.17</td>
<td>0.23</td>
<td>0.29</td>
<td>0.15</td>
<td>0.26</td>
</tr>
<tr>
<td>Ann. Relative Returns</td>
<td>-</td>
<td>0.89%</td>
<td>1.64%</td>
<td>-0.45%</td>
<td>1.22%</td>
</tr>
<tr>
<td>Ann. Tracking Error</td>
<td>-</td>
<td>6.62%</td>
<td>7.73%</td>
<td>5.28%</td>
<td>7.48%</td>
</tr>
<tr>
<td>Information Ratio</td>
<td>-</td>
<td>0.13</td>
<td>0.21</td>
<td>-0.09</td>
<td>0.16</td>
</tr>
<tr>
<td><strong>Developed Asia Pacific ex Japan</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ann. Returns</td>
<td>12.91%</td>
<td>15.31%</td>
<td>15.91%</td>
<td>16.12%</td>
<td>18.01%</td>
</tr>
<tr>
<td>Ann. Volatility</td>
<td>23.93%</td>
<td>23.08%</td>
<td>20.72%</td>
<td>25.45%</td>
<td>22.13%</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
<td>0.47</td>
<td>0.60</td>
<td>0.69</td>
<td>0.57</td>
<td>0.74</td>
</tr>
<tr>
<td>Ann. Relative Returns</td>
<td>-</td>
<td>2.40%</td>
<td>2.99%</td>
<td>3.21%</td>
<td>5.10%</td>
</tr>
<tr>
<td>Ann. Tracking Error</td>
<td>-</td>
<td>6.98%</td>
<td>7.55%</td>
<td>4.73%</td>
<td>6.85%</td>
</tr>
<tr>
<td>Information Ratio</td>
<td>-</td>
<td>0.34</td>
<td>0.40</td>
<td>0.68</td>
<td>0.74</td>
</tr>
</tbody>
</table>
3. Risk Allocation with Smart Factor Indices
3. Risk Allocation with Smart Factor Indices

In Section 2, we introduced the concept of a *smart factor index*, which can be regarded as an efficient investable proxy for a given risk premium. In a nutshell, a risk premium can be thought of as a combination of a risk (exposure) and a premium (to be earned from the risk exposure). Smart factor indices have been precisely engineered to achieve a pronounced factor tilt emanating from the stock selection procedure (right risk exposure), as well as high Sharpe ratio emanating from the efficient diversification of unrewarded risks related to individual stocks (fair reward for the risk exposure). The access to the fair reward for the given risk exposure is obtained through a well-diversified *smart-weighted* portfolio (as opposed to a concentrated cap-weighted portfolio) of the selected stocks so as to ensure that the largest possible fraction of individual stocks’ unrewarded risks is eliminated.

3.1. The Many Dimensions of Efficient Risk Allocation
Section 3 is dedicated to showing that such smart factor indices can be used as attractive building blocks in the design of an efficient allocation to the multiple risk premia to be harvested in the equity universe.

3.1.1. From Equally-Weighted to Efficient Risk Allocation to Smart Factor Indices
In the context of generating a “smart” (meaning efficient) allocation to the smart factor indices, a natural first, albeit naive, approach, consists in forming an equally-weighted portfolio of the selected smart factor indices, in this case the indices that serve as proxies for the value, mid cap, momentum and low volatility risk premia. The risk and return characteristics of this equally-weighted portfolio of the four selected smart factor indices for the US region are summarised in Exhibit 3.1.

The results we obtain suggest a massive outperformance (398 basis points per annum over the sample period), with an outperformance probability exceeding 80% over all 3-year periods and with a volatility lower than the CW index volatility. Overall, the Sharpe ratio is more than doubled, going from 0.24 for the CW reference index to 0.52 for the EW multi-smart beta portfolio. In a nutshell, this outperformance comes from an efficient response to the two main shortcomings of CW indices, namely their inefficient factor exposures and their insufficient reward for each given factor exposure.

While an equally-weighted scheme is the simplest approach one can use, it is likely that the use of more sophisticated weighting schemes could provide additional value, in particular when it comes to the management of the risks relative to the CW benchmark. Clearly, a unique allocation framework that generates optimal portfolios for all possible investors does not exist; the allocation methodology should instead be tailored to accommodate the preferences and constraints of each investor.

In an attempt to identify, and analyse the benefits of, the possible approaches to efficient risk allocation across the various smart factor indices, we identify four main dimensions that can be taken into consideration when designing a sophisticated allocation methodology (see Exhibit 3.2). These dimensions
3. Risk Allocation with Smart Factor Indices

define the allocation framework and the choices implemented in each one of these dimensions allow asset managers to perform the risk allocation exercise so as to best accommodate investors' needs.

The first dimension relates to whether risk is defined by the investor from an absolute perspective in the absence of a benchmark, or whether it is instead defined in relative terms with respect to an existing benchmark, which is more often than not a CW index. In the former situation, one would use volatility as a relevant risk measure (restricting the analysis to the second moment of the portfolio return distribution), while tracking error with respect to the CW index would instead be used in the latter case (restricting the analysis to the second moment of the portfolio excess return distribution with respect to the CW index).

The second dimension concerns whether one would like to incorporate views regarding factor returns in the optimisation process. While additional benefits can be obtained from the introduction of views on factor returns at various points of the business cycle, we focus only on approaches that are solely based on risk parameters, which are notoriously easier to estimate with a sufficient degree of robustness and accuracy (Merton (1980)).

Exhibit 3.1 – Implementation of EW Allocation across Smart Factor Indices. The table compares performance and risk of Scientific Beta Diversified Multi-Strategy indices. The Multi Beta Diversified Multi-Strategy index is the equal combination of the four Diversified Multi-Strategy indices with stock selection based on mid cap, momentum, low volatility, and value respectively. All statistics are annualised and daily total returns from 31-December-1972 to 31-December-2012 are used for the analysis. The CRSP S&P500 index is used as the cap-weighted benchmark. The yield on Secondary US Treasury Bills (3M) is used as a proxy for the risk-free rate.

<table>
<thead>
<tr>
<th>US Long Term (Dec 1972 - Dec 2012)</th>
<th>Diversified Multi-Strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CW Index</td>
</tr>
<tr>
<td>Ann. Returns</td>
<td>9.74%</td>
</tr>
<tr>
<td>Ann. Volatility</td>
<td>17.47%</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
<td>0.24</td>
</tr>
<tr>
<td>Max Drawdown</td>
<td>54.53%</td>
</tr>
<tr>
<td>Ann. Excess Returns</td>
<td>4.45%</td>
</tr>
<tr>
<td>Ann. Tracking Error</td>
<td>6.80%</td>
</tr>
<tr>
<td>95% Tracking Error</td>
<td>11.55%</td>
</tr>
<tr>
<td>Information Ratio</td>
<td>0.66</td>
</tr>
<tr>
<td>Outperf. prob (3Y)</td>
<td>74.12%</td>
</tr>
</tbody>
</table>

Exhibit 3.2 – The Various Dimensions of Allocation Methodologies across Assets or Risk Factors.
3. Risk Allocation with Smart Factor Indices

The third dimension is related to the objective of the allocation procedure. Indeed, there are several possible targets for the design of a well-diversified portfolio of factor exposures, depending upon whether one would like to use naive approaches (equal dollar allocation or equal risk allocation) or scientific approaches based on minimising portfolio risk (volatility in the absolute return context or tracking error in the relative return context). If views on expected returns are introduced, then one may also envision maximising risk-adjusted returns, either from an absolute perspective (maximising the portfolio Sharpe ratio) or from a relative perspective (maximising the portfolio information ratio).

The fourth and last dimension related to the presence of constraints is the optimisation. A simple constraint that is required by many investors is a long-only position in the smart factor index constituents. Additional weight constraints could be used so as to shrink a scientifically diversified portfolio towards a naively diversified portfolio (equal-dollar or equal-risk allocation) in an attempt to alleviate the concern over impact on parameter uncertainty on out-of-sample portfolio performance (see Jagannathan and Ma (2003) or DeMiguel et al. (2009) for competing forms of shrinkage towards the equally-weighted portfolio, and Deguest, Martellini and Meucci (2013) for shrinkage towards the risk parity portfolio). Other types of constraints can/should be added to enhance the investability of the portfolio, including constraints on maximum turnover, minimum liquidity or capacity levels, etc. In addition to such weight constraints, one may also envision the introduction of constraints on country, sector or factor exposures. In what follows, we precisely argue that measuring and controlling factor exposures is obviously highly relevant in the context of risk factor allocation.

3.1.2. On the Relevance of Measuring and Controlling Factor Exposures

One natural question that arises with smart factor indices is to analyse what their exposure to common equity factors is, including precisely the market factor, value factor, size factor, momentum factor and low volatility factor. Since the smart factor indices have been engineered to serve as proxy-replicating portfolios for these factors, one would ideally expect that each smart factor index has (in a long-short version) a beta of 1 with respect to the corresponding factor, and a beta of 0 with respect to other factors. Besides, applying this analysis to a CW index would lead to a better measurement of the factor biases (e.g. large cap and growth tilts) of this index. In what follows, we perform this analysis by regressing the CW index, as well as all eight selected smart factor indices (based on the selection of value stocks, but also growth stocks; mid cap stocks, but also large cap stocks; high momentum stocks, but also low momentum stocks; low volatility stocks, but also high volatility stocks) with respect to a set of long/short factor returns. In addition to the market factor, we consider for consistency the value factor, the size factor, the momentum factor and the low volatility factor.

It is important to note at this stage that we are not computing the factor exposure of the CW index and the smart factor indices with respect to a CW version of these factors, but instead with respect to an EW version of these factors. Generally speaking, there are at least three different contexts in which
3. Risk Allocation with Smart Factor Indices

Factor returns are being used in portfolio analysis. On the one hand, factors can be used in a performance attribution context. In this case, the academic consensus is in favour of the use of cap-weighted factors, which is consistent with the fact that the benchmark implicitly or explicitly used by investors is more often than not a cap-weighted index. On the other hand, factors can be used as investable benchmarks themselves. In this case, the focus should be on maximising efficiency measured in terms of risk-adjusted performance, and the market cap weighting scheme is no longer taken for granted since cap weighting often leads to high concentration, and therefore an excess of unrewarded risk. Finally, factors can be used in the context of risk budgeting strategies, where the focus is on measuring and adjusting the relative contribution of various factors to portfolio risk. In this context, the regression of the CW index onto a set of factors that would include the CW index factor itself would only give the trivial result that the CW factor has a beta of 1 with respect to itself and a beta of 0 with respect to other factors. So as to identify and measure the factor biases of the CW index, the choice of an EW scheme, which can be regarded as the most natural and neutral approach, is commonly used in academic research (see Plyakha, Uppal and Vilkov (2014) for a recent paper documenting the impact of the weighting scheme, EW versus CW, on standard cross-sectional as well as time-series asset pricing tests). In other words, it is equally valid to claim that the EW index has a value and small cap bias with respect to the CW reference, or to claim that the CW index has a growth and large cap bias with respect to an EW reference. Since our focus is on the shortcomings of the CW index, and possible remedies to these shortcomings, and because we use factors in an absolute risk budgeting exercise, it is only fitting that we use a set of EW factors as regressors.

The results of this analysis are shown in Exhibit 3.3, which leads to a number of interesting insights.

Exhibit 3.3 – Factor Exposure of Smart Factor-Tilted and CW Indices. The graph shows the betas computed with multivariate linear regressions using 2-year rolling windows of daily total returns from 31-December-1972 to 31-December-2012. The factors used to perform linear regressions are Max-Deconcentration smart factor US indices used as proxy for equally-weighted factors.
3. Risk Allocation with Smart Factor Indices
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The first finding is that the CW index shows a pronounced negative exposure to the mid cap factor (thus confirming its large cap bias) and a negative exposure to the value factor (thus confirming its growth bias). In addition, the CW index shows a positive exposure to the momentum factor and an exposure sometimes positive, but also sometimes negative to the low volatility factor. All in all, these results suggest that holding a CW index leads an investor to holding an inefficient and uncontrolled for bundle of factor exposures.

The second finding is that each factor index (e.g. the value smart factor index) has a beta fluctuating around a value of about 0.5 with respect to the corresponding factor (in this case the value factor) while the smart factor index corresponding to the opposite factor tilt (in this case the growth smart factor index) has a beta fluctuating around a value of about -0.5 with respect to the same factor. In this sense, an investor going long the smart value index factor and short the smart growth index factor would have a beta of about 1 with respect to the long-short value factor, with respect to the corresponding factor (in this case the value factor).

The third and last key finding is that while most smart factor indices have a low beta with respect to factors others than the one for which they serve as a factor replicating portfolio (with a positive or negative exposure), some of these betas are not quite zero. For example, most smart factor indices show a positive, albeit not extremely high, exposure to the low volatility factor, which is consistent with the fact that most weighting schemes that are used lead to a bias in favour of the least volatile stocks within a given stock selection.

Another explanation for the presence of residual factor exposures for the smart factor indices is the presence of non-zero correlations between the factors, as can be seen from the Exhibit 3.4, which shows the factor correlation estimates over the whole sample. These results confirm for example that low volatility firms tend to be more represented within large cap firms versus small cap firms (hence the negative correlation between the low volatility factor and the mid cap factor), that growth firms and past winners tend to be over represented within large cap firms (hence the positive correlation between the value and mid cap factors or the negative correlation between mid cap and momentum factors). In principle, one could try and orthogonalise the factors, but typical orthogonalisation procedures such as principal component analysis are typically highly intrusive, and even some of the least intrusive orthogonalisation techniques (see for example Meucci, Santangelo and Deguest (2013) for minimum linear torsion techniques) typically involve a substantial cost in terms of robustness. In Section 3.2.2, we show that the absence of total purity of the factor replicating portfolios (i.e. the fact that smart factor indices have betas not exactly equal to zero with respect to the factors that they are tracking) or of the factors themselves (i.e. the fact that the factors are correlated even in their long-short version), is not a strong problem per se as long as such biases can be measured and controlled for using suitably-designed factor risk budgeting constraints.
3. Risk Allocation with Smart Factor Indices

As a last comment, let us note that from the betas obtained in Exhibit 3.3, we can compute for each smart factor index, or portfolio of such factor indices, the specific risk $\sigma_\varepsilon$ defined as the mean-squared error of the multivariate linear regression with respect to the long-only market factor and the four standard long-short factors (Size, Value, Momentum and Low Vol):

$$r_p = \alpha_p + \beta_{pwm} \mu_m + \sum_{k=1}^{K} \beta_{pk} r_k + \varepsilon,$$

where

$$\beta_{pk} = \sum_{j=1}^{N} \beta_{pj} = (\beta_{1k} \ldots \beta_{Nk}) w$$

so as to obtain:

$$\sigma_\varepsilon = \sqrt{\text{var} \left( r_p - \alpha_p - \beta_{pwm} \mu_m - \sum_{k=1}^{K} \beta_{pk} r_k \right)}$$

We notice in Exhibit 3.5 that investing in an equally-weighted portfolio of smart factor indices leads to a strong decrease in unrewarded risks compared to the cap-weighted index. Similarly, investing in the equally-weighted allocation of smart factors also leads to a specific risk that is lower than the minimal specific risk values among the smart factor indices themselves. This decrease in specific, a priori unrewarded, risk translates into superior risk-adjusted performance, as already noted.

3.1.3. A Roadmap for Risk Allocation with Smart Factor Indices

To summarise the previous discussions, we shall sequentially consider in what follows the absolute return approach and the relative return approach, both with and without factor risk parity/budgeting constraints. In each case, we consider naive approaches to diversification (maximum deconcentration in terms of dollar or risk contributions) and scientific approaches.
(minimum risk from the absolute or the relative return perspective).

As previously mentioned, all these methodologies will be implemented without any active views (expected return forecasts) on constituents or factors; they generate portfolios that can be regarded as attractive starting points, with very substantial risk-adjusted outperformance benefits with respect to cap-weighted indices, to which additional benefits could be added by asset managers possessing skills for actively timing factor exposures.

We have performed two types of empirical experiments using two different investment universes: the US on the one hand, and world developed markets on the other hand.

First, we use a US dataset over a long period of time starting on 31-Dec-1972 and ending on 31-Dec 2012, and consider as smart factor indices the classical rewarded tilts over long-term period, namely mid cap, value, momentum and low volatility. For the weighting scheme of the constituents, as argued in Section 2, we adopt an agnostic point of view, and consider the Multi-Strategy smart factors which are equally-weighted combinations of the five weighting schemes available on the Scientific Beta platform: Max-Deconcentration, Max-Decorrelation, Efficient Minimum Volatility, Efficient Maximum Sharpe Ratio, and Diversified Risk-Weighting.

We also consider a second dataset over the 10-year period going from 31-Dec-2003 to 31-Dec 2013 using five sub-regions of the global developed universe: US, UK, Dev. Europe Ex UK, Japan, Dev. Asia Pac. Ex Japan. Using the same four smart multi-strategy indices as proxies for the value, size, momentum and volatility rewarded tilts in each sub-region as in the US experiment, we obtain a total of (5x4) 20 constituents. Since the results we obtain in both cases are qualitatively very similar, we focus on the international data in what follows, which allows us to identify and discuss additional relevant questions such as constraints on regional exposures. The results for the US case can be found in a dedicated Appendix.

3.2. Absolute Return Perspective

As argued previously, following an equally-weighted allocation is equivalent to holding an equal dollar allocation, which does not necessarily lead to an equal risk allocation. Formally, the risk contribution of a stock to the total risk of a portfolio is given by the weight of the stock in the portfolio times the marginal contribution of the stock to the total portfolio volatility. Qian (2006) shows that decomposing total portfolio volatility in terms of its constituents’ risk contributions is also related to the expected contributions to the portfolio losses, particularly when considering extreme losses. In what follows, we consider two approaches to the management of portfolio volatility—one approach based on minimising portfolio volatility (global minimum variance approach) and another approach based on imposing an equal contribution of all constituents to the portfolio volatility (heuristic equal risk contribution approach).
3. Risk Allocation with Smart Factor Indices

3.2.1. Absolute Risk Management without Factor Risk Exposure Constraints

In our attempt to design an efficient allocation to smart factor indices, we first impose that all constituents to the portfolio have the same contribution to portfolio risk. To do so, we first consider the decomposition of portfolio variance:

$$\sigma_p^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \sigma_{ij} = \sum_{i=1}^{N} w_i \sum_{j=1}^{N} w_j \sigma_{ij}$$

where $w_i$ is the (positive) portfolio weight of stock $i$ and $\sigma_p$ the portfolio volatility. A natural solution for imposing an equal risk allocation which is an agnostic approach in terms of risk rewarding, is to implement an equal-risk contribution portfolio (ERC). If we define the contribution to risk as:

$$c_i(w) = \frac{1}{N} \frac{\partial \sigma_p^2}{\partial w_i}$$

with

$$\sum_{i=1}^{N} c_i(w) = \sigma_p^2$$

The ERC portfolio is defined as the allocation $w$ that satisfies the following identity:

$$\frac{c_i(w)}{\sigma_p^2} = \frac{1}{N} \text{ for all } i$$

If one makes the explicit assumption that all pairwise correlation coefficients across constituents are identical, then the ERC weights can be obtained analytically and are proportional to the inverse volatility of the smart factor indices. In the general case, i.e. without the assumption of identical pairwise correlations across stocks, the risk parity methodology does not yield a closed-form solution. However, Maillard, Roncalli and Teiletche (2010) propose numerical algorithms to compute risk parity portfolios.

Overall, ERC and EW are two competing forms of implementation of agnostic diversification. When looking at the empirical analysis performed in the Global Developed Universe shown in Exhibit 3.6, we find that the allocation between the equally-weighted and the ERC schemes can exhibit strong discrepancies. For example, the largest average weight over the period under study is given to the Japan Low Volatility smart factor index (7.45%), whereas the lowest weight is given to the Dev. Europe Ex UK Value smart factor (3.78%). We also find that the equal risk contribution can lead to regional allocations that strongly deviate from the corresponding allocation within a cap-weighted index, where the larger markets (e.g. the US) strongly dominate smaller markets such as Japan for example. So as not to introduce overly strong biases with respect to the CW index, and even though the focus is not on relative risk management in this section, we introduce, in what follows, a set of constraints dedicated to ensure that each sub-region is not too strongly under- or over-represented with respect to its market capitalisation in the CW global developed index.

In order to do so, we impose the following constraints (with $\delta = 2$) in each region:

$$\frac{mcap_{Reg}}{\delta} \leq w_{Reg, MultCap} + w_{Reg, Value} + w_{Reg, HMMom}$$

$$w_{Reg, LowVol} \leq \delta mcap_{Reg}$$

where $mcap$ represents the market capitalisations of the different sub-regions, and $w_{Reg}$ is the weight in each smart factor index of the same corresponding sub-regions.
3. Risk Allocation with Smart Factor Indices

In Sample Absolute Risk Contributions by Asset in % - Global Dev. ERC MultiBeta

In Sample Absolute Risk Contributions by Asset in % - Global Dev. EW MultiBeta
3. Risk Allocation with Smart Factor Indices

These constraints at the regional level will be used in the context of the design of the scientifically diversified portfolio, in this case the GMV portfolio of the 20 constituents, which is the only efficient portfolio that does not require expected return inputs:

\[
\min \mathbf{w}^\top \mathbf{Cw}
\]

We notice in Exhibit 3.7 that the GMV allocation with geographical constraints leads to a portfolio almost exclusively invested in the lowest volatility smart index of each sub-region (on average, 52.47% in the low volatility smart factor US index, 8.60% in the low volatility smart factor UK index, 16.42% in the low volatility smart factor Dev. Europe Ex UK index, 12.68% in the low volatility smart factor Japan index, and 6.74% in the low volatility smart factor Dev. Asia Pacific Ex Japan index).

Exhibit 3.7 – GMV Allocations to Smart Factor Diversified Multi-Strategy Indices Under Geographical Constraints (Developed Universe). The graph shows the allocation and risk contributions of the GMV allocation invested in the 20 Diversified Multi-Strategy indices converted in US Dollars with stock selection based on Mid Cap, Momentum, Low Volatility, and Value in the five sub-regions: US, UK, Dev. Europe Ex UK, Japan and Asia Pacific Ex Japan. The period goes from 31-December-2003 to 31-December-2013.

In the end, this process leads to a dynamically managed portfolio of the 20 constituents that should achieve a low volatility, but that is highly concentrated.

Exhibit 3.8 shows that the portfolio variance is almost exclusively driven by the low volatility factor, an observation that stresses the need for the introduction of risk factor budgeting constraints in order to better balance the factor contributions to the risk of the portfolio.28

3.2.2. Introducing Risk Budgeting Constraints

Having an equal contribution of the constituents to the overall portfolio risk is not identical to having an equal contribution of the factors.29 In the following, we use the factor exposure of the smart factor indices presented in Section 3.1.2 to analyse the question. Again, we will compute exposure constraints.
3. Risk Allocation with Smart Factor Indices

Exhibit 3.8 – Factor Contribution to GMV Allocation (Developed Universe). The graph shows the contribution of the four long-short factors to the risk of the portfolio resulting from a GMV allocation to the 20 Diversified Multi-Strategy indices with stock selection based on Mid Cap, Momentum, Low Volatility, and Value in the five sub-regions: US, UK, Dev. Europe Ex UK, Japan and Dev. Asia Pacific Ex Japan. Both risk parity and geographical constraints are imposed onto the resulting portfolios. The period goes from 31-December-2003 to 31-December-2013.

with respect to the equally-weighted version of the factors, since they are the most neutral reference portfolios.

In order to measure the contribution of the factors to the portfolio variance, we go back to the decomposition of the portfolio return as the sum of $K+1$ factors leading to:

$$r_p = \alpha_p + \beta_{p,\text{ref}} \cdot \epsilon + \sum_{i=1}^{K} \beta_{pi} r_i + \epsilon,$$

where

$$\beta_{pi} = \sum_{j=1}^{K} \omega_j \beta_{ij} = (\beta_{i1}, \ldots, \beta_{iK})' \omega$$

Then, focusing only on the contribution of the $K$ long-short factors to the portfolio variance leads to the following expression for the contribution of factor $i$ to the variance coming from the $K$ factors:

$$c_{i,K}^{\text{var}}(w) = \beta_{i,\text{ref}} \sum_{j=1}^{K} \beta_{ij} \sigma_{ij}$$

As a neutral target, we may seek to impose an equal contribution of the factors to the variance coming from these $K$ factors. This extension of the ERC approach from the constituents to the factors leads to the following $K$ linear constraints in the design of the portfolio:

$$c_{i,K}^{\text{var}}(w) = c_{j,K}^{\text{var}}(w) \text{ for all } i, j \leq K$$

In what follows, we introduce factor risk budgeting constraints to the portfolio allocation process so as to avoid the domination of any one particular factor (such as the domination of the low volatility factor implicit in Exhibit 3.7). When the number of constituents $N$ is greater than the number of factors constraints $K$, and long-short solutions are allowed, an infinite number of portfolios satisfy a given set of factor risk budgets (e.g. factor risk parity exposure). In a long-only context, we may have zero or multiple solutions. When no solution exists, then one can start with the
3. Risk Allocation with Smart Factor Indices

long-short version and rescale the weights to avoid short positions.

On the other hand, when multiple solutions exist, one can address the diversification of specific risks, e.g. from a scientific perspective, by minimising portfolio variance subject to factor risk parity constraints:

\[
\min_{\mathbf{w}} \quad \mathbf{v}(\mathbf{w}) = \mathbf{w}^T \mathbf{Cw}
\]

such that

\[
c_i^{fu}(\mathbf{w}) = c_j^{fu}(\mathbf{w}), \text{ for all } i, j \leq K
\]

We may also maximise portfolio deconcentration, measured by the effective number of constituents, again subject to factor risk parity constraints:\[30\]

\[
\max_{\mathbf{w}} \quad \text{ENC} = \frac{1}{\mathbf{w}^T \mathbf{w}}
\]

such that

\[
c_i^{fu}(\mathbf{w}) = c_j^{fu}(\mathbf{w}), \text{ for all } i, j \leq K
\]

In Exhibit 3.9 we show the result of the Max-Deconcentration and GMV allocations under risk parity as well as geographical constraints.

First of all, we notice that factor risk parity is satisfied, and that the portfolio is no longer simply invested in the low-volatility constituents as in Exhibit 3.7.

Similarly to the allocation we obtained in Exhibit 3.7, we also notice that the aggregated weights in the different sub-regions appear to represent the sub-region market capitalisations more fairly than in the EW or ERC allocations of Exhibit 3.6, due to the presence of regional constraints. We also note that the Max-Deconcentration approach shows a more stable allocation over time compared to the GMV which is still sensitive

---

30 - Remember that in the absence of constraints, maximising deconcentration simply leads to giving a weight of $1/N$ to each constituent in the universe.

Exhibit 3.9 – Max Deconcentration and GMV Allocations Under Risk Factor and Geographical Constraints (Developed Universe). The graph shows the allocations and factor contributions of the Max-Deconcentration and GMV Diversified Multi-Strategy indices invested in the 20 Diversified Multi-Strategy indices with stock selection based on Mid Cap, Momentum, Low Volatility, and Value in the five sub-regions: US, UK, Dev. Europe Ex UK, Japan and Dev. Asia Pacific Ex Japan. Both risk parity and geographical constraints are imposed onto the resulting portfolios. The period goes from 31-December-2003 to 31-December-2013.
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3. Risk Allocation with Smart Factor Indices

to changes in input parameters. Also we see that the addition of factor risk parity constraints forces the allocations to spread more evenly the country weight among the different tilts compared to Exhibit 3.7.

In Exhibit 3.10 we report the risk and return characteristics of the various portfolios, and compare the results that have been obtained so far.

We note that the GMV allocation process leads to the lowest volatility. Also, we notice that the EW and ERC allocations have higher returns and higher volatilities than the GMV, as is often the case. Also we note that the introduction of factor risk parity constraints has led to a substantial improvement in information ratios, with an information ratio above 1 for the Max-Deconcentration allocation under geographical and risk parity constraints. This shows that the introduction of factor risk parity constraints leads to a stabilisation of the portfolio that has resulted in strong outperformance (3.37%) over the CW index with a tracking error hardly greater than 5% (see Section 3.3 for explicit management of relative risk leading to a further decrease in tracking error).

Interestingly we note that the introduction of factor risk parity constraints leads to 100% outperformance probabilities over a 3-year horizon.

In Exhibit 3.11, we analyse the performances in bull versus bear market regimes (defined as positive versus negative returns for the CW index). We observe that the addition of risk parity constraints to the GMV allocation tends to stabilise the returns across market conditions. For example, in the absence of factor risk parity constraint, the GMV allocation leads to a massive outperformance of 11.94% with respect to the CW index in bear markets, which is due to the almost exclusive domination of the low volatility factor, with a defensive bias that proves extremely useful in such market conditions. On the other hand, the relative return in bull markets is negative at -3.90%.

### Exhibit 3.10 – Multi Beta Allocations across Smart Factor Indices (Developed Universe)

The table shows the allocations of the EW, ERC, GMV under geographical constraints, and both the Max-Deconcentration and GMV Diversified Multi-Strategy indices under geographical and risk parity constraints, invested in the 20 Diversified Multi-Strategy indices with stock selection based on Mid Cap, Momentum, Low Volatility, and Value in the five sub-regions: US, UK, Dev. Europe Ex UK, Japan and Dev. Asia Pacific Ex Japan. The period goes from 31-December-2003 to 31-December-2013.

<table>
<thead>
<tr>
<th>Developed (2004-2013)</th>
<th>Diversified Multi-Strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CW (All Stocks)</td>
</tr>
<tr>
<td>Ann. Returns</td>
<td>7.80%</td>
</tr>
<tr>
<td>Ann. Volatility</td>
<td>17.09%</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
<td>0.36</td>
</tr>
<tr>
<td>Max Drawdown</td>
<td>57.13%</td>
</tr>
<tr>
<td>Excess Returns</td>
<td>-</td>
</tr>
<tr>
<td>Tracking Error (TE)</td>
<td>-</td>
</tr>
<tr>
<td>95% TE</td>
<td>-</td>
</tr>
<tr>
<td>Information Ratio</td>
<td>-</td>
</tr>
<tr>
<td>Outperf. Prob (3Y)</td>
<td>-</td>
</tr>
<tr>
<td>Max Relative Drawdown</td>
<td>-</td>
</tr>
</tbody>
</table>

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due to the performance drag associated with exclusively holding defensive equity exposure in bull market conditions. In this context, one key advantage of the introduction of factor risk parity constraints is that it leads to a much more balanced return profile across market conditions with positive outperformance in both bear and bull markets (at 2.66% and 3.18% respectively).

We have shown that simple allocations that do not balance their expositions to the factors may be too exposed to the low-volatility factor, which may lead to lower relative returns with respect to the cap-weighted index, particularly in bull market regimes. Another way to design allocation strategies that take into account the presence of a CW index benchmark is to directly perform risk management using relative returns.

### 3.3. Relative Return Perspective

It is often the case that investors maintain the cap-weighted index as a benchmark, which has the merit of macro-consistency and is well-understood by all stakeholders. In this context, a multi smart beta solution can be regarded as a reliable cost-efficient substitute to expensive active managers, and the most relevant perspective is not an absolute return perspective, but a relative perspective with respect to the cap-weighted index.

In what follows, we focus on two approaches:

- **Naive diversification**: relative equal risk allocation (R-ERC) portfolio, which focuses on equalising the contribution of the smart factor-tilted indices to the portfolio tracking error.
- **Scientific diversification**: relative global minimum variance portfolio (R-GMV), also known as minimum tracking error portfolio, which focuses on minimising the variance of the portfolio relative returns with respect to the cap-weighted index.

### Exhibit 3.11 – Multi Beta Allocations across Smart Factor Indices in Bull/Bear Regimes (Developed Universe)

The table shows the allocations of the EW, ERC, GMV under geographical constraints, and both the Max-Deconcentration and GMV Diversified Multi-Strategy indices under geographical and risk parity constraints, invested in the 20 Diversified Multi-Strategy indices with stock selection based on Mid Cap, Momentum, Low Volatility, and Value in the five sub-regions: US, UK, Dev. Europe Ex UK, Japan and Dev. Asia Pacific Ex Japan in bull and bear market regimes. The period goes from 31-December-2003 to 31-December-2013.

<table>
<thead>
<tr>
<th>Developed (2004-2013)</th>
<th>Diversified Multi-Strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Multi Beta EW Allocation</td>
</tr>
<tr>
<td>Ann. Ret. Bull</td>
<td>31.58%</td>
</tr>
<tr>
<td>Ann. Rel. Ret. Bull</td>
<td>2.50%</td>
</tr>
<tr>
<td>Tracking Error Bull</td>
<td>5.03%</td>
</tr>
<tr>
<td>Ann. Ret. Bear</td>
<td>-24.51%</td>
</tr>
<tr>
<td>Ann. Vol. Bear</td>
<td>21.33%</td>
</tr>
<tr>
<td>Ann. Rel. Ret. Bear</td>
<td>4.65%</td>
</tr>
<tr>
<td>Tracking Error Bear</td>
<td>9.64%</td>
</tr>
</tbody>
</table>
It should be noted that controlling for factor exposure biases from an absolute risk budgeting perspective is no longer a key required ingredient since the CW index now already provides a proper anchor point, which is an implicit (as opposed to explicit) reference set of factor exposures. In the same manner, we find that regional constraints are no longer needed, since a portfolio seeking to equalise the contributions of the 20 constituents to the portfolio tracking error, or seeking to minimise the tracking error, will not lead to a severe overweighting of smaller regions with respect to larger regions, in contrast to what has been found from an absolute risk perspective.

In Exhibit 3.12, we show the allocations of the relative GMV and relative ERC portfolios. First of all, we find again that the relative ERC allocation is more stable over time, which is due to the higher sensitivity of the relative GMV allocation to the parameter estimates, confirming a higher degree of robustness with the ERC approach. Even though both allocation strategies rely on risk parameter estimates, scientific diversification tends to over-use input information compared to the more agnostic risk budgeting diversification, which makes a more parsimonious use of input estimates (see Roncalli (2013) for more details and interpretations for the higher robustness of ERC portfolios with respect to errors in risk parameter estimates).

Secondly, by construction, we observe that the relative ERC leads to identical constituents’ contributions to the tracking error. However, the relative GMV portfolio involves non-equal time-varying contributions from various constituents to the tracking error of the portfolio, even though it is less exposed to the...
3. Risk Allocation with Smart Factor Indices

![Allocation by Asset in %: Global Dev. R-GMV MultiBeta](chart1)

![In Sample Relative Risk Contributions by Asset in %: Global Dev. R-ERC MultiBeta](chart2)

![In Sample Relative Risk Contributions by Asset in %: Global Dev. R-GMV MultiBeta](chart3)
low-volatility factor even without factor risk parity constraints than the GMV portfolio shown in Exhibit 3.7 because of the relative risk focus.

Finally, we display in Exhibit 3.13 the risk and return characteristics of the relative ERC and GMV allocation strategies.

We note that the focus on relative return leads to lower tracking error levels compared to the portfolios that had an absolute return focus. For example, the ex-post tracking error is around 2.50% for these portfolios (2.43% for the minimum tracking error portfolio and 2.56% for the relative equal risk contribution portfolio). Such low tracking error levels, associated with substantial outperformance (more than 300 basis points per annum for the R-ERC portfolio), eventually leads to exceedingly high information ratios. In particular, the relative ERC has an information ratio of 1.22, which is the highest level among all portfolio strategies tested so far, with an outperformance probability of 100% over any given three-year investment horizon during the same period. We also find that the focus on relative risk leads to lower tracking errors in bull and bear market regimes compared to their absolute risk counterparts.

3.4. Investability Considerations

The multi-factor allocations analysed do not only to provide efficient management of risk and return, but also allow for genuine investability. In fact, each of the smart factor indices has a target of 30% annual one-way turnover which is set through optimal control of rebalancing (with the notable exception of the momentum tilt, which has a minimal target of 60% turnover). In addition, the stock selections used to tilt the indices implement buffer rules in

order to reduce unproductive turnover due to small changes in stock characteristics. The component indices also apply weight and trading constraints relative to market-cap weights so as to ensure high capacity. Finally, these indices offer an optional High Liquidity feature which allows investors to reduce the application of the smart factor index methodology to the most liquid stocks in the reference universe. Amenc et al. (2014a, 2014b) present a more detailed explanation on how including carefully designed rules at different stages of the index design process eases implementation of investments in smart beta indices.

In addition to these implementation rules, which are applied at the level of each smart factor index, the multi-beta allocations provide a reduction in turnover (and hence of transaction costs) compared to a separate investment in each of the smart factor indices. This reduction in turnover arises from different sources. First, when the renewal of the underlying stock selections takes place, it can happen that a stock being dropped from the universe of one smart factor index is being simultaneously added to the universe of another smart factor index. Second, for constituents that are common to several smart factor indices, the trades to rebalance the weight of a stock in the different indices to the respective target weight may partly offset each other.

Exhibit 3.14 displays statistics relative to the investability of the multi-beta EW and relative ERC allocations along with the average of the mid cap, momentum, low volatility and value smart factor indices. For comparison, we also show the same analytics for their Highly Liquid counterparts. We see that the turnover of multi-beta indices is very reasonable. In fact, managing a mandate on each smart factor index separately would yield a turnover which is higher than the average turnover across the smart factor indices. This is due to the fact that rebalancing each component index to the allocation target would induce extra turnover. However, implementing the multi-beta index in a single mandate exploits the benefits of natural crossing arising across the different component indices, and it actually reduces the turnover below the average level observed for component indices. In the table, as opposed to managing the same allocations separately, we provide the amount of turnover that is internally crossed in multi-beta indices for each multi-beta allocation. We see that about 6% turnover is internally crossed by the EW allocation and that the ERC allocation which tends to generate more turnover also exploits natural crossing effects more than the EW allocation (around 7.8% is crossed internally). These cancelling trades result in an average one-way annual turnover that can be even lower than for the EW allocation, as is the case in the Developed universe.

In addition to turnover, the exhibit also shows the average capacity of the indices in terms of the weighted average market-cap of stocks in the portfolio. This index capacity measure indicates very decent levels with an average market-cap of around US$ 10bn for the multi-beta index, while the highly liquid version further increases capacity to levels exceeding US$ 15bn in the case of the US Long Term Track Records. In the case of the Developed universe, the weighted average market caps are higher since the period under scrutiny is more recent (last 10 years) – around US$ 16.3bn for the standard
3. Risk Allocation with Smart Factor Indices

Exhibit 3.14 – Implementation of Multi-Beta Allocations across Standard or Highly Liquid Factor-Tilted Indices

The analysis is based on daily total return data from 31-December 1972 to 31 December 2012 (40 years) in Panel A and from 31-December-2003 to 31-December-2013 (10 years) in Panel B. The S&P 500 index and SciBeta Developed CW index are used respectively as the cap-weighted reference for US Long Term Track Records and SciBeta Developed Investable Indices. Days to Trade is the number of days necessary to trade the total stock positions, assuming a US$1bn AUM and that 100% of the Average Daily Dollar Traded Volume can be traded every day. The weighted average market capitalisation of index is in $million and averaged over the 40-year period. All statistics are average values across 160 quarters (40 years). The net returns are the relative returns over the cap-weighted benchmark net of transaction costs. Two levels of transaction costs are used - 20 bps per 100% one-way turnover and 100 bps per 100% one-way turnover. The first case corresponds to the worst case observed historically for the large and mid-cap universe of our indices while the second case assumes 80% reduction in market liquidity and a corresponding increase in transaction costs. The risk-free rate is the return of the 3-month US Treasury Bill. (*)Due to data availability, the period is restricted to last 10 years of the sample for Scientific Beta US indices.

Panel A

<table>
<thead>
<tr>
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<tbody>
<tr>
<td></td>
<td>All Stocks</td>
</tr>
<tr>
<td></td>
<td>Average of 4 Smart Factor Indices</td>
</tr>
<tr>
<td>One-Way Turnover</td>
<td>34.19%</td>
</tr>
<tr>
<td>Internally Crossed Turnover</td>
<td>-</td>
</tr>
<tr>
<td>Days to Trade for $1 bn Initial Investment (Quantile 95%)*</td>
<td>0.02</td>
</tr>
<tr>
<td>Weighted Avg. Market Cap ($m)</td>
<td>9 378</td>
</tr>
<tr>
<td>Information Ratio</td>
<td>0.67</td>
</tr>
<tr>
<td>Relative Returns</td>
<td>3.90%</td>
</tr>
<tr>
<td>Relative Returns net of 20 bps transaction costs (historical worst case)</td>
<td>3.84%</td>
</tr>
<tr>
<td>Relative Returns net of 100 bps transaction costs (extreme liquidity stress scenario)</td>
<td>3.56%</td>
</tr>
</tbody>
</table>

Panel B

<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All Stocks</td>
</tr>
<tr>
<td></td>
<td>Average of 4 Smart Factor Indices</td>
</tr>
<tr>
<td>One-Way Turnover</td>
<td>45.69%</td>
</tr>
<tr>
<td>Internally Crossed Turnover</td>
<td>-</td>
</tr>
<tr>
<td>Days to Trade for $1 bn Initial Investment (Quantile 95%)*</td>
<td>0.048</td>
</tr>
<tr>
<td>Weighted Avg. Market Cap ($m)</td>
<td>16 047</td>
</tr>
<tr>
<td>Information Ratio</td>
<td>0.78</td>
</tr>
<tr>
<td>Relative Returns</td>
<td>2.57%</td>
</tr>
<tr>
<td>Relative Returns net of 20 bps transaction costs (historical worst case)</td>
<td>2.48%</td>
</tr>
<tr>
<td>Relative Returns net of 100 bps transaction costs (extreme liquidity stress scenario)</td>
<td>2.11%</td>
</tr>
</tbody>
</table>

Source: www.ScientificBeta.com
indices and US$ 23bn for the highly liquid ones. In both regions, we provide an estimate of the time that would be necessary to set up an initial investment (i.e. full weights) of US$1bn in the indices, assuming that the average daily dollar traded volume can be traded (100% participation rate) and that the number of days required grows linearly with the fund size. Overall, this does highlight the ease of implementation of the multi-beta indices and the effectiveness of the high liquidity option. Indeed, the Days to Trade required for the initial investment on US indices are very manageable (about 0.12 days for the standard multi-beta indices, and 0.07 days with the highly liquid feature). Even in the Developed universe, the highly liquid multi-beta indices would require about 0.09 days of trading. In addition, one should keep in mind that the number of days needed to rebalance the indices (i.e. trade the weight change rather than the full weight on each stock) would be much lower. Even though the excess return is reduced by a few basis points, which can be explained by a potential illiquidity premium, it should be noted that the highly liquid multi-beta indices do maintain the level of relative risk-adjusted performance (information ratio) of the standard multi-beta indices in the US case and provide even stronger information ratios in the Developed universe. Finally, even when assuming unrealistically high levels of transaction costs, all the smart factor indices deliver strong outperformance (from 2% to 3.69%) net of costs in both regions. Compared to the average stand-alone investment in a smart factor index, the multi-beta indices almost always result in higher average returns net of costs due to the turnover reduction through natural crossing effects across their component smart factor indices.
Conclusion
Conclusion

We find that well-rewarded factor-tilted indices constitute attractive building blocks for the design of an improved equity portfolio. First-generation smart beta investment approaches only provide a partial answer to the main shortcomings of cap-weighted indices. Multi-Strategy factor indices, which diversify away unrewarded risks and seek exposure to rewarded risk factors, address the two main problems of cap-weighted indices (their undesirable factor exposures and their heavy concentration) simultaneously. The adoption of a simple and consistent portfolio construction methodology allows for the avoidance of data mining risks. The results suggest that such Multi-Strategy factor indices lead to considerable improvements in risk-adjusted performance. For long-term US data, smart factor indices for a range of different factor tilts roughly double the Sharpe ratio of the broad cap-weighted index. Moreover, outperformance of such indices persists at levels ranging from 2.92% to 4.46%, even when assuming unrealistically high transaction costs. The outperformance of Multi-Strategy factor indices over CW factor indices is observed for other developed stock markets as well. By providing explicit tilts to consensual factors, such indices improve upon many current smart beta offerings where, more often than not, factor tilts result as unintended consequences of ad hoc methodologies.

Moreover, additional value can be added at the allocation stage, where the investor can control for the dollar and risk contributions of various constituents or factors to the absolute (volatility) or relative risk (tracking error) of the portfolio. As a result, extremely substantial levels of risk-adjusted outperformance (information ratios) can be achieved even on the absence of views on factor returns. The portfolio strategies we have presented in this paper can be regarded as robust attempts at generating an efficient strategic factor allocation process in the equity space. Active portfolio managers may generate additional value by incorporating forecasts of factor returns at various points of the business cycle in the context of tactical factor allocation decisions.

While our approach has been focused on an asset-only perspective, it is feasible, and potentially desirable, to design dedicated efficient multi-factor equity portfolios that are optimised from an asset-liability management perspective. For example, a mature pension fund facing a stream of bond-like pension obligations may find it useful to select stocks that show an above average degree of “liability-friendliness”, which can be measured, for example, in terms of their correlation or tracking error with respect to a liability proxy and/or their ability to pay a high and predictable stream of dividends. Once these stocks are selected, a dedicated efficient factor index can be designed, and used as an additional building block in allocation exercises dedicated to achieving the optimal trade-off between liability-hedging benefits and performance benefits (see Coqueret, Martellini and Milhau (2014) for a detailed analysis of these questions).
Appendix
Appendix

The resulting ERC allocation to the four selected US smart factor indices is shown in Exhibit A.1. We note that the ERC portfolio is not very different from the EW portfolio, which shows that the covariance structure of the constituents is quite homogeneous. Indeed, there is a relatively high level of correlation and a relatively low dispersion of volatility levels. However, we notice a slightly higher weight for the low-volatility constituent which is due to its lower volatility compared to the other constituents.

Also, as expected, the ERC portfolio generates an identical risk contribution of the four constituents as illustrated in the bottom graphs of Exhibit A.1.

Exhibit A.1 – EW and ERC Allocations to Smart Factor Diversified Multi-Strategy Indices and Risk Contributions (US Universe). The graph compares the allocation and risk contributions of Diversified Multi-Strategy indices: the equal combination of the four Diversified Multi-Strategy indices with stock selection based on mid cap, momentum, low volatility, and value, and the ERC combination of the same four constituents. The period goes from 31-December-1972 to 31-December-2012.
If we also look at the factor risk contributions to the EW and ERC portfolios (see Exhibit A.2) in the US universe, we notice that the factor contributions are far from being equal, and can even become negative. We note a strong domination of the low volatility factor in terms of risk factor contributions, which can be explained in particular by positive betas of all smart factor indices with respect to this factor.
Exhibit A.2 – EW and ERC Factor Risk Contributions (US Universe). The graph compares the factor risk contributions of Diversified Multi-Strategy indices: the Multi Beta Diversified Multi-Strategy with equal combination of the four Diversified Multi-Strategy indices with stock selection based on mid cap, momentum, low volatility, and value, and the ERC combination of the same four constituents. The period goes from 31-December-1972 to 31-December-2012.

The first three panels of Exhibit A.3, show the allocation of a Max-Deconcentration portfolio under risk parity constraints in a long-short framework. First, it is worth noting that since the number of constraints is equal to the number of constituents, then the allocation is fully driven by the constraints (no need for a diversification objective in this experiment, but it will be proven useful in the following ones). We notice that the short positions are quite small, and concern mostly the low-volatility constituent. Also, we confirm that the equality of the factor risk contributions is satisfied, i.e. that the risk parity constraints are not violated over the 40-year sample period.

If we now move to the last three panels of Exhibit A.3, where we impose no-short sale constraints on the allocation strategy, then we notice that there are periods where the
allocation to the low-volatility constituent is equal to 0 (1998-1999 and 2002-2003), which is not surprising since it coincides with periods of time where the long-short allocation was shorting the same constituent. However, if long-only constraints are imposed, we can no longer satisfy the factor risk parity constraints, and we notice that the contribution of the low-volatility factor is dominating the other contributions on the same periods of time where the allocation to the low-volatility factor is reduced to 0.

Given that short positions are often undesirable, one practical way to reduce the exposure of the low-volatility factor in a long-only context is to introduce a high-volatility constituent. In this case we would be dealing with a portfolio with N>K constituents, which would leave room for a diversification objective.

Exhibit A.3 – Max-Decentration Allocation under Risk Parity Constraints (US Universe). The graphs show the allocations, the risk contributions and the factor contributions of a Max-Decentration strategy under risk parity constraints in the four Diversified Multi-Strategy indices with stock selection based on mid cap, momentum, low volatility, and value. The first three panels show the case of a long-short allocation, whereas the last three panels focus on a long-only allocation. The period goes from 31-December-1972 to 31-December-2012.
Appendix
In Exhibit A.4, we consider a Max-Deconcentration allocation to five smart factors indices under risk parity constraints (in a long-only environment). We notice that the introduction of the high-volatility constituent allows us to recover the parity of the factors’ contributions. Also we see that the need for the high-volatility constituent is increased over the periods where the factor risk parity was being violated in the case of four constituents only.

Exhibit A.4 – Max-Deconcentration Allocation under Risk Parity Constraints [Lo with the HiVol Constituent]. The graphs show the allocations, the risk contributions and the factor contributions of a Max-Deconcentration strategy under risk parity constraints in the five Diversified Multi-Strategy indices with stock selection based on Mid Cap, Momentum, Low Volatility, High Volatility, and value. The period goes from 31-December-1972 to 31-December-2012.
In Exhibit A.5, we present the risk and return characteristics of the various allocation strategies to the four US smart factor indices. It is interesting to notice that the introduction of risk parity constraints has led to increases in excess returns, while maintaining a level of tracking error that is comparable to those of the EW and ERC allocations. This feature leads to a higher information ratio, which is desirable for investors that have the cap-weighted index as reference.

If we focus on the bull and bear market regimes (see Exhibit A.6), we observe that the Max-Deconcentration scheme with factor risk parity constraints leads to much higher annual returns in bull market regimes (more than 1.3% above the EW allocation), and displays slightly lower annual returns in bear market regimes (less than 1% below the EW allocation), which is consistent with the intuition that the introduction of factor risk parity...
constraints has led to a less defensive portfolio strategy. Besides, the information ratio of the strategy is much higher than those of the competing allocations (0.81 compared to 0.76 for the EW or 0.74 for the ERC).

Exhibit A.5 – Multi Beta Allocations across Smart Factor Indices (US Universe). The table compares performance and risk of Scientific Beta Diversified Multi-Strategy indices. We look at equally-weighted, ERC and Max-Deconcentration under risk parity constraints allocations in the four Diversified Multi-Strategy indices with stock selection based on mid cap, momentum, low volatility, and value respectively. All statistics are annualised and daily total returns from 31-December-1972 to 31-December-2012 are used for the analysis. CRSP S&P500 index is used as the cap-weighted benchmark. Yield on Secondary US Treasury Bills (3M) is used as a proxy for the risk-free rate.

<table>
<thead>
<tr>
<th>US Long Term (Dec 1972 - Dec 2012)</th>
<th>Diversified Multi-Strategy</th>
<th>Avg of 4 Smart Factor Indices</th>
<th>Multi Beta Equal Weight</th>
<th>Multi Beta ERC Allocation</th>
<th>Multi Beta MDec-Fact Allocation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ann. Returns</td>
<td>13.65%</td>
<td>13.72%</td>
<td>13.63%</td>
<td>14.01%</td>
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<tr>
<td>Ann. Volatility</td>
<td>15.99%</td>
<td>15.79%</td>
<td>15.67%</td>
<td>16.41%</td>
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<tr>
<td>Sharpe Ratio</td>
<td>0.51</td>
<td>0.52</td>
<td>0.52</td>
<td>0.52</td>
<td></td>
</tr>
<tr>
<td>Max Drawdown</td>
<td>53.91%</td>
<td>53.86%</td>
<td>53.62%</td>
<td>56.56%</td>
<td></td>
</tr>
<tr>
<td>Excess Returns</td>
<td>3.90%</td>
<td>3.98%</td>
<td>3.89%</td>
<td>4.27%</td>
<td></td>
</tr>
<tr>
<td>Tracking Error (TE)</td>
<td>5.92%</td>
<td>5.23%</td>
<td>5.25%</td>
<td>5.27%</td>
<td></td>
</tr>
<tr>
<td>95% TE</td>
<td>10.45%</td>
<td>8.95%</td>
<td>9.10%</td>
<td>8.69%</td>
<td></td>
</tr>
<tr>
<td>Information Ratio</td>
<td>0.67</td>
<td>0.76</td>
<td>0.74</td>
<td>0.81</td>
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<tr>
<td>Outperf. Prob (3Y)</td>
<td>78.61%</td>
<td>80.38%</td>
<td>80.43%</td>
<td>78.83%</td>
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<tr>
<td>Max Rel. Drawdown</td>
<td>33.87%</td>
<td>33.65%</td>
<td>43.46%</td>
<td>33.87%</td>
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Exhibit A.6 – Multi Beta Allocations across Smart Factor Indices in Bull/Bear Regimes (US Universe). The table compares performance and risk of Scientific Beta Diversified Multi-Strategy indices in bull and bear market regimes. We look at equally-weighted, ERC and Max-Deconcentration under risk parity constraints allocations in the four Diversified Multi-Strategy indices with stock selection based on mid cap, momentum, low volatility, and value respectively. All statistics are annualised and daily total returns from 31-December-1972 to 31-December-2012 are used for the analysis. CRSP S&P500 index is used as the cap-weighted benchmark. Yield on Secondary US Treasury Bills (3M) is used as a proxy for the risk-free rate.

<table>
<thead>
<tr>
<th>US Long Term (Dec 1972 - Dec 2012)</th>
<th>Diversified Multi-Strategy</th>
<th>Multi Beta Equal Weight</th>
<th>Multi Beta ERC Allocation</th>
<th>Multi Beta MDec-Fact Allocation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ann. Ret. Bull</td>
<td>34.83%</td>
<td>34.57%</td>
<td>36.14%</td>
<td></td>
</tr>
<tr>
<td>Ann. Rel. Ret. Bull</td>
<td>3.03%</td>
<td>2.76%</td>
<td>4.34%</td>
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<tr>
<td>Tracking Error Bull</td>
<td>4.45%</td>
<td>4.46%</td>
<td>4.54%</td>
<td></td>
</tr>
<tr>
<td>Ann. Rel. Ret. Bear</td>
<td>4.83%</td>
<td>4.96%</td>
<td>3.87%</td>
<td></td>
</tr>
<tr>
<td>Tracking Error Bear</td>
<td>6.57%</td>
<td>6.58%</td>
<td>6.53%</td>
<td></td>
</tr>
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</table>

We now turn to the relative return perspective and present in Exhibit A.7 the weights allocated to the four US smart factor indices by a relative GMV allocation strategy and by a relative ERC allocation strategy.
Appendix

Exhibit A.7 – Relative GMV and ERC Allocations to Smart Factor Indices and Risk Contributions (US Universe). The graph compares the allocation and risk contributions of Diversified Multi-Strategy indices: the relative GMV and ERC allocations invested in the four Diversified Multi-Strategy indices with stock selection based on mid cap, momentum, low volatility, and value, and the ERC combination of the same four constituents. The Relative GMV strategy has been derived with the following additional weight constraints: \(1/N < w < N/N\), where \(N = 4\) constituents and \(\delta = 2\). The period goes from 31-December-1972 to 31-December-2012.
When analysing the risk and performance indicators in Exhibit A.8, we observe lower levels of tracking error for relative approaches than absolute approaches shown in Exhibit A.5. Moreover, we see that the relative GMV, which is supposed to minimise the tracking error, indeed achieves its goal when compared to other strategies.

Furthermore, we see that the levels of information ratios of relative strategies are higher than their absolute counterparts, but remain lower than allocations implemented with risk parity constraints. Finally, when we compare the levels of tracking error obtained by the relative approaches in the bull and bear regimes with those obtained in Exhibit A.8, we see that relative approaches leads to lower tracking error levels in each regime. We also find that they lead to positive outperformance in both bull and bear markets, with stronger outperformance in the latter market conditions, as expected.
Exhibit A.8 – Relative ERC and GMV Allocation to the CW Index across Smart Factor Indices (US Universe). The table compares performance and risk of Scientific Beta Diversified Multi-Strategy indices. We look at relative ERC and relative GMV allocations in the four Diversified Multi-Strategy indices with stock selection based on mid cap, momentum, low volatility, and value respectively. All statistics are annualised and daily total returns from 31-December-1972 to 31-December-2012 are used for the analysis. CRSP S&P500 index is used as the cap-weighted benchmark. Yield on Secondary US Treasury Bills (3M) is used as a proxy for the risk-free rate.

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<tbody>
<tr>
<td></td>
<td>Multi Beta Relative ERC</td>
<td>Multi Beta Relative GMV Allocation</td>
<td>Multi Beta Relative ERC</td>
</tr>
<tr>
<td></td>
<td>15.69%</td>
<td>15.60%</td>
<td>Ann. Vol. Bull</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
<td>0.51</td>
<td>0.51</td>
<td>Ann. Rel. Ret. Bull</td>
</tr>
<tr>
<td>Max Drawdown</td>
<td>53.30%</td>
<td>52.64%</td>
<td>Tracking Error Bull</td>
</tr>
<tr>
<td>Excess Returns</td>
<td>3.79%</td>
<td>3.71%</td>
<td>Ann. Ret. Bear</td>
</tr>
<tr>
<td>Tracking Error (TE)</td>
<td>4.91%</td>
<td>4.79%</td>
<td>Ann. Vol. Bear</td>
</tr>
<tr>
<td>95% TE</td>
<td>8.11%</td>
<td>7.99%</td>
<td>Ann. Rel. Ret. Bear</td>
</tr>
<tr>
<td>Information Ratio</td>
<td>0.77</td>
<td>0.77</td>
<td>Tracking Error Bear</td>
</tr>
<tr>
<td>Outperf. Prob (3Y)</td>
<td>80.90%</td>
<td>81.31%</td>
<td>Tracking Error Bear</td>
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<tr>
<td>Max Rel. Drawdown</td>
<td>28.74%</td>
<td>27.00%</td>
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</table>
References
References

References

References

References


References
About Amundi ETF & Indexing
About Amundi ETF & Indexing

Amundi
Amundi ranks first in Europe¹ and ninth worldwide² in the asset management industry with AUM of close to €800 billion worldwide².

Located at the heart of the main investment regions in almost 30 countries, Amundi offers a comprehensive range of products covering all asset classes and major currencies.

Amundi has developed savings solutions to meet the needs of more than 100 million retail clients worldwide and designs innovative, high-performing products for institutional clients which are tailored specifically to their requirements and risk profile.

The group contributes to funding the economy by orienting savings towards company development.

Amundi has become a leading European player in asset management, recognised for:
• Product performance and transparency;
• Quality of client relationships based on a long-term advisory approach;
• Efficiency in its organisation and teams’ promise to serving its clients;
• Commitment to sustainable development and socially responsible investment policies.

Amundi ETF & Indexing
With a long-standing experience combined with a strong pricing power, we offer first-class replication on more than 100 indices to internationally renowned institutions.

The Indexing expertise is built on the search for value-added sources within strict risk framework. It comprises a wide range of open-ended funds as well as having the capacity to implement customised mandates, including SRI and smart beta approaches.

In the ETF segment, Amundi has also successfully become a major player thanks to its strategy of competitive prices, innovation and high-quality tracking. Our ETF business has been growing consistently in recent years to rank¹ among the top five European providers by assets under management.

Amundi ETF & Indexing experienced team of dedicated index fund managers is based in Europe and in Japan, with a recognized track record, and benefiting from Amundi dealing capabilities and research teams’ excellence.

1 - No.1 of the Portfolio Manager having their registered office in Europe - Amundi group’s total assets under management - Source IPE “Top 400 asset managers active in the European marketplace” published in June 2013, based on figures as at December 2012. Interviews of asset management companies on their assets as at end-December 2012 (open-end funds, dedicated funds, mandates).

2 - Amundi Group figures as at 31st March 2014 Issued by Amundi - Société anonyme with a share capital of €596 262 615 - Portfolio manager regulated by the AMF under number GP04000036 - Head office: 90 boulevard Pasteur – 75015 Paris – France – 437 574 452 RCS Paris

1 - Amundi ETF/ Bloomberg as at 31 December 2013
About EDHEC-Risk Institute
About EDHEC-Risk Institute

The Choice of Asset Allocation and Risk Management
EDHEC-Risk structures all of its research work around asset allocation and risk management. This strategic choice is applied to all of the Institute’s research programmes, whether they involve proposing new methods of strategic allocation, which integrate the alternative class; taking extreme risks into account in portfolio construction; studying the usefulness of derivatives in implementing asset-liability management approaches; or orienting the concept of dynamic “core-satellite” investment management in the framework of absolute return or target-date funds.

Academic Excellence and Industry Relevance
In an attempt to ensure that the research it carries out is truly applicable, EDHEC has implemented a dual validation system for the work of EDHEC-Risk. All research work must be part of a research programme, the relevance and goals of which have been validated from both an academic and a business viewpoint by the Institute’s advisory board. This board is made up of internationally recognised researchers, the Institute’s business partners, and representatives of major international institutional investors. Management of the research programmes respects a rigorous validation process, which guarantees the scientific quality and the operational usefulness of the programmes.

Six research programmes have been conducted by the centre to date:
• Asset allocation and alternative diversification
• Style and performance analysis
• Indices and benchmarking
• Operational risks and performance
• Asset allocation and derivative instruments
• ALM and asset management

These programmes receive the support of a large number of financial companies. The results of the research programmes are disseminated through the EDHEC-Risk locations in Singapore, which was established at the invitation of the Monetary Authority of Singapore (MAS); the City of London in the United Kingdom; Nice and Paris in France; and New York in the United States.

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• Regulation and Institutional Investment, in partnership with AXA Investment Managers
• Asset-Liability Management and Institutional Investment Management, in partnership with BNP Paribas Investment Partners
• Risk and Regulation in the European Fund Management Industry, in partnership with CACEIS
• Exploring the Commodity Futures Risk Premium: Implications for Asset Allocation and Regulation, in partnership with CME Group

Founded in 1906, EDHEC is one of the foremost international business schools. Accredited by the three main international academic organisations, EQUIS, AACSB, and Association of MBAs, EDHEC has for a number of years been pursuing a strategy of international excellence that led it to set up EDHEC-Risk Institute in 2001. This institute now boasts a team of over 95 permanent professors, engineers and support staff, as well as 48 research associates from the financial industry and affiliate professors.
About EDHEC-Risk Institute

• Asset-Liability Management in Private Wealth Management, in partnership with Coutts & Co.
• Asset-Liability Management Techniques for Sovereign Wealth Fund Management, in partnership with Deutsche Bank
• The Benefits of Volatility Derivatives in Equity Portfolio Management, in partnership with Eurex
• Structured Products and Derivative Instruments, sponsored by the French Banking Federation (FBF)
• Optimising Bond Portfolios, in partnership with the French Central Bank (BDF Gestion)
• Asset Allocation Solutions, in partnership with Lyxor Asset Management
• Infrastructure Equity Investment Management and Benchmarking, in partnership with Meridiam and Campbell Lutyens
• Investment and Governance Characteristics of Infrastructure Debt Investments, in partnership with Natixis
• Advanced Modelling for Alternative Investments, in partnership with Newedge Prime Brokerage
• Advanced Investment Solutions for Liability Hedging for Inflation Risk, in partnership with Ontario Teachers’ Pension Plan
• The Case for Inflation-Linked Corporate Bonds: Issuers’ and Investors’ Perspectives, in partnership with Rothschild & Cie
• Solvency II, in partnership with Russell Investments
• Structured Equity Investment Strategies for Long-Term Asian Investors, in partnership with Société Générale Corporate & Investment Banking

The philosophy of the Institute is to validate its work by publication in international academic journals, as well as to make it available to the sector through its position papers, published studies, and conferences.

Each year, EDHEC-Risk organises three conferences for professionals in order to present the results of its research, one in London (EDHEC-Risk Days Europe), one in Singapore (EDHEC-Risk Days Asia), and one in New York (EDHEC-Risk Days North America) attracting more than 2,500 professional delegates.

EDHEC also provides professionals with access to its website, www.edhec-risk.com, which is entirely devoted to international asset management research. The website, which has more than 65,000 regular visitors, is aimed at professionals who wish to benefit from EDHEC’s analysis and expertise in the area of applied portfolio management research. Its monthly newsletter is distributed to more than 1.5 million readers.

EDHEC-Risk Institute:
Key Figures, 2011-2012

<table>
<thead>
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<th>Nbr of permanent staff</th>
<th>90</th>
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<tr>
<td>Nbr of research associates</td>
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<td>Nbr of affiliate professors</td>
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<td>Nbr of conference delegates</td>
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<tr>
<td>Nbr of participants at EDHEC-Risk Institute Executive Education seminars</td>
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</tr>
</tbody>
</table>
About EDHEC-Risk Institute

The EDHEC-Risk Institute PhD in Finance

The EDHEC-Risk Institute PhD in Finance is designed for professionals who aspire to higher intellectual levels and aim to redefine the investment banking and asset management industries. It is offered in two tracks: a residential track for high-potential graduate students, who hold part-time positions at EDHEC, and an executive track for practitioners who keep their full-time jobs. Drawing its faculty from the world’s best universities, such as Princeton, Wharton, Oxford, Chicago and CalTech, and enjoying the support of the research centre with the greatest impact on the financial industry, the EDHEC-Risk Institute PhD in Finance creates an extraordinary platform for professional development and industry innovation.

Research for Business

The Institute’s activities have also given rise to executive education and research service offshoots. EDHEC-Risk’s executive education programmes help investment professionals to upgrade their skills with advanced risk and asset management training across traditional and alternative classes. In partnership with CFA Institute, it has developed advanced seminars based on its research which are available to CFA charterholders and have been taking place since 2008 in New York, Singapore and London.

In 2012, EDHEC-Risk Institute signed two strategic partnership agreements with the Operations Research and Financial Engineering department of Princeton University to set up a joint research programme in the area of risk and investment management, and with Yale School of Management to set up joint certified executive training courses in North America and Europe in the area of investment management.

As part of its policy of transferring know-how to the industry, EDHEC-Risk Institute has also set up ERI Scientific Beta. ERI Scientific Beta is an original initiative which aims to favour the adoption of the latest advances in smart beta design and implementation by the whole investment industry. Its academic origin provides the foundation for its strategy: offer, in the best economic conditions possible, the smart beta solutions that are most proven scientifically with full transparency in both the methods and the associated risks.

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- Deguest, R., and L. Martellini. Improved Risk Reporting with Factor-Based Diversification Measures (February).

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- Blanc-Brude, F., Cocquemas, F., Georgieva, A. Investment Solutions for East Asia’s Pension Savings - Financing lifecycle deficits today and tomorrow (May)
- Blanc-Brude, F. and O.R.H. Ismail. Who is afraid of construction risk? (March)
- Deguest, R., L. Martellini, and V. Milhau. The benefits of sovereign, municipal and corporate inflation-linked bonds in long-term investment decisions (February).

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• Almeida, C., and R. Garcia. Robust assessment of hedge fund performance through nonparametric discounting (June).
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• Martellini, L., V. Milhau, and A. Tarelli. Dynamic investment strategies for corporate pension funds in the presence of sponsor risk (March).
• Sender, S. Shifting towards hybrid pension systems: A European perspective (March).
• Blanc-Brude, F. Pension fund investment in social infrastructure (February).
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• Charbit, E., Giraud J. R., F. Goltz, and L. Tang Capturing the market, value, or momentum premium with downside Risk Control: Dynamic Allocation strategies with exchange-traded funds (July).

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• Till, H. A review of the G20 meeting on agriculture: Addressing price volatility in the food markets (July).