Equity Hedge Fund ABS Models: Choosing the Volatility Factor

David E. Kuenzi
Head of Risk Management and Quantitative Research, Glenwood Capital Investments
Research Associate with the EDHEC Risk and Asset Management Research Centre

Xu Shi
Quantitative Risk Analyst, Glenwood Capital Investments
Abstract
The use of asset-based style analysis (ABS) in the context of hedge fund investments continues to take hold within the industry. Many of the factors used in performing this analysis are straightforward and well-accepted—particularly in the area of equity hedge funds, where a long market index factor, a small-minus-large factor, and a value-minus growth factor seem to be well-accepted components of an equity hedge fund ABS model. Little attention, however, has been given to understanding the most relevant volatility factors and the relative merits of various instruments in this context. The purpose of this paper is to explore the effectiveness of a variety of volatility factors and to identify those that seem to provide the best explanatory power and the best intuitions concerning the exposures of equity-related hedge fund managers.

We would like to thank John B. Rowsell, President of Glenwood Capital Investments, LLC. for helpful comments and insights that significantly improved the paper.

EDHEC is one of the top five business schools in France owing to the high quality of its academic staff (104 permanent lecturers from France and abroad) and its privileged relationship with professionals that the school has been developing since its establishment in 1906. EDHEC Business School has decided to draw on its extensive knowledge of the professional environment and has therefore concentrated its research on themes that satisfy the needs of professionals.

EDHEC pursues an active research policy in the field of finance. Its “Risk and Asset Management Research Centre” carries out numerous research programs in the areas of asset allocation and risk management in both the traditional and alternative investment universes.
Introduction

The use of asset-based style analysis (ABS) in the context of hedge fund investments continues to take hold within the industry. Many of the factors used in performing this analysis are straightforward and well-accepted—particularly in the area of equity hedge funds, where a long market index factor, a small-minus-large factor, and a value-minus-growth factor seem to be well-accepted components of an equity hedge fund ABS model. Little attention, however, has been given to understanding the most relevant volatility factors and the relative merits of various instruments in this context. The purpose of this paper is to explore the effectiveness of a variety of volatility factors and to identify those that seem to provide the best explanatory power and the best intuitions concerning the exposures of equity-related hedge fund managers.

Asset-based style analysis (ABS) for hedge funds has been growing in popularity—both as a tool for practitioners and as a research topic for academics. The emphasis of the published research has generally been on developing models with significant explanatory power and on choosing the “right” factors. Many models have focused on the overall hedge fund industry (see Agarwal and Naik [2000a], Fung and Hsieh [2002a], Kazemi and Schneeweis [2003], Fung and Hsieh [2004], Daniel and Hubner [2005], among others). Other models have focused on particular hedge fund strategies (see Fung and Hsieh [2002b], Mitchell and Pulvino [2001], and Agarwal, Fung, and Loon [2004], among others). To date there have been few studies that focus primarily on the role of options and volatility within the ABS model. The purpose of this paper is to explore the effectiveness of a variety of volatility factors and to identify those that seem to provide the best explanatory power and the best intuitions concerning the exposures of equity-related hedge fund managers.

One of the key challenges in applying ABS to hedge funds is to capture the non-linear exposures within the hedge fund being analyzed. We emphasize that these nonlinear exposures can be driven either by dynamic trading strategies or by strategies involving the actual purchase and sale of options or other volatility-related instruments. The first article introducing ABS for hedge funds (Fung and Hsieh [1997]) focuses on the importance of nonlinear payoffs in hedge fund and CTA (commodity trading advisor) strategies. The authors note that “there exist nonlinear correlations between...style factors and some of the standard asset classes, which can give rise to optionlike payouts” (Fung and Hsieh [1997], p. 288). The follow-up to this article re-emphasizes this point:

“To fully capture hedge fund risk, we must model the nonlinear relationships between these style factors and the markets in which hedge funds trade” (Fung and Hsieh [2001], p. 314). The volatility component of an ABS model allows the analyst to capture not only the explicit purchase or sale of options or other volatility-related products, but to understand the dynamic trading strategies of managers and groups of managers. As such, from a theoretical perspective, a well-constructed ABS model should make optimal use of options or other volatility-based factors.

As the use of ABS models evolves, they will increasingly be used for the following purposes:

1. In the due diligence process. ABS output for a single manager may be examined and then used in order to have a more informed conversation with the manager.

2. Risk monitoring. Aggregating exposures across managers will allow fund of funds managers to obtain a more concrete understanding of their systematic risk exposures.

3. Hedging and hedge fund replication. ABS output can be used in order to replicate hedge fund returns and to thus hedge portfolios of hedge fund exposures.

While the accuracy of the volatility factor is critical in all three applications, the intuitions available from these factors is perhaps most important with reference to the first. As such, the focus of our article involves not only the statistical significance of the factor, but the extent to which it provides one with significant insights into manager strategies.
The directional and spread factors used in equity-related ABS models almost invariably involve at least two of the following:

1. A market directional factor (e.g., returns of the S& P 500)

2. A spread factor representing small-cap exposure minus large-cap exposure (e.g., Russell 2000 Index return minus S& P 500 return)

3. A spread factor representing value minus growth exposure (e.g., Russell 1000 Value return minus Russell 1000 Growth return)

The volatility factor, however, often varies from author to author and is chosen with only limited commentary as to why one particular factor is chosen over another. The volatility- and option-related factors that have been used include: out-of-the-money (OTM) and at-the-money (ATM) puts and calls on equity indices, changes in the VIX,4 S& P 500 ATM straddles, and convertible bonds (see Agarwal and Naik [2000b], Schneeweis, Kazemi, and Martin [2003], Hill, Mueller, and Balasubramanian [2004], and Jaeger and Wagner [2005], respectively), while other models don’t include a volatility factor at all (Fung and Hsieh [2004], for instance). We consider various volatility factors systematically as a way of determining which provide the most explanatory power and the best intuitions related to manager strategies.

The remainder of the article is organized as follows. First, we describe our data and methodology. Second, we perform ABS analysis on a constant universe of equity-oriented hedge funds using the same directional and spread factors, but with different volatility factors in order to determine the implications of using each. Finally we conclude and offer ideas for further research.

**Data and Methodology**

The hedge fund data used in this study are drawn from the HFR database and includes all Equity Hedge, Equity Market Neutral, and Equity Non-Hedge managers that have at least $300 million in assets under management and at least 65 months of returns as of December 2005. The market data used in the study includes return data for the S& P 500 for the same period, return data for a Small minus Large factor (composed of the returns of the Russell 2000 index minus those of the S& P 500), a Value minus Growth factor (composed of the returns of the Russell 1000 Value Index minus the returns of the Russell 1000 Growth Index), the VIX index, relevant Libor series, and the dividend yield of the S& P 500. The market data was obtained from Bloomberg. All options and straddles involve at-the-money options with 3-months to expiration, with values derived using the VIX index for the volatility input to the Black Scholes model. The options, straddle and other volatility instruments are assumed to be initiated at the beginning of a three-month period and are then revalued at the end of the first, second, and third months in order to obtain returns. At the end of the third month, another such position with three months to expiry/maturity is initiated.

The methodology is to perform 24-month rolling regressions of the relevant factors against an aggregation of the fund returns from the indexes listed above. From this regression we obtain coefficients, which we interpret as hedge fund exposures. For all models, three factors will remain constant: S& P 500 returns, Small minus Large, and Value minus Growth. The volatility factors will then be different for each model.

While regressions for the entire period are helpful for determining which factors, on average, add the most value to the model, they are not able to identify factors (or hedge fund strategies) that are extremely relevant during limited periods—that perhaps managers use, discard, and then return to again some months later. While Agarwal and Naik [2000b], for instance, perform some very thoughtful work on determining volatility factors using stepwise regression for the entire period, we take a somewhat different approach. As a test for significance we rely on the collection of p-values of a particular factor for each of the regressions performed over a relatively long rolling period. The p-values show the probability of obtaining our factor allocation percentages if actual

---

4 - The VIX index, calculated by the Chicago Board of Trade, is an index measuring the one-month implied volatility of S&P 500 options contracts.
allocation to this factor across our universe of managers is actually zero. This way, we are able to detect the extent to which a particular instrument may have been highly reflective of a particular hedge fund strategy during sub-periods of the total data sample examined.

Volatility Factors in an ABS Model for Equity-Related Hedge Funds

Model 1: Straddle as Volatility Factor
The first volatility factor we consider is the straddle. This is an intuitively good factor, as it initially (when first purchased or sold) has very little in the way of delta or directional exposure but contains both vega and gamma.

We perform the rolling regressions using the three standard factors identified above (S&P 500 returns, Small minus Large returns, and Value minus Growth returns) as well as a 3-month ATM straddle. The results are shown in Exhibit 1. Panel 1 shows the estimated exposures across time, while Panel 2 provides information concerning the p-values relating to the various coefficients. There are two things to note. First, the allocation to the straddle position appears to be near zero. This is driven by the leverage provided by options (a 1% position in an ATM S&P 500 straddle as of April 2006 would have represented approximately 21% in notional). Second, the p-values of the straddle are large. While the model provides for overall high explanatory power (as shown in the rolling r-squared values in Exhibit 2), the straddle seems to contribute little to this.

In order to focus purely on the volatility factor (the straddle) we present the same information in Exhibit 3 but with the linear factors removed (from the graph, but not from the regression). This enables us to see that the estimated straddle exposures range between -1% and 1%. (As we look at exposures to other volatility factors, we will not include the exposures to the directional and spread factors in the graphs, as the level and significance of these factors generally change very little across the various volatility factors.)

There are three potential issues with the straddle:
1. As soon as the underlying moves in either direction, the straddle’s delta changes substantially; it will then include directional rather than purely volatility exposures.
2. If the underlying moves substantially in either direction, the straddle will have very little in the way of vega or gamma exposures—what we think of as volatility exposures.
3. There is no flexibility in varying the number of calls or puts—it precludes the use of options for, say, covered call strategies.

The other factors we consider try to alleviate some of these issues.

Model 2: Variance Swaps as Volatility Factor
Variance swaps are one of these instruments. Variance swaps, which are often touted as pure volatility plays, are being traded with ever-increasing volume. The payoff of a variance swap is as follows:

\[
Payoff = N\left(\frac{1}{T-t_0} \sum_{i=1}^{M} \left( \frac{S_i - S_{i-1}}{S_{i-1}} \right)^2 - K_{var}\right)
\]

\[
= N\left(\frac{1}{T-t_0} \sum_{i=1}^{M} (R_i^2) - K_{var}\right)
\]

\[
= N\left(V(t_0,T) - K_{var}\right)
\]

where \(M\) is the total number of monitoring periods between swap inception at time \(t_0\) to swap maturity at time \(T\). If we assume daily monitoring, \(M\) would be the number of trading days between swap inception and maturity. \(S_i\) is the price of the underlying on day \(i\), \(R_i\) is return of the underlying on day \(i\), \(V(t_0,T)\) is the realized variance.

---

5 - When calculating p-values in the context of traditional ABS analysis, in which portfolio weights are constrained to be positive and sum to 1, we would recommend the method found in Lobosco and DiBartolomeo (1997). In the case here—an unconstrained regression—we use the standard method for calculating p-values for ordinary least- squares regression as described, for instance, in Johnston and DiNardo (1997), pp. 90-95.

6 - Delta, gamma, and vega are three of the principle option "Greeks." Delta is the change in value of the option for a $1 change in the value of the underlying, gamma is the change in delta for a $1 change in the value of the underlying, and vega is the change in the value of an option for a one percentage point change in volatility.
over the period, $K_{\text{var}}$ is the initially agreed upon variance strike expressed in volatility points squared, and $N$ is the notional amount.

The use of variance swaps solves the first two concerns noted with the use of straddles—there is no delta exposure no matter what happens with the underlying, and the vega and gamma exposure remain generally quite stable for moves in the underlying. From this perspective, it would seem to be quite an improvement over the straddle. Exhibit 4 shows the exposures and p-values. Panel 1 of Exhibit 4 is somewhat inconsistent with Panel 1 of Exhibit 3 (the straddle data), and the p-values of the variance swap (as shown in Panel 2 of Exhibit 4) show that these exposures are not providing much in the way of statistical significance.

This may be driven by one of the key problems with variance swaps—the vega is not constant across time, but moves from very large at inception to nearly zero just before maturity. This can easily be seen by considering the equation for the mark-to-market value (MTM) of a variance swap:

$$MTM = Ne^{-r(T-t)}\left[\lambda(V(t_0, t) - K_{\text{var}}) + (1 - \lambda)(K_t - K_{\text{var}})\right]$$

where $V(t_0, t)$ is the realized variance from time $t_0$ to time $t$, $K_t$ is the strike for a new variance swap running from time $t$ to time $T$ (where $T$ is swap maturity), and $\lambda$ is the proportion of the total time to expiry that has elapsed between $t_0$ and $t$. If we ignore the notional and the discounting and focus only on the two terms in the brackets, it is clear that near swap inception the second term will receive more weight (more vega exposure) and that near expiry it will receive very little weight (almost no vega exposure). So using this instrument as a factor is very difficult as the exposures it represents will change across time. This drawback may be a driver of the poor p-values.

Model 3: VIX Futures and Gamma Derivatives as Volatility Factors

The problem of non-constant vega in the variance swap contract was addressed by Kuenzi (2005). The author shows that a variance swap combined with a daily accreted amount of a contract akin to a VIX futures contract would provide for a position with stable vega across time; as a variance swap loses a little bit of vega exposure each day, this vega leakage can be stopped by purchases a small amount of VIX futures every day. The author then shows that this portfolio of a variance swap plus a daily accreting amount of VIX futures can be disaggregated into a single position in the VIX futures and a single position in a gamma derivative. The payoff of this gamma derivative is as follows:

$$\text{Gamma Derivative Payoff} = \frac{1}{T-t_0} \sum_{i=1}^{n} \left( R_i^2 - K_{\text{var}(i-1)} \right) \frac{T-t_0}{M}$$

where $R_i^2$ is the squared return of the underlying on day $i$, and $K_{\text{var}(i-1)}$ is the one-day fair variance strike as of the previous night’s close. Since the variance strike is reset each day, there is no vega exposure, but only gamma exposure. In this way, an investor can invest in vega (VIX futures, which have vega but no gamma) and gamma (gamma derivative, which has gamma but no vega) separately.

These instruments would be ideal in the context of ABS for hedge fund exposures, as one could track pure volatility exposure of managers as described by vega (exposure to changes in implied volatility) and gamma (exposure to realized volatility as compared to ex-ante implied volatility). Additionally, use of these instruments as factors would address many of the concerns listed in the previous two sections—the volatility positions have no directional exposure and exhibit generally stable vega and gamma exposures regardless of movements in the underlying or movement of time toward expiry/maturity. Exhibit 5 shows the exposures and p-values when the VIX futures and the gamma derivative are used as volatility factors.

---

7 - Gamma exposure in this context is akin to the gamma exposure inherent in a delta-hedged straddle position. Gamma and vega can be thought of as the quintessential volatility exposures.
8 - We calculate the "return" of a variance swap as the P&L per $1 notional amount. We use the VIX index as a proxy for the variance strike (as of September 22, 2003, the VIX calculation changed such that it is akin to the variance strike of a one-month variance swap). We calculate the "return" on the VIX futures and gamma derivatives similarly.
9 - Given the relatively short life of VIX futures, we use the VIX itself as a proxy for VIX futures.
The results, shown in Exhibit 5, present some interesting intuitions. It appears as though managers in this universe are largely long gamma during the period covering late 2000 through mid 2004, and are then short gamma for the remainder of the period (Panel 1), where the short gamma exposure corresponds with the recently stable and low-volatility environment. At the same time, the managers seem to be long vega across the board. In the most recent period, this combination of long vega and short gamma could be interpreted as being short short-dated options and long long-dated options. This strategy is intuitive in the recent low volatility environment (average VIX of 12.8% in 2005) in two respects. First, this may be representative of managers explicitly selling short-term options in what may be perceived as a persistently low volatility environment and in turn buying long-term options to take advantage of a perhaps eventual reversion of volatility to its long-term mean (average lifetime level of 19.5% as of December 31, 2005). Second, this may reflect choppy market trading strategies that are implicitly short volatility—such as selling rallies and then legging back into stocks as the market declines. This would perhaps lead to a short gamma exposure and little vega exposure.

While significance levels (Exhibit 5, Panel 2) represent a marginal improvement over the previous two approaches, they are still not satisfactory. The gamma derivative shows airly consistent significance at the 90% level in late 2002 and early 2003, but the VIX shows little in the way of significance. So while the possible intuitions that could be drawn from this combination of instruments is interesting, the limited statistical significance is not supportive of these factors.

Model 4: Calls and Puts as Volatility Factors

While the use of calls and puts as volatility factors present many of the same issues involved with the use of straddles (delta can be non-zero and volatility exposure can deteriorate if there are large moves in the underlying), they have the advantage of being more versatile. They can potentially pick up a wider variety of manager strategies—protective puts, covered call writing, as well as straddle exposures. It is in this context that they may prove to be desirable factors.

At first glance, it seems that using simply a call and a put for volatility factors has one significant disadvantage—put-call parity:

\[
C - P = S e^{-D(t - \tau)} - K e^{-(r - \delta) t}
\]

Put-call parity says that a long call and a short put are approximately equal to a long forward contract. In the context of the ABS model, this means that we have two instruments that are linearly related to one another (the directional factor—the S&P 500—on the one hand, and the put-call combination—possibly a forward on the S&P 500—on the other). Combinations of calls and puts may in such a case simply replace a portion of the directional exposure and add nothing to the analysis. In order to avoid this situation, we have developed a simple solution. If the call and the put exposure have opposite signs, we net them out in an amount equal to the smaller exposure by absolute value and add this netted amount to the directional factor. For example, if the model shows call exposure of 1.5%, and put exposure of -1%, we net these based on equation (4) to get a levered 1% exposure to a forward, a 0.5% exposure to a call option, and a 0% exposure to the put. The 1% exposure to the forward (after being multiplied by the leverage factor) is then added back to our directional factor. Therefore, if the model shows exposure to the S&P 500 of 20% and the leverage factor is 35, we add back a 35% forward contract exposure so that we will have 55% exposure to the S&P 500. The total exposures would now be 55% S&P 500, 0.5% call option, and 0% put option. In this way, we can be assured that the call and put exposures being identified are not related to put-call parity.

Exhibit 6 shows the model results using calls and puts. The results from Panel 1 are very intuitive. Earlier in the period (through the Iraq invasion and just after), it seems that managers generally show a long straddle exposure—long both calls and puts. As the market begins to rally through late 2003 and 2004, managers show long call exposure—perhaps in an effort to enhance gains. Then, as equity markets flattened out and implied volatilities came in and stabilized at the end of 2004 and through 2005, they show short put exposure. This is perhaps reflective of an attempt to collect option premiums in a flat and low volatility market. (Again, we emphasize our belief that a good portion of this short put exposure has involved trading strategies in which
managers attempt to buy into soft equity markets and then sell the securities when the market begins to strengthen.) The p-values in Panel 2 of Exhibit 6 support this story. Between early 2003 and late 2004, the call shows no significance, while the put is showing marginal significance. Then from late 2004 through the end of 2005, the call is shown to be highly significant, while the put shows no significance. All in all, we are able to draw sensible intuitions based on both the exposure information as well as the p-value information.

Ideal versus Practical Considerations and Model Recommendation

The above results show that choice of the volatility factors can lead to substantially different conclusions concerning manager exposures. While there seems to be some consistency between the four models, there are some notable differences—e.g., the negative volatility exposure of the straddle in 2002. Also, the difference in statistical significance between the four models suggests that, in fact, the choice of the volatility factor does matter.

Ideally, we would prefer to use Model 3. This model enables us to estimate manager exposures to two very pure components of volatility—vega and gamma. Not only is it appealing to track exposure to vega and gamma separately, but this particular model works quite well in terms of generating intuitions concerning likely manager exposures. Unfortunately, however, the VIX futures position is not showing statistical significance while the gamma derivative is showing only limited significance. Another drawback is that gamma derivatives are not yet traded, so actually replicating the returns is impossible. In our view, a significant strength of the ABS approach is that a component of hedge fund returns is actually replicable with a fairly straightforward strategy. Using an instrument that is not yet traded (despite strong intuitions) runs counter to this core strength of the methodology.

The best model is Model 4 (the call and the put). While the factors are not as "pure" as for Model 3, one can derive solid intuitions and there is greater statistical significance than with all other models. Additionally, one can easily implement a replicating portfolio using exchange-traded instruments, which make any results extraordinarily useful in the context of risk management and performance measurement (calculation of alphas). For completeness, we show all exposures and p-values for the recommended model in Exhibit 7.

Conclusion

We have explored the use of several volatility factors in the context of equity-related hedge fund exposures. We find that these factors are less clear cut than directional and spread factors, as they often involve several different types of Greek exposures that vary across time and for various values of the underlying. While the most pure and versatile instruments were VIX futures and gamma derivatives, we cannot recommend these to researchers and analysts as the statistical significance was not adequate and the gamma derivative is not yet traded. Rather, we recommend a simple model consisting of calls and puts, with the caveat that forward exposures be netted and added to the directional factor. Calls and puts generate strong intuitions concerning manager exposures and show reasonable significance as compared to the other volatility factors.

In the face of limited transparency from hedge funds, and in some cases limited alpha, asset-based style analysis is a powerful tool for hedge fund investors and is likely to continue to evolve. Its applications in terms of hedging, synthetic replication, and performance measurement (identification of true alpha) are in their nascent stages. Research involving volatility exposures is likely to evolve in tandem with innovations in the derivatives market itself, with new instruments allowing new types of exposure analysis. Exposure to CDX swaptions among credit managers, for instance, may prove an interesting direction for future research.

11 - CDX swaptions are options on the credit spread of an index of corporate bond (credit default swap) exposures.
REFERENCES


• Kuenzi, David E. “Variance Swaps and Non-Constant Vega.” Risk, October 2005, pp. 79-84.


EXHIBIT 1
Exposures of US Equity Long/Short Funds and Related P-Values (Using a Straddle as the Volatility Factor)
June 2002 to December 2005

Panel 1. Exposures:

Panel 2. P-Values:
EXHIBIT 2
R-Squared of US Equity Long/Short ABS Model (Using Straddle as the Volatility Factor)
June 2002 to December 2005
EXHIBIT 3
Straddle Exposures of US Equity Long/Short Funds and Related P-Values
June 2002 to December 2005

Panel 1. Exposures:

Panel 2. P-Values:
EXHIBIT 4
Variance Swap Exposures of Equity Long/Short Funds and Related P-Values
June 2002 to December 2005

Panel 1. Exposures:

Panel 2. P-Values:
EXHIBIT 5
Vix Futures & Gamma Derivatives Exposures of US Equity Long/Short Funds and Related P-Values
June 2002 to December 2005

Panel 1. Exposures:

Panel 2. P-Values:
EXHIBIT 6
Call & Put Exposures of US Equity Long/Short Funds (Calls & Puts Netted) and Related P-Values
June 2002 to December 2005

Panel 1. Exposures:

Panel 2. P-Values:
EXHIBIT 7
Exposures of US Equity Long/Short Funds (Calls & Puts Netted) and Related P-Values (Using Calls & Puts as the Volatility Factors)
June 2002 to December 2005

Panel 1. Exposures:

Panel 2. P-Values: