<table>
<thead>
<tr>
<th>Table of Contents</th>
</tr>
</thead>
<tbody>
<tr>
<td>Executive Summary ........................................................................................................... 5</td>
</tr>
<tr>
<td>1. Introduction .............................................................................................................. 9</td>
</tr>
<tr>
<td>3. Dynamic Asset Allocation Decisions for Sovereign Wealth Funds .................................. 17</td>
</tr>
<tr>
<td>4. Empirical Analysis ..................................................................................................... 23</td>
</tr>
<tr>
<td>5. Conclusion .................................................................................................................... 31</td>
</tr>
<tr>
<td>Appendices ....................................................................................................................... 35</td>
</tr>
<tr>
<td>References ....................................................................................................................... 45</td>
</tr>
<tr>
<td>About EDHEC-Risk Institute ............................................................................................ 49</td>
</tr>
<tr>
<td>About Deutsche Bank ....................................................................................................... 61</td>
</tr>
</tbody>
</table>

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The opinions expressed in this study are those of the authors and do not necessarily reflect those of EDHEC Business School.
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This publication contains the results of the first-year research work conducted at EDHEC-Risk Institute within the EDHEC-Deutsche Bank research chair on asset-liability management (ALM) techniques for sovereign wealth fund management. Under the responsibility of Lionel Martellini, the scientific director of EDHEC-Risk Institute, this chair examines optimal allocation policies for sovereign wealth funds (SWFs). SWFs have become a dominant force on international financial markets, and a better understanding of optimal investment policy and risk management practices for such long-term investors is needed.

The present publication proposes a formal analysis of these questions, which can be regarded as the extension to sovereign wealth funds of the liability-driven investing (LDI) paradigm recently developed in the pension fund industry. This work highlights in particular the need for a hedge against the risk emanating from fluctuating revenues to the fund. In an empirical application, the authors investigate the composition of this revenue-hedging portfolio for an oil-based sovereign fund. This analysis has important potential implications in terms of the emergence of genuinely dedicated ALM and risk management solutions for sovereign wealth funds.

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Executive Summary
It is now widely recognised that sovereign wealth funds (SWFs) are a dominant force on international financial markets. By some estimates, the total size of sovereign wealth funds currently stands at more than $3 trillion, more than twice the estimated size of the world’s hedge fund industry (around $1.5 trillion of assets under management) but only a seventh of the global investment-fund industry (around $21 trillion of assets under management). While total assets managed by sovereign wealth funds have fallen substantially after 2008 because of collapsing asset values during the financial crisis, with cash withdrawals from sovereign funds also expected to increase due to the need to inject liquidities into weakening economies, the growth of sovereign wealth funds is in fact likely to continue, and is expected to reach around $5 trillion in the next five years and $10 trillion within the next decade.\(^1\) The rapid growth of sovereign wealth funds and its implications pose a series of challenges for the international financial markets and for sovereign states. In particular, an outstanding challenge is to improve our understanding of optimal investment policy and risk management practices for sovereign wealth funds.

This paper proposes a quantitative dynamic asset allocation framework for sovereign wealth funds, modelled as large long-term investors that manage fluctuating revenues typically emanating from budget or trade surpluses in the presence of stochastic investment opportunity sets. The optimal asset allocation strategy takes into account the stochastic features of the sovereign fund endowment process (where is the money coming from), the stochastic features of the sovereign fund’s expected liability value (what the money is going to be used for), and the stochastic features of the assets held in its portfolio. Our results suggest that the investment strategy for an SWF should involve a state-dependent allocation to three building blocks, a performance-seeking portfolio (PSP, typically heavily invested in equities), an endowment-hedging portfolio (EHP, customised to meet the risk exposure in the sovereign wealth fund endowment streams), and a liability-hedging portfolio (LHP, heavily invested in bonds for interest rate hedging motives, and in assets exhibiting attractive inflation-hedging properties, when the implicit or explicit liabilities of the sovereign wealth funds exhibit inflation indexation), as well as separate hedging demands for risk factors impacting the investment opportunity set, most notably interest rate risk and equity expected return risk.

While the first PSP building block is the standard highest risk-reward component in any investor’s portfolio, the EHP and LHP building blocks must be customised to meet the tailored needs of each specific sovereign wealth fund. In an application to oil-based sovereign funds with inflation-linked benchmarks, we conduct an empirical analysis of the oil- and inflation-hedging properties of several traditional and alternative asset classes that can be used as ingredients within this building block using a restricted vector autoregressive (VAR) model. Overall, it appears that the development of an asset-liability management analysis of sovereign wealth funds has potential important implications in terms of the emergence of new forms of financial engineering techniques for the design of customised building blocks aiming at facilitating the

implementation of genuinely dedicated ALM and risk management solutions for these long-term investors. The PSP/EHP/LHP approach can in fact be seen as the extension to sovereign wealth funds of the liability-driven investing (LDI) paradigm recently developed in the pension fund industry.

In terms of implementation, a number of challenges remain, including the need to reconcile the top-down asset allocation decisions with bottom-up security selection decisions. Indeed, the asset allocation decisions analysed in this paper relate to the design of the long-term strategic allocation for SWFs, with an associated optimal exposure to rewarded risk factors. Additionally, it is legitimate for SWFs to seek alpha opportunities, and/or consider reaching strategic stakes in selected target companies. In fact, long-term equity holdings can be a natural source of alpha generation for sovereign wealth funds given that SWFs are better placed to benefit from any temporary mispricing opportunity than hedge funds thanks to a longer-term investment horizon, and also better placed than pension funds thanks to higher margin for error and the absence of regulatory constraints. Eventually, however, such security selection decisions can lead to a strong bias, such as a recent overweighted exposure to financials. These unintended bets on market, sector, and style returns are unfortunate as they can have a very significant, positive or negative, impact on the portfolio return. As a result, these biases need to be quantitatively measured, and optimised. Alternatively, these biases can be adjusted for in a multi-factor setting through a completeness portfolio designed to fill in differences between portfolio allocation and the long-term strategic benchmark allocation. In this context, index futures can be used as cost-efficient vehicles for dynamic adjustment of portfolio exposure to market risk.

Our work can be extended in several directions. On the one hand, more work is needed for a better understanding of the composition of the endowment hedging building block. For example, in the case of a sovereign wealth fund managing commercial surpluses, the endowment stream is related to worldwide economic growth, the fluctuations of which are not replicable by a traded asset. On the other hand, it is also critically important to account for the presence of short-term risk constraints which are faced by SWFs despite their long investment horizon. While many sovereign funds were built on the idea that they could hold on to investments over long time horizons, some have had to draw back from investments abroad, even if they are marked at a loss at the time, in order to finance investments in their home country. These elements suggest that beyond the elements described above SWFs would also benefit from implementing dynamic risk-controlled allocation strategies that are designed to help long-term investors meet a number of short-term goals and constraints. Finally, the approach proposed in this paper needs to be extended towards the inclusion of other sovereign assets as well as sovereign liabilities. In particular, the size of local- and foreign-currency-denominated debt, as well as contingent liabilities towards pension systems or industries, relative to foreign reserves and sovereign assets will, for example, determine sovereign leverage and is expected to have
a material impact on optimal sovereign asset management. We leave these, and other interesting questions for further research.
1. Introduction
1. Introduction

It is now widely recognised that sovereign funds are a dominant force on international financial markets. By some estimates, the total size of sovereign wealth funds currently stands at more than $3 trillion, more than twice the estimated size of the world’s hedge fund industry (around $1.5 trillion of assets under management) but only a seventh of the global investment-fund industry (around $21 trillion of assets under management). Although the total assets managed by sovereign wealth funds have fallen substantially since 2008 because of collapsing asset values during the financial crisis, with cash withdrawals from sovereign funds also expected to increase due to the need to inject liquidities into weakening economies, the growth of sovereign wealth funds is in fact likely to continue, and is expected to reach around $5 trillion in the next five years and $10 trillion within the next decade.² Broadly speaking, there are three main kinds of sovereign wealth funds. The first group contains the natural resources funds, with an estimated 70% of total sovereign wealth fund asset holdings in the hands of resource-rich countries such as the United Arab Emirates and Norway. The focus of these funds is to maintain economic stability against commodity price fluctuations and ensure that future generations are not disadvantaged by the exploitation of natural resources by the current generation. The second group relates to the foreign reserve funds and includes a number of Asian countries such as China, Korea, and Singapore. The focus of these funds should be to hedge away the impact of risk factors behind these commercial surpluses, and to generate higher returns than local sterilisation bond costs related to the issuance of sovereign debt aiming at reducing the monetary-base expansion related to capital inflows. The last group of funds, which accounts for a more marginal fraction of total sovereign wealth, contains the pension reserve funds for countries such as New Zealand, France, or Ireland, which have set aside a portion of their pension funds and manage them separately to prepare for an ageing society.

This rapid growth of sovereign wealth funds and its implications pose a series of challenges for the international financial markets and for sovereign states. In particular, an outstanding challenge is to improve our understanding of optimal investment policy and risk management practices for sovereign wealth funds. The purpose of this paper is to focus on improving our understanding of optimal investment policy risk management practices for SWFs. More specifically, we aim to analyse the optimal investment policy of an SWF in a dynamic ALM framework that will allow us to formalise the impact on the optimal allocation policy induced by the presence of risk factors affecting the dynamics of the state’s surplus and/or the state’s investment and consumption objectives. Recent advances in dynamic asset pricing theory have in fact paved the way for a better understanding of optimal dynamic asset allocation decisions for such long-term investors in the presence of stochastic opportunity sets, starting with the seminal contributions of Merton (1969, 1971), and subsequently extended by a variety of authors, including Brennan et al. (1997), Brennan and Xia (2002), Campbell et al. (2003), Campbell and Viceira (1999), Kim and Omberg (1996), Liu (2007), Lynch (2001), or Wachter (2002), among many others. Recently, several authors have analysed the impact of endowment risk (Cairns et al. 2006;
1. Introduction

Munk and Sørensen (2005) or liability risk (Martellini and Milhau 2008) on optimal long-term allocation decisions.

Our ambition is to contribute to the literature by introducing a formal dynamic asset allocation model that will incorporate the most salient factors in SWF management, and in particular take into account the stochastic features of the sovereign fund endowment process (where the money is coming from), the stochastic features of the sovereign fund’s expected consumption streams, e.g., in the form of an inflation-linked investment benchmark (what the money is going to be used for), and the stochastic features of the assets held in its portfolio.

We find that the optimal asset allocation strategy for a sovereign state fund involves a state-dependent allocation to various building blocks, including (i) a performance-seeking portfolio (PSP), typically heavily invested in equities, (ii) an endowment-hedging portfolio (EHP), customised to meet the risk exposure in the sovereign wealth fund endowment streams, (iii) an inflation-hedging portfolio (IHP), heavily invested in assets exhibiting attractive inflation-hedging properties, when the implicit or explicit liabilities of the sovereign wealth funds exhibit inflation indexation, as well as (iv) separate hedging demands for risk factors impacting the investment opportunity set, and most notably interest rate risk and equity expected return (or Sharpe ratio) risk. While the first PSP building block is the standard highest risk-reward component in any investor’s portfolio, the EHP building block needs to be customised to meet the tailored needs of each sovereign wealth fund. For example, in the case of the Norwegian sovereign fund, which is a natural resource fund that has been set up to help meet future pension payments, the optimal allocation strategy should involve a short position in oil/gas commodity futures, or a long position in stocks of companies such as airlines that benefit from decreases in oil prices, so as to diversify away some of the risk exposure in the country’s revenues. Additionally, the allocation policy should include a long position in inflation-linked securities, which will help the sovereign state to hedge away some of the inflation uncertainty in future pension payments.

Our paper is closely related to a series of recent work on asset allocation decisions for SWFs by Gintschel and Scherer (2008) and Scherer (2009b, 2009c). These papers solve for the optimal allocation policy of an oil-based sovereign wealth fund in a static setting, and find the oil price hedging demand to be an important component of the optimal allocation decision. Taking the problem to a dynamic setting is critically needed given that static portfolio analysis can hardly be justified for long-term investors, and does not allow one to analyse intertemporal hedging demands in the presence of a stochastic opportunity set. A first step in that direction has been taken by Scherer (2009a), who considers a discrete-time model similar to that of Campbell et al. (2003), and finds that a hedging demand against shocks to the short-term risk-free rate is optimally required, in addition to the oil price hedging demand. We further extend these results by proposing a comprehensive continuous-time dynamic asset allocation model, which also encompasses a mean-reverting equity.

3 - Also related is a paper by Gray et al. (2007), who propose to use modern contingent claims analysis to measure, analyse, and manage sovereign risk, and discuss the implications for sovereign wealth management, with hedging against the impact of unexpected changes in risk factors impacting the revenues of the sovereign state as a key ingredient in the optimal allocation and risk management policy.
1. Introduction

premium, an important ingredient in any long-term investment problem.

The rest of the paper is organised as follows. In section 2, we introduce a formal continuous-time model of sovereign wealth fund management. In section 3, we present the solution in a simplified context with complete markets. In section 4, we present an empirical analysis of oil price hedging as well as inflation-hedging properties of various asset classes that can be used in endowment-hedging and inflation-hedging portfolios. Section 5 concludes. Mathematical details are relegated to a dedicated appendix.

We introduce in this section a formal model of asset–liability management, and discuss its application within a sovereign wealth management context. This stochastic control approach to ALM is appealing in spite of its highly stylised nature because it leads to a tractable solution, allowing one to fully and explicitly understand the various mechanisms affecting the optimal allocation strategy.

2.1 State Variables and Asset Returns

We consider an investor with a finite horizon $T$, which is typically a long horizon for an SWF. Uncertainty in the economy is represented through a standard probability space $(\Omega, \mathcal{F}, \mathbb{P})$ endowed with a filtration $\mathcal{F}_t$ such that $\mathcal{F}_T = \mathcal{F}$. All processes relevant to decision making are assumed to be progressively measurable with respect to this filtration.

The sources of risk in our model are: equity return uncertainty $z_S$, equity risk premium (Sharpe ratio) uncertainty $z_\lambda$, nominal interest rate uncertainty $z_R$, inflation uncertainty $z_\Phi$, and income rate uncertainty $z_e$. The processes $z_S$, $z_\lambda$, $z_R$, $z_\Phi$, and $z_e$ are standard Wiener processes such that the vector $(z_S, z_\lambda, z_R, z_\Phi, z_e)$ is Gaussian.

4 - In the presence of interest rate risk, the cash is risky over a non-infinitesimal period because wealth is reinvested at uncertain rates. In this context, the risk-free asset for an investor with finite horizon $T$ whose preferences are expressed over nominal wealth is precisely the nominal zero-coupon bond (see Wachter 2003 for a general proof of this result). But, of course, the zero-coupon bond is risky over any sub-holding period because its value fluctuates with the short-term rate.

5 - In appendix E, we briefly survey the case where the asset mix also includes the indexed bond.

The set of traded assets includes at least two risky securities, the stock index and one zero-coupon bond of maturity $T$, as well as an instantaneous risk-free cash asset paying the continuously compounded nominal risk-free rate $R_t$. Trading in the two risky assets allows one to span the sources of risk $z_S$ and $z_R$. If an inflation-indexed zero-coupon bond (with maturity $T$) were also traded, the risk $z_\Phi$ would be spanned as well. Given the lack of capacity of inflation-linked bond markets, one may alternatively envision investing in other risky securities with attractive inflation-hedging properties to try to hedge out inflation uncertainty (see section 4 for an empirical analysis of inflation-hedging properties of asset classes including stocks and bonds, commodities or real estate). Finally, we distinguish in what follows two cases, depending on whether the income risk $z_e$ is traded or not. In general, because of the presence of non-tradable risk factors, the market is potentially dynamically incomplete, in the sense of Duffie (2001).

We now present our assumptions regarding the dynamics of the processes involved in this paper. We assume that the stock index price evolves as:

$$\frac{dS_t}{S_t} = (R_t + \sigma_S \lambda_t^S) \, dt + \sigma_S \, dz_t^S \quad (2.1)$$

and that the other state variables follow stochastic processes:

$$d\lambda_t^S = \kappa(\bar{\lambda} - \lambda_t^S) \, dt + \sigma_\lambda \, dz_t^\lambda \quad (2.2)$$

$$dR_t = a(b - R_t) \, dt + \sigma_R \, dz_t^R \quad (2.3)$$

\[
\frac{d\Phi_t}{\Phi_t} = \pi \, dt + \sigma_\Phi \, dz^\Phi_t \tag{2.4}
\]

where \( R \) is the nominal short-term interest rate, \( \pi \) the assumed constant expected inflation rate, \( \Phi \) the price index, and \( \lambda \) the Sharpe ratio of the stock index. Following Wachter (2002) and Munk et al. (2004), we will assume throughout this paper that there is perfect anti-correlation between Sharpe ratio risk and stock risk. As a consequence, we have

\[
dz^\lambda_t = -dz^S_t.
\]

The SWF also receives an endowment flow \( e \equiv (e_t)_{t \in [0,T]} \), which is, for example, due to exploitation of oil or gas resources in the case of a natural resource SWF. \( e \) follows a positive stochastic process that we assume to be described by:

\[
de_t = e_t [\mu_e \, dt + \sigma_e \, dz^e_t] \tag{2.5}
\]

where \( \mu_e \) and \( \sigma_e \) are assumed to be constant for simplicity.

One can introduce a more synthetic notation for these processes, using a five-dimensional Brownian motion \( z \) that summarises all relevant risks in the economy:

\[
\frac{dS^S_t}{S^S_t} = (R_t + \sigma^S \lambda^S_t) \, dt + \sigma^S_t \, dz_t \tag{2.6}
\]

\[
\frac{d\lambda^S_t}{\lambda^S_t} = \kappa (\lambda - \lambda^S_t) \, dt + \sigma^\lambda_t \, dz_t \tag{2.7}
\]

\[
\frac{dR_t}{R_t} = a (b - R_t) \, dt + \sigma^R_t \, dz_t \tag{2.8}
\]

\[
\frac{d\Phi_t}{\Phi_t} = \pi \, dt + \sigma^\Phi_t \, dz_t \tag{2.9}
\]

\[
de_t = e_t [\mu_e \, dt + \sigma^e_t \, dz_t] \tag{2.10}
\]

Given that Sharpe ratio risk is spanned by the stock index, the volatility vector in (2.7) is \( \sigma^\lambda = -\frac{\sigma^S}{\sigma^S} \sigma^S \). We let \( B(t, T) \) be the price at time \( t \) of the nominal zero-coupon bond with maturity date \( T \) and \( I(t, T) \) be the price of the indexed zero-coupon bond with the same maturity. Closed-form expressions can be derived for bond prices, as shown in appendix A. From these expressions, we get that the volatility vector of the nominal bond with maturity date \( T \) is:

\[
\sigma_B(t, T) = b_a(T - t) \sigma_R
\]

where \( b_a(s) = -\frac{1 - e^{-as}}{a} \). Similarly, the volatility vector of \( I \) is given by:

\[
\sigma_I(t, T) = \sigma_\Phi + b_a(T - t) \sigma_R
\]

The volatility matrix of traded assets, \( \sigma_\nu \), is obtained by concatenating the volatility vectors of these assets (so far the nominal bond and the stock). Hence this is a \( 5 \times 2 \) matrix.

We define \( \lambda_t \) as the unique market price of the risk vector spanned by the traded assets. Since the market is incomplete, there are infinitely many prices of risk, although \( \lambda_t \) is the only to be spanned by the volatility matrix of traded assets. As shown by He and Pearson (1991) (henceforth HP91), the market prices of risk are those vector processes of the form \( \lambda_t + v \), where \( v \) is such that \( \sigma^\nu_t = 0 \) holds for all \( t \). Each \( v \) defines a pricing kernel, \( M^{\lambda + v} \), through:

\[
d[M_t^{\lambda + v}] = -M_t^{\lambda + v} [R_t \, dt + (\lambda_t + v_t) \, dz_t]
\]

By definition, the price of any traded asset multiplied by any of the pricing kernels follows a martingale under \( \mathbb{P} \).

Each pricing kernel defines in turn a risk-adjusted probability measure under which the discounted price of any traded asset is a martingale. Formally, \( Q^{\lambda + \nu} \) is defined by its density with respect to \( \mathbb{P} \):

\[
\frac{dQ^{\lambda + \nu}}{d\mathbb{P}} = \exp \left[ - \int_t^T \left( R_s + \frac{\langle \lambda_s + \nu_s \rangle^2}{2} \right) ds - \int_t^T (\lambda_s + \nu_s)' dz_s \right]
\]

2.2 Financial Wealth of the Investor and Preferences

Let the vector \( \theta_t \) denote the allocation (in monetary value, as opposed to percentage terms) to the traded assets, with the following convention: the first component is the wealth allocated to the nominal bond with maturity \( T \), the second component is the amount invested in the stock index. When the inflation-linked bond is traded, \( \theta_t \) has a third component, the investment in this bond. The strategy can equivalently be described by the vector of weights \( \omega_t = \frac{\theta_t}{A_t} \), where \( A_t \) is the financial wealth at date \( t \). Given the presence of a non-financial income process \( e \), SWF wealth \( A \) evolves as:

\[
dA_t = (A_t R_t + \theta_t' \sigma_t' \lambda_t) dt + \theta_t' \sigma_t' dz_t + e_t dt
\]  

(2.11)

Regarding the objective of the investor, we need to account for the possible presence of liabilities beyond date \( T \). Most SWFs, however, do not have explicit liabilities in that they are not managed in view of financing a well-defined schedule of expenses. On the other hand, it is typically at least implicitly understood that the savings should serve to protect the purchasing power of future generations.

As a result, we argue that the preferences of the SWF should be expressed over real, as opposed to nominal, wealth. This can be mathematically stated as:

\[
\max_{\theta_t} \mathbb{E} \left[ u \left( \frac{A_T}{\Phi_T} \right) \right]
\]

(2.12)

for some utility function \( u \), subject to the budget constraint (2.11). Throughout this paper, we will take \( u \) to be the constant relative risk aversion (CRRA) function, that is \( u(x) = \frac{x^{1-\gamma}}{1-\gamma} \). The objective (2.12) has been extensively studied in the literature; see Brennan and Xia (2002), Munk et al. (2004), or Sangvinatsos and Wachter (2005). When utility is from real wealth, the risk-free asset over the investment period is no longer a nominal zero-coupon bond, because the real payoff of this bond is uncertain, but instead the inflation-indexed zero-coupon bond paying \( \Phi_T \) at time \( T \). If such a bond does not exist, then there is no risk-free asset, which we assume to be the case in this paper (see appendix E for a review of how results are modified when a real risk-free asset is introduced). We solve the optimal allocation decision problem for the SWF (programme (2.12)) by using the martingale approach in incomplete markets, as developed by HP91.
3. Dynamic Asset Allocation Decisions for Sovereign Wealth Funds
In what follows, we present a dynamic asset allocation strategy, with a fund separation theorem, typical of optimal asset allocation decisions in the presence of stochastic opportunity sets, which appears as a parsimonious way to capture some of the complexity involved in optimal sovereign wealth investment decisions. We first solve the problem in a complete market setting, and then turn to the more complex incomplete market case.

3.1 The Complete Market Case

In this section, we consider the case where income risk is spanned by traded securities, i.e., the vector $\sigma_e$ in (2.10) is spanned by the volatility matrix of traded assets $\sigma_t$. In this case, the income stream $e$ can be valued as the dividend process of a traded security, whose price would be:

$$H_t = \mathcal{H}(t, R_t, \lambda^S_t, e_t)$$

One can obtain from Ito’s lemma the volatility vector of $H$:

$$H_t \sigma_{H,t} = \mathcal{H}_R \sigma_R + \mathcal{H}_\lambda \sigma_\lambda + \mathcal{H}_e \sigma_e$$

Since we have assumed that the three risk factors that affect $H$ (interest rate, equity premium, and non-financial income) are spanned by traded assets (stocks, nominal bonds and possibly indexed bonds), the volatility vector $\sigma_{H,t}$ is spanned by their volatility matrix $\sigma_t$.

Hence there is a portfolio process $\theta_t$ such that $H_t \sigma_{H,t} = \sigma_t \theta_{H,t}$ for all $t$. In appendix B, we derive the dynamics of $H$:

$$\text{d}H_t = [R_t H_t + \theta'_t \lambda_{H,t} \sigma'_t \lambda_t] \text{d}t + \theta'_t \sigma'_t \text{d}z_t - e_t \text{d}t$$

which implies that $A + H$ evolves as:

$$\text{d}(A_t + H_t) = [R_t (A_t + H_t) + (\theta_t + \theta_{H,t})' \sigma'_t \lambda_t] \text{d}t + (\theta_t + \theta_{H,t})' \sigma'_t \text{d}z_t$$

This process describes the dynamics of a self-financing strategy that invests $\theta_t + \theta_{H,t}$ in the stock index and the bond at time $t$. In fact, one important implication is that the SWF behaves as if it had access to the total wealth $A_t + H_t$, as is shown in the following proposition.
3. Dynamic Asset Allocation Decisions for Sovereign Wealth Funds

Proposition 1  The optimal payoff in (2.12) is given by:

\[ A_t^* = \frac{A_0 + H_0}{E \left( (M^{\lambda + \nu} \Phi_t)^{1/2} \right)} \left( M^{\lambda + \nu} \right)^{-1/2} \Phi_t^{-1/2} \]

where \( M^{\lambda + \nu} \) is the minimax pricing kernel.

The optimal wealth process reads:

\[ A_t^* = \frac{A_0 + H_0}{E \left( (M^{\lambda + \nu} \Phi_t)^{1/2} \right)} \Phi_t^{1/2} \left( M^{\lambda + \nu} \right)^{-1/2} g(t, \xi_t, \lambda_t^S) - H_t \]

and the optimal portfolio strategy reads:

\[ \theta_t^* = \left( A_t^* + H_t \right) \omega_t^0 - \theta_{H,t} \] \hspace{1cm} (3.4)

where \( \omega_t^0 \) is the optimal vector of weights in the absence of non-financial income, that is:

\[ \omega_t^0 = \frac{1}{\gamma} \left( \sigma_t \right)^{-1} \sigma_t \lambda_t - \frac{1}{3} \left( A_0 - A_0 \right) \left( \sigma_t \right)^{-1} \sigma_t \lambda_t
- \left( 1 - \frac{1}{\gamma} \right) \left[ A_1(T - t) + A_2(T - t) \lambda_t^S + \frac{1}{2} A_4(T - t) \lambda_t^S \right] \left( \lambda_t^S \right)^2 \] \hspace{1cm} (3.5)

The function \( g \) is given by:

\[ g(t, R_t, \lambda_t^S) = \exp \left[ \frac{1}{\gamma} \left( A_1(T - t) + A_2(T - t) R_t + A_3(T - t) \lambda_t^S + \frac{1}{2} A_4(T - t) \lambda_t^S \right)^2 \right] \] \hspace{1cm} (3.6)

and the functions \( A_2 \), \( A_3 \), and \( A_4 \) are solutions to ODE written in appendix C. The indirect utility function reads:

\[ J(t, A_t, R_t, \lambda_t^S, \Phi_t, H_t) \]

\[ = \frac{1}{1 - \gamma} \left( \frac{A_t + H_t}{\Phi_t} \right)^{1-\gamma} g(t, R_t, \lambda_t^S) \gamma \] \hspace{1cm} (3.7)

Proof. See appendix C.

The optimal allocation (3.4) involves the familiar speculative demand for the risky assets, \( A_t^* + H_t \left( \sigma_t \right)^{-1} \sigma_t \lambda_t \). This term is the solution to the standard Merton’s problem (Merton 1969), and involves a focus on Sharpe ratio maximisation within the PSP building block. The other terms in the allocation are hedging demands of different nature. First, the terms in \( \left( \sigma_t \right)^{-1} \sigma_t \) and \( \left( \sigma_t \right)^{-1} \sigma_t \lambda_t \) are hedging demands in the sense of Merton (1971) that arise because the opportunity set is stochastic, that is, because the rate of return on the cash and the equity premium evolve randomly over time. As shown by Merton (1971), investors modify their demands for the risky assets to account for the presence of such uncertainty. Note that the portfolio hedging unexpected changes in the short-term rate is fully invested in the nominal zero-coupon bond, while the portfolio that best hedges against fluctuations in the stock index expected return is fully invested in the stock index itself. This is because of the assumed perfect anti-correlation between changes in the stock price and changes in the expected return (Wachter 2002; Munk et al. 2004) for a justification of this assumption). Hence the hedging demand can be rewritten as:

\[ - \left( 1 - \frac{1}{\gamma} \right) \left[ A_1(T - t) + A_2(T - t) \lambda_t^S \right] \left( \sigma_t \right)^{-1} \sigma_t \lambda_t \]

\[ = \left( 1 - \frac{1}{\gamma} \right) \left[ A_1(T - t) + A_2(T - t) \lambda_t^S \right] \left( 0 \right) \] \hspace{1cm} (3.8)
3. Dynamic Asset Allocation Decisions for Sovereign Wealth Funds

is stochastic. Since the SWF is averse to uncertainty over terminal real wealth, it is optimal to hedge at least partially against inflation risk by investing in a portfolio that best replicates the fluctuations of the price index. If an inflation-indexed zerocoupon bond is available, the inflation-hedging portfolio will be fully invested in this bond. Finally, the term in $\theta_{H,t}$ is the hedging demand against endowment risk, which is present because the investor perceives a non-financial random income over the investment period.

Hedging demands against interest rate, Sharpe ratio, and inflation risks depend on the relative risk aversion $\gamma$. They vanish in the case where $\gamma = 1$, that is, when the CRRA utility function boils down to the logarithmic function. Moreover, the hedging demand against $\lambda_S$ is state-dependent since it depends on the current level of expected return. One can show that when the investor is more risk-averse than the logarithmic investor (i.e., when $\gamma > 1$), $A_t$ is a positive increasing function of time horizon. This can be interpreted in view of (3.8) in a very intuitive way: the higher the expected excess return, the higher the hedging demand for the stock, and this effect is more pronounced for long-term investors. Only the hedging demand against endowment risk, $-\theta_{H,t}$, is preference-free. We emphasise that this hedging demand is negative, which means that the investor holds a short position in the portfolio that replicates the value of the capitalised income. The intuition for this result is as follows. A sovereign wealth fund from an oil-rich country should hold a short position in oil prices to hedge away implicit long positions in endowment flows. In the absence of the short position in the endowment hedging portfolio, investing in the standard PSP plus holding the long position in oil implicit in surplus streams would lead to overinvestment in oil, which needs to be compensated for. In the context of a fund with funding from exports of goods and services other than natural resources (for which investable proxies are more readily available), the question of the design of the endowment hedging portfolio is a complex challenge that requires a quantitative econometric model of the key risk factors impacting the fund revenues.

Finally, it is worth noting that the indirect utility, that is, the welfare achieved by an investor following the optimal strategy, can be written as:

$$J(t, A_t, R_t, \lambda_t^S, \Phi_t, H_t) = J(t, A_t + H_t, R_t, \lambda_t^S, \Phi_t, 0)$$

(3.9)

On the right hand side of this equation, we find the highest utility level for a SWF with the same characteristics (horizon and risk aversion) but no exogenous endowment streams. This shows in particular that at any date $t$ the SWF manager has no preference for either of the following two possibilities: i) start from initial wealth $A_t$, receive the income stream $e$ and implement the strategy $\theta$; ii) start from the larger initial wealth $A_t + H_t$, receive no income and implement the strategy $\theta$. Hence, $H_t$ can be interpreted as the indifference price at time $t$ of the income stream $(e_s)_{s \geq t}$, in the sense of Hodges and Neuberger (1989).
3. Dynamic Asset Allocation Decisions for Sovereign Wealth Funds

3.2 The Incomplete Market Case
In the previous section we have assumed that the endowment process risk could be replicated by a suitably designed portfolio of traded assets. As a consequence, the SWF manager optimally acts as if the SWF wealth were given by the sum of the SWF’s current accumulated wealth $A_t$ and the present value of future endowments $H_t$. In reality, the income process for SWFs emanates from primary surpluses, foreign trade surpluses, or the exploitation of natural resources, and it is in general highly unlikely that a perfect hedging portfolio exists for these revenues, which implies the introduction of a non-zero unspanned volatility component in the dynamics of $e$ (see equation (2.5)). As a consequence, the income risk is no longer spanned by traded securities, which introduces a form of dynamic incompleteness beyond the incompleteness arising from possibly unspanned inflation risk. Optimal asset allocation decisions in the presence of non-tradable income risk have been studied by, among others, Viceira (2001) in a discrete-time model, and by Munk and Sørensen (2005) in a continuous-time model closer to ours.

Because endowment risk is not spanned, it is no longer possible to define unambiguously the present value of receiving the income flow $e$ as we did in (3.1). The valuation of the future endowment process will be specific to the investor, and will depend in particular on the SWF’s horizon and degree of risk aversion. By analogy with (3.9), one can define an indifference price for the income stream by the following equality:

$$J(t, A_t + H_t^{\text{ind}}, R_t, \lambda_t^S, \Phi_t, 0)$$

$$\equiv \sup_{(\theta_s)_{s\geq t}} \mathbb{E}_t \left[ u \left( \frac{A_T}{\Phi_T} \right) \right]$$

Because this indifference price process is not necessarily replicable by dynamically trading in the available assets, a vector playing the role of $\theta_e$ is not immediately defined. In this context, Munk and Sørensen (2005) compute the optimal strategy by a numerical method, which allows them to impose non-negativity constraints on the financial wealth $A$ as well as short-selling constraints on the amounts $\theta_t$. In appendix D we write the optimal portfolio strategy $\theta^*$ in terms of the value function, as well as the partial differential equation satisfied by the value function.
3. Dynamic Asset Allocation Decisions for Sovereign Wealth Funds
4. Empirical Analysis
4. Empirical Analysis

In this section, we propose an empirical analysis of the previous dynamic asset allocation model for an SWF with revenues emanating from the exploitation of oil resources. As explained in section 3, the optimal investment strategy (see (3.4) and (3.5)) is found to involve a state-dependent allocation to dedicated building-blocks, in addition to the standard performance-seeking portfolio and hedging demand components. The problem of the design of a maximum Sharpe ratio PSP is not specific to SWFs, and will not be addressed here. Instead, we will focus on the composition of the dedicated hedging demands. The first dedicated building block is an endowment-hedging portfolio customised to meet the risk exposure in the sovereign wealth fund’s endowment streams. The second dedicated building block, relevant for sovereign funds endowed with implicit or explicit inflation benchmarks, is a liability-hedging portfolio invested in asset classes exhibiting attractive inflation-hedging properties. In an application to oil-based sovereign funds, we conduct an empirical analysis of the oil-price- and inflation hedging properties of various traditional and alternative asset classes that can be used as ingredients in these two building blocks, and therefore focus on the two hedging portfolios that deserve further investigation: the portfolio against income risk (corresponding to the term \( \theta_{H,t} \) in (3.4)) and the portfolio against inflation risk (corresponding to the term \( (\sigma_p, \sigma_I)^{-1} \sigma_p, \sigma_P \) in (3.5)).

4.1 Description of the Data

In the empirical analysis, we extend the set of tradable assets beyond that which was considered in the previous section; in addition to stocks and bonds, we also consider a commodity index and a real estate index, which might prove useful in terms of their inflation and/or oil-price-hedging properties. The stock index is represented by the CRSP NYSE/Alternext/NASDAQ Value-Weighted index, including dividends. Long-term bonds are proxied by the CRSP twenty-year constant maturity bond index, and we use the yield on the three-month US Treasury bills as a proxy for the short-term risk-free rate. The alternative asset classes are real estate (proxied by the FTSE/NAREIT index) and commodities (proxied by the S&P Goldman Sachs Commodity index). The oil price is proxied by the crude oil Brent FOB. Returns on alternative classes and oil investment have been obtained from Datastream. Inflation is represented by changes in the consumer price index (CPI). In the empirical analysis, we also include state variables that have been shown to have some predictive power of stock returns in the literature: (i) the dividend yield, computed as the logarithm of the sum of the dividends paid over the last four quarters, divided by the current price; (ii) the credit spread, obtained as the difference between Moody’s BAA corporate bond rate and the ten-year Treasury constant maturity rate; (iii) the term spread, proxied by the difference between the ten-year and the three-month constant maturity rates. All time series are considered at quarterly frequency.

4.2 Calibration Method

Let \( \epsilon_{S,t+1} \) (resp. \( r_{S,t+1} \)) denote the vector of excess log-returns (resp. of log-returns) on the risky asset classes.
(stocks, long-term bonds, real estate and commodities) over the T-bill rate between \( t \) and \( t+1 \), and let \( r_{TB,t+1} \) denote the log-return on T-bills. Let \( z_t \) be the ten-dimensional vector containing all state variables and asset excess returns at time \( t \); we seek to estimate the following VAR(1) model:

\[
\mathbf{z}_{t+1} = \mathbf{\Phi}_0 + \mathbf{\Phi}_1 \mathbf{z}_t + \mathbf{\varepsilon}_t \tag{4.1}
\]

where \( \mathbf{\varepsilon} \) is an error term with mean zero, constant covariance matrix \( \mathbf{\Sigma} \), and no serial auto-correlation. Most of our time series are available from 1962 to 2008, with the notable exception of the commodity, real estate, and oil price indices, for which data is available with a shorter history. For example, oil prices are available only from 1983 to 2008. If we were to estimate an unrestricted first-order VAR over the longest common history, we would therefore use data only from 1983 to 2008. This is arguably a concern, since using an excessively small sample is not appropriate for the analysis of the long-term behavior of asset returns. To address this problem, we follow the procedure proposed by Hoevenaars et al. (2008), and we impose suitable restrictions on the VAR model (4.1). First we define a core set of variables, containing the returns on traditional asset classes (stocks, long bonds and T-bills) as well as the predictors for stock returns (dividend yield, term spread, credit spread), and consumer inflation. For these core variables, data is available from 1962 to 2008, and we can estimate an unrestricted VAR model on this sample period:

\[
\mathbf{z}_{1,t+1} = \mathbf{\Phi}_{0,1} + \mathbf{\Phi}_{1,1} \mathbf{z}_{1,t} + \mathbf{\varepsilon}_{1,t} \tag{4.2}
\]

The remaining variables are commodity, real estate, and oil price index returns, for which we fit the following VAR model:

\[
\mathbf{z}_{2,t+1} = \mathbf{\Phi}_{0,2} + \mathbf{\Theta}_1 \mathbf{z}_{1,t+1}
+ \mathbf{\Theta}_2 \mathbf{z}_{1,t} + \mathbf{\Phi}_{1,2} \mathbf{z}_{2,t} + \mathbf{\varepsilon}_{2,t} \tag{4.3}
\]

where we impose that the matrix \( \mathbf{\Phi}_{1,2} \) as well as the covariance matrix of the error term \( \mathbf{\varepsilon}_2 \) be diagonal. Hence, out-of-core variables have no cross-effects on each other, but they can be impacted by the contemporaneous and lagged values of the core variables. The last restriction that we impose is that the error terms \( \mathbf{\varepsilon}_1 \) and \( \mathbf{\varepsilon}_2 \) be cross-sectionally uncorrelated. Together with (4.2), this implies that there is no feedback effect from out-of-core variables on core variables. If we assume that the core equations are written as the first seven equations of the VAR, the parameters in (4.1) can be computed from the parameters of sub-models (4.2) and (4.3) as:

\[
\mathbf{\Phi}_0 = \begin{pmatrix} \mathbf{\Phi}_{0,1} \\ \mathbf{\Theta}_1 \mathbf{\Phi}_{0,1} + \mathbf{\Phi}_{0,2} \end{pmatrix},
\]

\[
\mathbf{\Phi}_1 = \begin{pmatrix} \mathbf{\Phi}_{1,1} & 0 \\ \mathbf{\Theta}_1 \mathbf{\Phi}_{1,1} + \mathbf{\Theta}_2 & \mathbf{\Phi}_{1,2} \end{pmatrix},
\]

\[
\mathbf{\Sigma} = \begin{pmatrix} \mathbf{\Sigma}_{\varepsilon,1} & \mathbf{\Sigma}_{\varepsilon,1} \mathbf{\Theta}_1' \\ \mathbf{\Theta}_1 \mathbf{\Sigma}_{\varepsilon,1} & \mathbf{\Theta}_1 \mathbf{\Sigma}_{\varepsilon,1} \mathbf{\Theta}_1' + \mathbf{\Sigma}_{\varepsilon,2} \end{pmatrix}
\]

Tables 1 and 2 display the estimated matrix \( \mathbf{\Phi}_1 \) and the estimated correlation matrix of residuals.
4.3 Term Structure of Risk and Correlation

Given the structure of the VAR model (4.1), one can easily derive implied variances and covariances at any horizon \( T \) for excess returns. Since we are using log-returns, returns at horizon \( T \) years are simply obtained by summing up \( 4T \) quarterly returns. The covariance matrix for the long-term excess returns, \( \Sigma \), can then be obtained as a sub-matrix of:

\[
\Sigma = \begin{pmatrix}
\sum_{t=1}^{4T} \Phi_t \sum_{i=0}^{k-1} \Phi_i & \sum_{i=0}^{k-1} \Phi_i \\
\sum_{i=0}^{k-1} \Phi_i & \sum_{i=0}^{k-1} \Phi_i
\end{pmatrix}
\]

The variances of gross returns, as opposed to excess returns, for assets other than T-bills can be inferred from the above covariance matrix, through the textbook equality:

\[
\text{Var}(r_{t,t+T}) = \text{Var}(e_{r,t+T}) + \text{Var}(r_{T,t+T}) + 2\text{Cov}(e_{r,t+T}, r_{T,t+T})
\]

where \( r_{t,t+T} \) denotes the gross return on some asset \( S \) over the period \([t, t+T] \), \( r_{T,t+T} \) is the return on the T-bill and \( e_{r,t+T} \) is the excess return of \( S \) over the T-bill. Similarly, the covariance of \( r_{t,t+T} \) with \( r_{T,t+T} \) can be computed from the entries of matrix (4.4) as:

\[
\text{Cov}(r_{t,t+T}, r_{T,t+T}) = \text{Cov}(e_{r,t+T}, r_{T,t+T}) + \text{Cov}(e_{r,t+T}, r_{T,t+T})
\]

Figure 1 shows the term structure of risk, i.e., the annualized standard deviation of gross returns as a function of the horizon \( T \). Panel (a) displays the term structure of volatility for gross returns. Like the existing literature (Campbell and Viceira 2005), we find that stock markets are less risky over the long run: the volatility on the stock index is approximately divided by two when the investment horizon increases from one year to twenty-five years. This effect is explained by the presence of mean-reversion in stock returns, captured by the predictive relationship between stock returns and dividend yields (Campbell and Viceira 2001). Other asset classes exhibit a different pattern. T-bills, for example, are found to be riskier over the long run than over the short run because of increased uncertainty involved in the roll-over at uncertain interest rates. Investment in long bonds, real estate, and commodities also shows an upward-sloping term structure of implied volatility, at least until a ten-year horizon beyond which the estimated term structure of annualised volatility is approximately flat. Changes in oil prices show by far the highest risk level, with an annualised volatility that starts at 50% for a one-year horizon to reach 70% for a twenty-five-year investment period. The magnitude of oil price uncertainty highlights the need for SWFs to hedge against unexpected changes in oil prices. Panel (b) takes a slightly different perspective, by focusing on excess returns. This is useful because the volatility of excess returns will be involved in the construction of portfolios hedging oil risk and inflation risk (see subsections 4.4 and 4.5 below).

In figure 2, we investigate the hedging properties of the asset classes under consideration with respect to oil price uncertainty. Panel (a) shows the correlations between gross returns on risky
4. Empirical Analysis

assets and gross returns on the oil price index. Unsurprisingly, commodity returns are found to have a positive correlation with oil returns at all horizons. This comes in part from an endogeneity effect since Brent crude oil is part of the GSCI commodity index. T-bills also appear to be strongly correlated with oil at horizons beyond ten years. Long bonds and real estate, on the other hand, have negative correlations with oil at all horizons. Finally, stocks show mixed hedging properties as a function of the investment horizon. At short horizons (less than five years), they are negatively correlated with oil, but at long horizons (more than ten years), the correlation becomes positive. In panel (b), we display the correlations between excess returns. As far as commodities, T-bills, and long bonds are concerned, the signs of these correlations are the same as in panel (a): this is because the correlations between gross returns have already high absolute values, so taking excess returns does not alter their sign. Gross returns on stocks are positively correlated with gross oil returns in the long run, but this correlation is weaker than for T-bills and commodities, which may explain why the correlation becomes negative when it comes to excess returns.

Figure 3 shows the correlations between gross returns and realised inflation. It appears that T-Bills and commodities have the best hedging properties against inflation risk over the long run. Stocks are negatively correlated with inflation on the short run, but they have appealing hedging properties at long horizons. Our finding of a negative short-term relationship between expected stock returns and expected inflation is consistent with previous empirical analysis (Fama and Schwert 1977; Gultekin 1983; and Kaul 1987, among others) and is also consistent with the intuition that higher inflation leads to lower economic activity, thus depressing stock returns (Fama 1981). On the other hand, higher future inflation leads to higher dividends and thus higher returns on stocks (Campbell and Shiller 1988), and thus equity investments should offer significant inflation protection over longer horizons, a fact that has also been confirmed by a number of recent empirical academic studies (Boudoukh and Richardson 1993; Schotman and Schweitzer 2000). On the other hand, real estate and long-term bonds appear to be negatively correlated with inflation at all horizons.

4.4 Hedging Demand against Oil Price Uncertainty in SWF Allocation Decisions

As we have seen in the case where income risk is spanned (see proposition 1), the presence of a stochastic non-financial endowment induces the presence of a dedicated preference-free hedging demand in the optimal allocation. This additional term represents a short position in the portfolio \( \theta_{H,t} \) that replicates the process \( H_t \) which is the present value of forthcoming income streams, a portfolio which is given by:

\[
\theta_{H,t} = H_t (\sigma_t^i \sigma_t^{-1}) \sigma_t^i \sigma_{H,t}
\]

where \( \sigma_{H,t} \) is itself given by (3.2). In particular, \( \theta_{H,t} \) combines a hedging demand against interest rate risk, a second hedging demand against equity premium risk, and a third hedging demand against
The first two portfolios are trivial: interest rate risk is spanned by the nominal bond, and Sharpe ratio risk is spanned by the stock under the maintained assumption of a perfect negative correlation between stock returns and changes in the stock index Sharpe ratio. The only component that remains to be analysed is therefore the portfolio hedging shocks to the endowment process, a portfolio that is defined as the one that maximises the instantaneous correlation with the return on oil. In the application to natural resources SWFs, where the main risk factor is assumed to be related to unexpected changes in oil prices, we thus seek to maximise the correlation between returns on oil and the return on a portfolio of traded assets given asset return dynamics consistent with the calibrated VAR model. Such a portfolio can be described by a vector \( \omega_t \) of weights allocated to stocks, bonds, real estate and commodities, with the remainder, invested in T-bills. Following Campbell et al. (2003), we approximate the excess log-return on the portfolio over a short period \([t, t+1]\) by a linear function of portfolio weights:

\[
er^A_{t,t+1} = \omega' \left( e^{S}_{t,t+1} + \frac{T}{2} \sigma^{S}_{t,t+1} \right)
- \frac{T}{2} \omega' \Sigma^{S}_{t,t+1} \omega
\]  

where \( \Sigma^{S}_{t,t+1} \) is the covariance matrix of excess log-returns over the period \([t, t+1]\) and \( \sigma^{S}_{t,t+1} \) is its diagonal, i.e., the vector of variances. We now assume a fixed-mix allocation within the endowment hedging portfolio, which means that the vector of weights \( \omega_t \) remains constant over time and can thus be denoted by \( \omega \). Summing up equation (4.5) and assuming that the matrix \( \Sigma_{t,t+1} \) is time-independent, we get that the log-return over a \( T \)-year period is:

\[
\frac{1}{T} \log \frac{V_t}{V_{t+T}} = \omega' \left( e^{S}_{t,t+T} + \frac{T}{2} \sigma^{S}_{t,t+1} \right)
- \frac{T}{2} \omega' \Sigma^{S}_{t,t+1} \omega
\]

The correlation between the \( T \)-period excess return on oil and the \( T \)-period excess return on the portfolio can then be written as:

\[
\text{Corr}_t \left( e^{S}_{t,t+T}, e^{S \text{ill}}_{t,t+T} \right) = \frac{\omega' \text{Cov}_t \left( e^{S}_{t,t+T}, e^{S \text{ill}}_{t,t+T} \right)}{\sqrt{\omega' \text{Var}_t \left( e^{S}_{t,t+T} \right)} \sqrt{\omega' \text{Var}_t \left( e^{S \text{ill}}_{t,t+T} \right)}}
\]

(4.6)

Here \( \text{Cov}_t \left( e^{S}_{t,t+T}, e^{S \text{ill}}_{t,t+T} \right) \) is a four-dimensional vector gathering the covariances of the excess returns on all assets with oil excess returns. A standard argument shows that the correlation (4.6) is maximum for the following portfolio:

\[
\omega = (\text{Var}_t \left( e^{S}_{t,t+T} \right))^{-1} \text{Cov}_t \left( e^{S}_{t,t+T}, e^{S \text{ill}}_{t,t+T} \right)
\]

(4.7)

The covariance matrix of the asset returns as well as the covariance vector with oil returns can be computed from the VAR-implied covariance matrix (4.4).
4. Empirical Analysis

of equation (4.7) reveals that the weight optimally allocated to a given asset class is a function of its covariance with oil returns, its volatility, but also of the covariances between other assets. There is no reason for the weights computed from (4.7) to fall between 0 and 1. If one wants to impose short-sale constraints, then the correlation has to be numerically maximised over the set of those portfolios that satisfy the weight constraints. In the next numerical exercise, we perform both the unconstrained and the constrained maximisation.

Panel (a) in table 3 shows the composition of the portfolio (4.7) for different investment horizons $T$, as well as the corresponding weight in T-bills. As could be expected from figures 1 and 2, we find that this unconstrained portfolio is largely dominated by T-bills. This is because a roll-over of T-bills delivers a high correlation with oil returns, while having the lowest volatility of all asset classes. Commodities also prove to be a good hedge against oil risk, but they are penalised by a much higher volatility than T-bills. As a result, they obtain a positive weight, but this weight is much smaller than that of T-bills. In contrast, long-term bonds have a large negative weight, which is consistent with their strong negative correlation with oil excess returns. From panel (b) in figure 2, excess returns on stocks and real estate have negative correlation with excess oil returns at all horizons, but this correlation is lower in magnitude compared to the correlation of bonds, T-bills and commodities. The analysis of the sign of the allocation to a given asset class in the correlation-maximising portfolio needs to be cast in a multi-variate context, and can prove to be counter-intuitive with respect to a naive standalone analysis. For example, the weight allocated to stocks is an increasing function of the time-to-horizon, and it turns out to be positive beyond a fifteen-year horizon. Also, real estate investment receives a positive weight for all investment horizons, and thus in spite of its negative correlation with oil returns. Since the SWF should hold a short position in the portfolio (4.7) to hedge against oil risk, table 3 implies that the fund should, in principle, hold for oil-price-hedging motives large long positions in bonds and short positions in commodities, real estate, and T-bills; it should be long stocks at horizons less than fifteen years, and short stocks beyond. These findings are similar to those of Scherer (2009b), except for the sign of the position in real estate. Since the unconstrained portfolio is highly leveraged and is short bonds, one expects the portfolio with no short-sales to be very different. Panel (b) in table 3 shows that this is indeed the case. In fact, when the weights are constrained to fall between 0 and 1, the correlation-maximising portfolio is entirely invested in commodities, except for short investment horizons (less than five years), where the very low volatility of T-bills makes them highly attractive and justifies a positive allocation.

4.5 Hedging Demand against Inflation Uncertainty in SWF Allocation Decisions

We now turn to the the analysis of the portfolio hedging inflation risk. As for oil risk, we compute the fixed-mix portfolio that maximises the correlation with
4. Empirical Analysis

realised inflation over a $T$-year investment period:

$$\max \omega \frac{\omega' \text{Cov}_t \left( e_{r_t, t+T}^S, r_{t, t+T}^\Phi \right)}{\sqrt{\omega' \mathbb{V}_t \left[ e_{r_t, t+T}^S \right] \omega \sqrt{\mathbb{V}_t \left[ r_{t, t+T}^\Phi \right]}}}$$

where $r_{t, t+T}$ denotes realised inflation over the period $[t, t+T]$. Maximizing this correlation with respect to $\omega$ yields:\(^{12}\)

$$\omega = \left( \mathbb{V}_t \left[ e_{r_t, t+T}^S \right] \right)^{-1} \text{Cov}_t \left( e_{r_t, t+T}^S, r_{t, t+T}^\Phi \right)$$

(4.8)

Table 4 shows the composition of the unconstrained portfolio (4.8) and that of the portfolio that maximizes the correlation subject to the no short-sales constraint. We find that the unconstrained portfolio is invested mostly in T-bills, which comes as no surprise in view of panel (b) in figure 3. Commodities and real estate have a slightly positive weight, while the weights of stocks and bonds are negative at all horizons. When short-sales constraints are imposed, the correlation-maximising portfolio is invested entirely in commodities.
5. Conclusion
In this paper, we have introduced a formal dynamic asset allocation model that incorporates the most salient factors in SWF management, and in particular that takes into account the stochastic features of the sovereign fund endowment process (where the money is coming from), the stochastic features of the sovereign fund expected liability value (what the money is going to be used for), and the stochastic features of the assets held in its portfolio. We find that the optimal asset allocation strategy for a sovereign state fund involves a state-dependent allocation to three building blocks, a performance-seeking portfolio (PSP, typically heavily invested in equities), an endowment-hedging portfolio (EHP, customised to meet the risk exposure in the sovereign wealth fund endowment streams), and a liability-hedging portfolio (LHP, heavily invested in bonds for interest rate hedging motives, and in assets exhibiting attractive inflation-hedging properties, when the implicit or explicit liabilities of the sovereign wealth funds exhibit inflation indexation), as well as separate hedging demands for risk factors impacting the investment opportunity set. While the first building block is the standard highest risk-reward component in any investor’s portfolio, the other two building blocks need to be customised to meet the needs of each sovereign wealth fund. This PSP/EHP/LHP approach, which is yet another example of a fund separation theorem, can be seen as the extension to sovereign wealth funds of liability-driven investment strategies (LDI) recently developed in the pension fund industry. Overall, it appears that the development of an asset-liability management analysis of sovereign wealth funds has potentially important implications in terms of the emergence of new forms of financial engineering techniques for the design of customised building blocks aiming at facilitating the implementation of genuinely dedicated ALM and risk management solutions for these long-term investors.

In terms of implementation, a number of challenges remain, including the need to reconcile the top-down asset allocation decisions with bottom-up security selection decisions. Indeed, the asset allocation decisions analysed in this paper relate to the design of the long-term strategic allocation for SWFs, with an associated optimal exposure to rewarded risk factors. Additionally, it is legitimate for SWFs to seek alpha opportunities, and/or consider reaching strategic stakes in selected target companies. In fact, long-term equity holdings can be a natural source of alpha generation for sovereign wealth funds given that SWFs are better placed to benefit from any temporary mispricing opportunity than hedge funds thanks to a longer-term investment horizon, and better placed than pension funds thanks to higher margins for error and the absence of regulatory constraints. Eventually, however, such security selection decisions can lead to a strong bias, such as a recent overweighted exposure to financials. These unintended bets on market, sector, and style returns are unfortunate as they can have a very significant, positive or negative, impact on the portfolio return. As a result, these biases need to be quantitatively measured, and optimised. Alternatively, these biases can be adjusted for in a multi-factor setting through a completeness portfolio designed to fill in differences between
5. Conclusion

portfolio allocation and the long-term strategic benchmark allocation. In this context, index futures can be used as cost-efficient vehicles for dynamic adjustment of portfolio exposure to market risk.

Our work can be extended in several directions. On the one hand, more work is needed for a better understanding of the composition of the endowment-hedging building block. In general, uncertainty in the endowment stream is not entirely spanned by existing securities. For example, in the case of a sovereign wealth funds managing commercial surpluses, the endowment stream is related to worldwide economic growth, the fluctuations of which are not replicable by a traded asset. This induces a specific form of market incompleteness, which makes the dynamic asset allocation problem more complex. It also raises the challenge of designing investable proxies allowing for the hedging of unexpected changes in risk factors that would be likely to impact the revenues flowing into the fund. For example, in the case of a foreign reserves sovereign wealth fund, where revenues are related to trade balance surpluses from the sovereign country (e.g., China or Singapore), the risk factors impacting the contributions to the sovereign wealth funds would be related to world economic growth, inflation differentials, and changes in currency rates, among others. A first step in the direction of a formal analysis of the magnitude and relative importance of oil price shocks relative to other sources of macroeconomic risk for oil stabilisation funds has been taken by Scherer (2009b), and this work needs to be extended to foreign reserve funds. On the other hand, it is also critically important to account for the presence of short-term risk constraints which are faced by SWFs despite their long investment horizon. Many sovereign funds were built on the idea that they could hold on to investments over long time horizons and that they could cash in a premium for investing in illiquid assets. The recent economic crisis is illustrating, however, that this strategy might not always work out. With increasing economic problems in the home country, the calls for investing in distressed assets at home become louder. At the same time, contributions to many funds are likely to decrease, given a drop in demand for natural resources and shrinking fiscal sources. As a consequence, some sovereign funds have to draw back from investments abroad, even if they are marked at a loss at the time, to finance investments in their home country. Furthermore, states are calling more and more on their funds to bridge gaps in their housekeeping or to finance economic stimulus packages. This again might make ill-timed divestments necessary. These elements suggest that beyond the elements described above SWFs would also benefit from implementing dynamic risk-controlled allocation strategies that are designed to help long-term investors meet a number of short-term goals and constraints. These insights are becoming increasingly used by pension funds, where a shift from static to dynamic LDI strategies is taking place (Martellini and Milhau 2008), and they are likely to impact SWF allocation policies in the years ahead. Finally, the approach proposed in this paper needs to be extended towards the inclusion of other sovereign assets as well as sovereign liabilities. In particular, the size of local- and foreign-currency-denominated debt, as well as contingent liabilities towards
5. Conclusion

pension systems or industries, relative to foreign reserves and sovereign assets will, for example, determine sovereign leverage and is expected to have a material impact on optimal sovereign asset management. We leave these, and other interesting questions for further research.
Appendices
A. Prices of Bonds

The price for the nominal zero-coupon bond is standard given that the nominal term-structure is driven by a single factor, which is the nominal short-term rate \( R \) and \( R \) follows the Vasicek (1977) model:

\[
B(t, T) = \exp [b_0(T - t)R_t + a_1(T - t)]
\]

where:

\[
b_0(\tau) = \frac{1 - e^{-\alpha \tau}}{\alpha}, \quad a_1(\tau) = \left( b - \frac{\sigma \lambda R}{a} \right) \left[ \frac{1 - e^{-\alpha \tau}}{\alpha} - \tau \right] + \frac{\sigma^2}{2a^2} \left[ \tau - 2 \frac{1 - e^{-\alpha \tau}}{\alpha} + \frac{1 - e^{-2\alpha \tau}}{2a^2} \right]
\]

To derive the price of the indexed bond, we write it as:

\[
I(t, T) = \Phi_t \exp \left[ \int_t^T \left( \frac{\partial}{\partial s} \left( R_s + b \right) \right) ds - \frac{1}{2} \int_t^T \sigma_s^2 ds \right]
\]

From the dynamics of \( R \), we have that:

\[
\int_t^T \sigma_s^2 ds = \int_t^T b_s(T - s) \sigma_s^2 \text{d}z_s
\]

We then set \( \nu_t = \sigma \phi \lambda t + b(T - t) \sigma_R \) and use the equality:

\[
\mathbb{E}_t \left[ \int_t^T \nu_s^2 \text{d}z_s - \frac{1}{2} \int_t^T \| \nu_s \|^2 \text{d}s \right] = 1
\]

After some algebra, we obtain:

\[
I(t, T) = \Phi_t \exp \left[ b_0(T - t)R_t + \frac{\sigma^2}{2a^2} (T - t) \right. \\
\left. + 2b_0(T - t) - b_0(T - t) \right] \\
\left. + \left[ b + \frac{\sigma^2}{a} (\sigma \phi R + \lambda \sigma) \right] (T - t) + b_0(T - t) \right]
\]

B. Dynamics of Capitalised Income \( H \)

We apply Clark-Ocone’s formula to obtain:

\[
\int_t^T M_s^\nu \text{d}s = M_t^\nu H_t + \int_t^T \mathbb{E}_s \left[ \int_t^T \mathcal{D}_u (M_u^\nu \text{d}s) \right] \text{d}z_u
\]

where \( \mathcal{D}_u \) denotes Malliavin derivative at time \( u \). This holds for any \( t \) in \([0, T]\), whence:

\[
\int_0^T M_s^\nu \text{d}s = H_0 - M_T^\nu H_T + \int_0^T \mathbb{E}_s \left[ \int_u^T \mathcal{D}_u (M_u^\nu \text{d}s) \right] \text{d}z_u \quad \text{(B.1)}
\]

Let the dynamics of \( H \) read

\[
dH_t = H_t \left[ \mu_{H, t} dt + \sigma_{H, t} \text{d}z_t \right]
\]

for some progressively measurable processes \( \mu_H \) and \( \sigma_H \). Then, applying Ito’s lemma to both sides of (B.1), we obtain that:

\[
\mu_{H, t} = R_t - \frac{c_t}{H_t} + \sigma_{H, t} \lambda_t, \quad \sigma_{H, t} = \lambda_t + \frac{\lambda_s}{M_T^\lambda H_T} \mathbb{E}_t \left[ \int_t^T \mathcal{D}_u (M_u^\lambda \text{d}s) \right]
\]

whence (3.3).

C. Proof of Proposition 1

For notational convenience, we define a state variable vector \( X_t \) as:

\[
X_t = \begin{pmatrix}
R_t \\
\lambda_t^S
\end{pmatrix}
\]

and the parameters \( \mu_X \) and \( \sigma_X \) as:

\[
\mu_X = \begin{pmatrix}
ab \\
\kappa \lambda
\end{pmatrix} + \begin{pmatrix}
a & 0 \\
0 & -\kappa
\end{pmatrix} X_t,
\]

\[
\sigma_X = \begin{pmatrix}
\sigma_R & \sigma_\lambda
\end{pmatrix}
\]
Appendices

so that $X$ evolves as:

$$dX_t = \mu_X dt + \sigma'_X d z_t$$

We then transform the original, dynamic, programme (2.12) into a static one as:

$$\max_{A_T} \mathbb{E} \left[ \frac{1}{1 - \gamma} A_T^{1 - \gamma} \right],$$

s.t. $\mathbb{E} \left[ M_T^{\lambda + \nu^*} A_T \right] = A_0 + H_0$

where $M^{\lambda + \nu^*}$ defines the minimax pricing kernel in the sense of HP91.

The optimal terminal payoff, as computed from (C.1), is $A_T^* = (\eta M^{\lambda + \nu^*})^{-1/\gamma} \Phi_T^{-1/\gamma}$ where $\eta$ is a Lagrange multiplier. The budget constraint yields

$$\eta^{-1/\gamma} = \frac{A_0 + H_0}{\mathbb{E} \left[ \left( M^{\lambda + \nu^*} \right)^{1-1/\gamma} \right]}$$

Following HP91, we assume that conditions for $(\Phi, M^{\lambda + \nu^*}, X)$ to be a Markov process are satisfied. Hence, the optimal wealth process can be written as:

$$A_t^* = \eta^{-1/\gamma} E_t \left[ \left( \frac{M^{\lambda + \nu^*}}{M^{\lambda + \nu^*}} \right)^{1-1/\gamma} \right] - H_t$$

$$= F(t, M_t^{\lambda + \nu^*}, \Phi_t, X_t) - H_t$$

We assume that $F$ is separable in the following sense:

$$F(t, M_t^{\lambda + \nu^*}, \Phi_t, X_t)$$

$$= \eta^{-1/\gamma} F_t^{-1/\gamma} \left( M_t^{\lambda + \nu^*} \right)^{-1/\gamma} g(t, X_t)$$

where $g(t, X_t)$ is thus equal to

$$\mathbb{E}_t \left[ \frac{M_t^{\lambda + \nu^*} \Phi_t}{M^{\lambda + \nu^*} \Phi_t} \right]$$

Matching the diffusion term of $A$ and the diffusion term of $F(t, M_t^{\lambda + \nu^*}, \Phi_t, X_t) - H_t$ yields the volatility vector of $A^*$:

$$\sigma_{A^*} = \left( 1 + \frac{H_t}{A_t^*} \right) \left[ \frac{1}{\gamma} (\lambda_t + \nu_t) + \frac{\sigma_X g_t}{g} \right] + \left( 1 - \frac{1}{\gamma} \right) \sigma_{\phi}$$

$$= \frac{H_t}{A_t^*} \sigma_{\phi}$$

(C.3)

Since $A^*$ is the value process of some trading strategy, its volatility vector must be spanned by the volatility matrix of traded securities, $\sigma$. This can be written as $I_2 - \sigma_t (\sigma'_t)_{-1} \sigma'_t = 0$. Given that $\sigma_{\phi}$ is spanned by $\sigma$, we get the vector $\nu_t^*$:

$$\nu_t^* = \left[ I_2 - \sigma_t (\sigma'_t)_{-1} \sigma'_t \right] \left[ (1 - \gamma) \sigma_{\phi} - \frac{\sigma_X g_t}{g} \right]$$

(C.4)

We then match the drift terms of $A^* + H$ and $F(t, M_t^{\lambda + \nu^*}, \Phi_t, X_t)$. The drift term of $F$ follows from application of Ito’s lemma to the right-hand side (rhs) of (C.2). To preclude arbitrage opportunities, it must be the case that:

$$\mu A^* = \mu A_t^* + \sigma'_A^* (\lambda_t + \nu_t^*)$$

This leads to:

$$\mu \frac{A_t^*}{A_t} + \frac{\sigma_{A^*}}{A_t^* \sigma_{\mu}} \left( \lambda_t + \nu_t^* \right) + \frac{g_t}{g} \left( \frac{\mu \chi + \frac{1}{2} \sigma_{X} (\lambda_t + \nu_t^*) + \left( \frac{1 - \frac{1}{\gamma}}{\gamma} \right) \sigma_{\phi} \sigma_{\phi} \right)$$

Rearranging terms and substituting (C.3) and (C.4), we arrive at:
This PDE and the associated terminal condition \(g(T, x) = 1\) involve only \(t\) and \(x\), which, ex post, justifies our assumption (C.2) on the separability of \(F\). The indirect utility is defined by:

\[
J(t, \lambda_t^*, R_t, \Phi_t, H_t) = \frac{1}{1 - \gamma} E_t \left[ \frac{\sigma_t^*}{\Phi_T} \right]^{1 - \gamma}
\]

so that:

\[
J(t, \lambda_t^*, R_t, \Phi_t, H_t) = \frac{1}{1 - \gamma} \left( \lambda_t^* + H_t \right) \frac{1}{\Phi_t} g(t, X_t) \gamma
\]

The optimal portfolio strategy is then obtained by pre-multiplying \(\sigma_t^*\) by \(\sigma_t(\sigma_t')^{-1}\) in (C.3), which obtains:

\[
\theta_t^* = (\lambda_t^* + H_t) \left[ \frac{1}{\gamma} (\sigma_t(\sigma_t')^{-1})^{-1} \right] A_t^* + (\sigma_t(\sigma_t')^{-1})^{-1} \frac{\sigma_t(\sigma_t')^{-1} A_t^*}{g} + \left( 1 - \frac{1}{\gamma} \right) \frac{\sigma_t(\sigma_t')^{-1} A_t^*}{\Phi_t}
\]

We now consider a candidate \(g\) of the form (3.6) and we impose the initial constraints \(A_i(0) = 0\) for \(i = 1, \ldots, 4\). We then take the relevant partial derivatives of \(g\) and substitute them into (C.5). The rhs of (C.5) can then be seen as an affine function of \((R_t, \lambda_t^*, (\lambda_t^*)^2)\) with time-dependent coefficients. Writing that each coefficient must be zero, we obtain a series of ODEs involving the \(A_i\) functions. Canceling the constant term leads to the ordinary differential equation that \(A_1\) solves (note, however, that this function is not involved in the optimal strategy \(\theta^*\)):

\[
A_1(T - t) = -a A_2(T - t) + 1
\]

This ODE can be solved in closed form with the initial condition \(A_1(0) = 0\), whence

\[
A_2(T - t) = \frac{1 - e^{-a(T - t)}}{a}
\]

We decompose \(\lambda_t = \lambda_1 + \lambda_2 A_2\), where:

\[
\lambda_1 = (\sigma_t(\sigma_t')^{-1})^{-1} \left[ \begin{array}{c} b_0(T - t) \sigma_t^* A_t \\
0 \end{array} \right]
\]

The \((\lambda_t^*)^2\) term must also be zero, whence:

\[
A_3(T - t) = \frac{1}{\gamma} \left[ \frac{1}{\gamma} - \gamma \sigma_t^* A_t - \kappa \right] A_3(T - t) + \frac{1}{\gamma} \gamma \sigma_t^* A_4(T - t)
\]

Cancelling the \((\lambda_t^*)^2\) term yields the ODE satisfied by \(A_3\):

\[
A_3(T - t) = \frac{1}{\gamma} \left[ \frac{1}{\gamma} - \gamma \sigma_t^* A_t - \kappa \right] A_3(T - t) + \frac{1}{\gamma} \gamma \sigma_t^* A_4(T - t)
\]

Finally, cancelling the constant term leads to the ordinary differential equation that \(A_4\) solves (note, however, that this function is not involved in the optimal strategy \(\theta^*\)):

\[
A_4(T - t) = -a A_2(T - t) + 1 - \frac{1}{\gamma} \gamma \sigma_t^* A_3(T - t)
\]

We now consider a candidate \(g\) of the form (3.6) and we impose the initial constraints \(A_i(0) = 0\) for \(i = 1, \ldots, 4\). We then take the relevant partial derivatives of \(g\) and substitute them into (C.5). The rhs of (C.5) can then be seen as an affine function of \((R_t, \lambda_t^*, (\lambda_t^*)^2)\) with time-dependent coefficients. Writing that each coefficient must be zero, we obtain a series of ODEs involving the \(A_i\) functions. Canceling the \(R_t\) term yields:

\[
A_2(T - t) = -a A_2(T - t) + 1
\]
Appendices

D. Asset Allocation with Unspanned Endowment Risk
Let $K(t, A_t, R_t, \lambda^S_t, \Phi_t, e_t)$ the new indirect utility function. By definition of the indifference price, we thus have that:

$$K(t, A_t, R_t, \lambda^S_t, \Phi_t, e_t) = J(t, A_t + H_t^{\text{ind}}, R_t, \lambda^S_t, \Phi_t, 0)$$

where the rhs is also equal to

$$\frac{1}{1-\gamma} \left( \frac{A_t + H_t^{\text{ind}}}{\phi_t} \right)^{1-\gamma} g(t, R_t, \lambda^S_t)^{\gamma}.$$

The dynamic programming principle (Merton, 1971) implies that:

$$\sup_{\theta_t} \left\{ K[A_t R_t + \theta_t \sigma_t \lambda_t + e_t] + K_{R_t}(b - R_t) + K_{\lambda_t}(\lambda - \lambda^S_t) + K_{\lambda_t} \theta_t \sigma_t + K_{e_t} \right\}$$

$$+ \frac{1}{2} K_{\lambda_t} \sigma_t^2 + \frac{1}{2} K_{e_t} \sigma_t^2 + \frac{1}{2} K_{\lambda_t} \sigma_t \theta_t \sigma_t + \frac{1}{2} K_{R_t} \theta_t \sigma_t \lambda_t + K_{\lambda_t} \theta_t \sigma_t + K_{\lambda_t} \theta_t \sigma_t \lambda_t$$

$$+ K_{\lambda_t} \sigma_t \theta_t \sigma_t + K_{\lambda_t} \lambda_t \sigma_t \lambda + K_{\lambda_t} \sigma_t \sigma_t = 0 \quad (D.1)$$

The first-order condition gives the optimal portfolio in the absence of short-sales constraints:

$$\theta_t^* = \frac{K_{AA} (\sigma_t \sigma_t - \sigma_t \lambda_t)}{K_{AA} (\sigma_t \sigma_t - \sigma_t \lambda_t) - K_{AR} (\sigma_t \sigma_t - \sigma_t \lambda_t) - K_{A\lambda} (\sigma_t \sigma_t - \sigma_t \lambda_t) - K_{Ae} (\sigma_t \sigma_t - \sigma_t \lambda_t)}$$

Substituting $\theta_t^*$ into (D.1) yields the Hamilton-Jacobi-Bellman (HJB) PDE:

$$0 = K_{AA} R_t + [K_{AA} + K_{AR} \nu_t + K_{A\lambda} (\lambda - \lambda^S_t) - \frac{K_{AA}^2}{2 K_{AA}} |\lambda_t|^2 - \frac{K_{AA} K_{AR}}{K_{AA}} \sigma_t \lambda_t]$$

$$- \frac{K_{AA} K_{AR}}{K_{AA}} \sigma_t \lambda_t - \frac{K_{AA} K_{AR}}{K_{AA}} \sigma_t \lambda_t + \frac{1}{2} \left( K_{AR} K_{AA} - K_{AA}^2 \right) \lambda_t^2 + \frac{1}{2} \left( K_{AR} K_{AA} - K_{AA}^2 \right) \lambda_t \sigma_t + K_{AA} \sigma_t \lambda_t$$

$$+ K_{AA} \sigma_t \sigma_t + K_{AR} \lambda_t \sigma_t + K_{A\lambda} \lambda_t \sigma_t + K_{Ae} \lambda_t \sigma_t + K_{AR} \lambda_t \sigma_t + K_{A\lambda} \lambda_t \sigma_t$$

$$+ K_{AA} \lambda_t \sigma_t + K_{AR} \lambda_t \sigma_t + K_{A\lambda} \lambda_t \sigma_t + K_{Ae} \lambda_t \sigma_t$$

The optimal terminal wealth still reads:

$$A^*_T = \frac{A_0 + H_0}{E \left( M^{\lambda + \nu^*} \phi_T \right)^{1-\gamma} g(t, R_T, \lambda^S_T)^{\gamma}} \quad (E.1)$$

with the terminal condition:

$$K(T, A, R, \lambda, \Phi, e) = \frac{1}{1-\gamma} \left( \frac{A}{\phi} \right)^{1-\gamma}$$

and

$$\lambda_T = A_1 + \lambda^S T A_2$$

E. Asset Allocation with Spanned Inflation Risk
In this section we restate, without proof, the proposition giving the optimal allocation to the risky assets, when the asset mix includes an inflation-indexed bond and the endowment risk is traded. Now the volatility matrix of the traded assets is:

$$\sigma_t = \begin{pmatrix} \sigma_B(t, T) & \sigma_S & \sigma_I(t, T) \end{pmatrix}$$

The optimal terminal wealth still reads:

$$A^*_T = \frac{A_0 + H_0}{E \left( M^{\lambda + \nu^*} \phi_T \right)^{1-\gamma} g(t, R_T, \lambda^S_T)^{\gamma}} \quad (E.1)$$

where $M^{\lambda + \nu^*}$ is the new minimax pricing kernel. The corresponding wealth process is:

$$A_t = \frac{A_0 + H_0}{E \left( M^{\lambda + \nu^*} \phi_T \right)^{1-\gamma} g(t, R_T, \lambda^S_T)^{\gamma}} \quad (E.1)$$

The optimal portfolio strategy is still given by (3.4) and (3.5), with the matrix $\sigma_t$ being as in (E.1). The functions $A_1(T - t)$, $A_2(T - t)$ and $A_4(T - t)$ are also modified. They are implicitly given as the solutions to a series of ODEs. The ODEs for $A_4$ and $A_2$ are the same as in (C.6) and (C.7). The ODE for $A_1$ is a slightly simplified version of (C.8), in which we use the fact that $N_{\sigma}\phi = 0$. 
Appendices

We write this modified version below:

\[ A'_1(T-t) = -\pi + \frac{\|\Lambda_1 - \sigma_{\Phi}\|^2}{2\gamma} + \sigma_{\Phi}'\Lambda_1 + \left[ ab + \frac{1-\gamma}{\gamma} \sigma_R'(\Lambda_1 - \sigma_{\Phi}) \right] \\
+ \left[ \kappa \bar{\lambda} + \frac{1-\gamma}{\gamma} \sigma_{\lambda}'(\Lambda_1 - \sigma_{\Phi}) \right] A_3(T-t) + \frac{1-\gamma}{2\gamma} \sigma_{\lambda}^2 A_3(T-t)^2 + \frac{1}{\gamma} \sigma_R \sigma_{\lambda} \]
Figures

**Figure 1: Term structure of risk**

(a) Standard deviations of gross returns.

Quarterly excess returns are fitted to the VAR model (4.1) over the period 1962.Q1-2008.Q4. Panel (a) shows the term structure of standard deviations of gross returns as implied by the VAR model; and panel (b) shows the term structure of standard deviations of excess returns for assets other than T-bills. All standard deviations are annualised.

(b) Standard deviations of excess returns.

**Figure 2: Term structure of correlations with oil**

(a) Gross returns on asset classes with gross returns on oil.

Quarterly excess returns are fitted to the VAR model (4.1) over the period 1962.Q1-2008.Q4. Panel (a) plots the term structure of correlations between gross returns as implied by the VAR model. Panel (b) displays the term structure of correlations between excess returns on assets other than T-bills and excess returns on oil.

(b) Excess returns on asset classes with excess returns on oil.
Figure 3: Term structure of correlations with inflation

(a) Gross returns on asset classes.

(b) Excess returns on asset classes.

Quarterly excess returns are fitted to the VAR model (4.1) over the period 1962.Q1-2008.Q4. This figure plots the term structure of correlations between gross returns and realised inflation, as implied by the VAR model.
### Table 1: Estimated matrix $\Phi_1$

<table>
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<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
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<td>1 Credit Spread</td>
<td>0.91</td>
<td>0.04</td>
<td>−0.01</td>
<td>−0.00</td>
<td>0.05</td>
<td>−0.01</td>
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<td>0.03</td>
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<td>6 20Y Bond</td>
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<td>1.54</td>
<td>−0.07</td>
<td>−0.02</td>
<td>0.23</td>
<td>−0.09</td>
<td>0.34</td>
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<td>7 3M T-Bill</td>
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<td>0.00</td>
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<td>0.05</td>
<td>0.60</td>
<td>−0.00</td>
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</tr>
</tbody>
</table>

This table shows the estimated entries of the matrix $\Phi_1$. Quarterly asset returns are fitted to the restricted VAR model (4.1) over the period 1962.Q1-2008.Q4. All returns are in excess of the T-bills, except for the T-bills themselves. Blank elements are zero by construction of the VAR.

### Table 2: Estimated correlation matrix of residuals

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<tbody>
<tr>
<td>1 Credit Spread</td>
<td>0.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 Term Spread</td>
<td>−0.06</td>
<td>0.01</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>3 Stocks</td>
<td>−0.17</td>
<td>0.02</td>
<td>0.09</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 Div. Yield</td>
<td>−0.10</td>
<td>−0.13</td>
<td>−0.01</td>
<td>0.02</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 CPI</td>
<td>−0.30</td>
<td>−0.10</td>
<td>−0.13</td>
<td>0.22</td>
<td>0.01</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6 20Y Bond</td>
<td>0.67</td>
<td>0.04</td>
<td>0.11</td>
<td>−0.05</td>
<td>−0.40</td>
<td>0.05</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7 3M T-Bill</td>
<td>0.14</td>
<td>0.23</td>
<td>−0.08</td>
<td>0.00</td>
<td>0.06</td>
<td>0.25</td>
<td>0.00</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>8 Commodities</td>
<td>−0.25</td>
<td>0.03</td>
<td>−0.07</td>
<td>0.18</td>
<td>0.43</td>
<td>−0.22</td>
<td>−0.04</td>
<td>0.11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9 Oil</td>
<td>−0.36</td>
<td>−0.15</td>
<td>−0.22</td>
<td>0.24</td>
<td>0.51</td>
<td>−0.49</td>
<td>−0.11</td>
<td>0.25</td>
<td>0.23</td>
<td></td>
</tr>
<tr>
<td>10 Real Estate</td>
<td>−0.20</td>
<td>0.15</td>
<td>0.64</td>
<td>−0.05</td>
<td>0.00</td>
<td>0.16</td>
<td>0.05</td>
<td>0.01</td>
<td>−0.15</td>
<td>0.09</td>
</tr>
</tbody>
</table>

This table shows the estimated entries of the correlation matrix of the residuals $\varepsilon$ in (4.1). Quarterly asset returns are fitted to the restricted VAR model (4.1) over the period 1962.Q1-2008.Q4. All returns are in excess of the T-bills, except for the T-bills themselves. Off-diagonal elements are correlations, and diagonal elements are standard deviations.
Tables

Table 3: Portfolios maximising correlation with oil returns
(a) No short-sale constraints

<table>
<thead>
<tr>
<th>Assets</th>
<th>Horizon (years)</th>
<th>1</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stocks</td>
<td>−0.53</td>
<td>−0.22</td>
<td>−0.05</td>
<td>0.05</td>
<td>0.11</td>
<td>0.15</td>
<td>0.18</td>
<td></td>
</tr>
<tr>
<td>20Y Bond</td>
<td>−2.64</td>
<td>−3.35</td>
<td>−3.50</td>
<td>−3.50</td>
<td>−3.48</td>
<td>−3.46</td>
<td>−3.44</td>
<td></td>
</tr>
<tr>
<td>Commodities</td>
<td>0.36</td>
<td>0.47</td>
<td>0.49</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
<td>0.51</td>
<td></td>
</tr>
<tr>
<td>Real Estate</td>
<td>0.26</td>
<td>0.26</td>
<td>0.31</td>
<td>0.35</td>
<td>0.38</td>
<td>0.41</td>
<td>0.42</td>
<td></td>
</tr>
<tr>
<td>3M T-Bill</td>
<td>3.55</td>
<td>3.85</td>
<td>3.75</td>
<td>3.59</td>
<td>3.48</td>
<td>3.40</td>
<td>3.33</td>
<td></td>
</tr>
</tbody>
</table>

(b) With short-sale constraints.

<table>
<thead>
<tr>
<th>Assets</th>
<th>Horizon (years)</th>
<th>1</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stocks</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>20Y Bond</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>Commodities</td>
<td>0.94</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>Real Estate</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>3M T-Bill</td>
<td>0.06</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td></td>
</tr>
</tbody>
</table>

These tables report the composition of the portfolio that maximises the correlation with oil returns. In panel (a), no short-sale constraints are imposed, while all weights are restricted to fall between 0 and 1 in panel (b).

Table 4: Portfolios maximising correlation with realized inflation
(a) No short-sale constraints

<table>
<thead>
<tr>
<th>Assets</th>
<th>Horizon (years)</th>
<th>1</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stocks</td>
<td>−0.04</td>
<td>−0.11</td>
<td>−0.15</td>
<td>−0.16</td>
<td>−0.16</td>
<td>−0.15</td>
<td>−0.14</td>
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</tr>
<tr>
<td>20Y Bond</td>
<td>−0.09</td>
<td>−0.15</td>
<td>−0.18</td>
<td>−0.23</td>
<td>−0.23</td>
<td>−0.25</td>
<td>−0.26</td>
<td></td>
</tr>
<tr>
<td>Commodities</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td></td>
</tr>
<tr>
<td>Real Estate</td>
<td>0.02</td>
<td>0.03</td>
<td>0.02</td>
<td>−0.01</td>
<td>−0.01</td>
<td>−0.02</td>
<td>−0.02</td>
<td></td>
</tr>
<tr>
<td>3M T-Bill</td>
<td>1.09</td>
<td>1.21</td>
<td>1.29</td>
<td>1.38</td>
<td>1.38</td>
<td>1.40</td>
<td>1.42</td>
<td></td>
</tr>
</tbody>
</table>

(b) With short-sale constraints

<table>
<thead>
<tr>
<th>Assets</th>
<th>Horizon (years)</th>
<th>1</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stocks</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>20Y Bond</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>Commodities</td>
<td>0.98</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>Real Estate</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>3M T-Bill</td>
<td>0.02</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td></td>
</tr>
</tbody>
</table>

These tables report the composition of the portfolio that maximises the correlation with realized inflation. In panel (a), no short-sale constraints are imposed, while all weights are restricted to fall between 0 and 1 in panel (b).
References
References


References


References

About EDHEC-Risk Institute
About EDHEC-Risk Institute

The Choice of Asset Allocation and Risk Management
EDHEC-Risk structures all of its research work around asset allocation and risk management. This issue corresponds to a genuine expectation from the market. On the one hand, the prevailing stock market situation in recent years has shown the limitations of diversification alone as a risk management technique and the usefulness of approaches based on dynamic portfolio allocation. On the other, the appearance of new asset classes (hedge funds, private equity, real assets), with risk profiles that are very different from those of the traditional investment universe, constitutes a new opportunity and challenge for the implementation of allocation in an asset management or asset-liability management context. This strategic choice is applied to all of the centre’s research programmes, whether they involve proposing new methods of strategic allocation, which integrate the alternative class; taking extreme risks into account in portfolio construction; studying the usefulness of derivatives in implementing asset-liability management approaches; or orienting the concept of dynamic “core-satellite” investment management in the framework of absolute return or target-date funds.

An Applied Research Approach
In an attempt to ensure that the research it carries out is truly applicable, EDHEC has implemented a dual validation system for the work of EDHEC-Risk. All research work must be part of a research programmeme, the relevance and goals of which have been validated from both an academic and a business viewpoint by the centre’s advisory board. This board is made up of internationally recognised researchers, the centre’s business partners and representatives of major international institutional investors. The management of the research programmes respects a rigorous validation process, which guarantees the scientific quality and the operational usefulness of the programmes.

Six research programmes have been conducted by the centre to date:
- Asset allocation and alternative diversification
- Style and performance analysis
- Indices and benchmarking
- Operational risks and performance
- Asset allocation and derivative instruments
- ALM and asset management

These programmes receive the support of a large number of financial companies. The results of the research programmes are disseminated through the three EDHEC-Risk locations in London, Nice, and Singapore.

In addition, EDHEC-Risk has developed close partnerships with a small number of sponsors within the framework of research chairs. These research chairs involve a three-year commitment by EDHEC-Risk and the sponsor to research themes on which the parties to the chair have agreed.

Source EDHEC (2002) and Ibbotson, Kaplan (2000)
About EDHEC-Risk Institute

The following research chairs have been endowed to date:

• Regulation and Institutional Investment, in partnership with AXA Investment Managers (AXA IM)
• Asset-Liability Management and Institutional Investment Management, in partnership with BNP Paribas Investment Partners
• Risk and Regulation in the European Fund Management Industry, in partnership with CACEIS
• Structured Products and Derivative Instruments, sponsored by the French Banking Federation (FBF)
• Private Asset-Liability Management, in partnership with ORTEC Finance
• Dynamic Allocation Models and New Forms of Target-Date Funds, in partnership with UFG
• Advanced Modelling for Alternative Investments, in partnership with Newedge Prime Brokerage
• Asset-Liability Management Techniques for Sovereign Wealth Fund Management, in partnership with Deutsche Bank
• Core-Satellite and ETF Investment, in partnership with Amundi ETF
• The Case for Inflation-Linked Bonds: Issuers’ and Investors’ Perspectives, in partnership with Rothschild & Cie
• Advanced Investment Solutions for Liability Hedging for Inflation Risk, in partnership with Ontario Teachers’ Pension Plan

Each year, EDHEC-Risk organises a major international conference for institutional investors and investment management professionals with a view to presenting the results of its research: EDHEC Risk Institutional Days.

EDHEC also provides professionals with access to its website, www.edhec-risk.com, which is entirely devoted to international asset management research. The website, which has more than 40,000 regular visitors, is aimed at professionals who wish to benefit from EDHEC’s analysis and expertise in the area of applied portfolio management research. Its monthly newsletter is distributed to more than 500,000 readers.

EDHEC-Risk Institute: Key Figures, 2008–2009

<table>
<thead>
<tr>
<th>Category</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of permanent staff</td>
<td>47</td>
</tr>
<tr>
<td>Number of research associates</td>
<td>17</td>
</tr>
<tr>
<td>Number of affiliate professors</td>
<td>5</td>
</tr>
<tr>
<td>Overall budget</td>
<td>€8,700,000</td>
</tr>
<tr>
<td>External financing</td>
<td>€5,900,000</td>
</tr>
<tr>
<td>Number of conference delegates</td>
<td>1,950</td>
</tr>
<tr>
<td>Number of participants at EDHEC Risk Executive Education seminars</td>
<td>371</td>
</tr>
</tbody>
</table>

Research for Business

The centre’s activities have also given rise to executive education and research service offshoots.

EDHEC-Risk’s executive education programmes help investment professionals to upgrade their skills with advanced risk and asset management training across traditional and alternative classes.
About EDHEC-Risk Institute

The EDHEC-Risk Institute PhD in Finance
The EDHEC-Risk Institute PhD in Finance at EDHEC Business School is designed for professionals who aspire to higher intellectual levels and aim to redefine the investment banking and asset management industries. It is offered in two tracks: a residential track for high-potential graduate students, who hold part-time positions at EDHEC Business School, and an executive track for practitioners who keep their full-time jobs. Drawing its faculty from the world’s best universities and enjoying the support of the research centre with the greatest impact on the European financial industry, the EDHEC-Risk Institute PhD in Finance creates an extraordinary platform for professional development and industry innovation.

The EDHEC-Risk Institute MSc in Risk and Investment Management
The EDHEC-Risk Institute Executive MSc in Risk and Investment Management is designed for professionals in the investment management industry who wish to progress, or maintain leadership in their field, and for other finance practitioners who are contemplating lateral moves. It appeals to senior executives, investment and risk managers or advisors, and analysts. This post graduate programme is designed to be completed in seventeen months of part-time study and is formatted to be compatible with professional schedules.

The programme has two tracks: an executive track for practitioners with significant investment management experience and an apprenticeship track for selected high-potential graduate students who have recently joined the industry. The programme is offered in Asia—from Singapore—and in Europe—from London and Nice.

FTSE EDHEC-Risk Efficient Indices
FTSE Group, the award winning global index provider, and EDHEC-Risk Institute launched the FTSE EDHEC Risk Efficient Indices at the beginning of 2010. The index series aims to capture equity market returns with an improved risk/reward efficiency compared to cap-weighted indices. The weighting of the portfolio of constituents achieves the highest possible return-to-risk efficiency by maximising the Sharpe ratio (the reward of an investment per unit of risk).

EDHEC-Risk Alternative Indexes
The different hedge fund indexes available on the market are computed from different data, according to diverse fund selection criteria and index construction methods; they unsurprisingly tell very different stories. Challenged by this heterogeneity, investors cannot rely on competing hedge fund indexes to obtain a “true and fair” view of performance and are at a loss when selecting benchmarks. To address this issue, EDHEC Risk was the first to launch composite hedge fund strategy indexes as early as 2003.

The thirteen EDHEC-Risk Alternative Indexes are published monthly on www.edhec-risk.com and are freely available to managers and investors.

2010
• Martellini, L., and V. Milhau. From deterministic to stochastic life-cycle investing: implications for the design of improved forms of target date funds (September).
• Amenc, N., F. Goltz, Martellini, L., and V. Milhau. New frontiers in benchmarking and liability-driven investing (September).
• Martellini, L., and V. Milhau. Capital structure choices, pension fund allocation decisions and the rational pricing of liability streams (July).
• Sender, S. EDHEC survey of the asset and liability management practices of European pension funds (June).
• Amenc, N., and S. Sender. Are hedge-fund UCITS the cure-all? (March).
• Amenc, N., F. Goltz, and A. Grigoriu. Risk control through dynamic core-satellite portfolios of ETFs: Applications to absolute return funds and tactical asset allocation (January).

2009
• Sender, S. Reactions to an EDHEC study on the impact of regulatory constraints on the ALM of pension funds (October).
• Amenc, N., L. Martellini, V. Milhau, and V. Ziemann. Asset-liability management in private wealth management (September).
• Amenc, N., F. Goltz, A. Grigoriu, and D. Schroeder. The EDHEC European ETF survey (May).
• Sender, S. The European pension fund industry again beset by deficits (May).
• Martellini, L., and V. Milhau. Measuring the benefits of dynamic asset allocation strategies in the presence of liability constraints (March).
• Le Sourd, V. Hedge fund performance in 2008 (February).
• La gestion indicielle dans l’immobilier et l’indice EDHEC IEIF Immobilier d’Entreprise France (February).
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• Amenc, N., L. Martellini, and V. Ziemann. Alternative investments for institutional investors: Risk budgeting techniques in asset management and asset-liability management (December).
• Goltz, F., and D. Schroeder. Hedge fund reporting survey (November).
• Amenc, N., and D. Schroeder. The pros and cons of passive hedge fund replication (October).
• Amenc, N., F. Goltz, and D. Schroeder. Reactions to an EDHEC study on asset-liability management decisions in wealth management (September).
• Le Sourd, V. Hedge fund performance in 2007 (February).

2007
• Ducoulombier, F. Etude EDHEC sur l’investissement et la gestion du risque immobiliers en Europe (November/December).
• Ducoulombier, F. EDHEC European real estate investment and risk management survey (November).
• Goltz, F., and G. Feng. Reactions to the EDHEC study “Assessing the quality of stock market indices” (September).
• Le Sourd, V. Hedge fund performance in 2006: A vintage year for hedge funds? (March).
• Amenc, N., L. Martellini, and V. Ziemann. Asset-liability management decisions in private banking (February).
• Le Sourd, V. Performance measurement for traditional investment (literature survey) (January).

2010
- Amenc, N., and V. Le Sourd. The performance of socially responsible investment and sustainable development in France: An update after the financial crisis (September).
- Lioui, A. Spillover effects of counter-cyclical market regulation: evidence from the 2008 ban on short sales (March).

2009
- Till, H. Has there been excessive speculation in the US oil futures markets? (November).
- Amenc, N., and S. Sender. A welcome European Commission consultation on the UCITS depositary function, a hastily considered proposal (September).
- Sender, S. IAS 19: Penalising changes ahead (September).
- Amenc, N. Quelques réflexions sur la régulation de la gestion d’actifs (June).
- Giraud, J.-R. MiFID: One year on (May).
- Lioui, A. The undesirable effects of banning short sales (April).

2008
- Amenc, N., and S. Sender. Les mesures de recapitalisation et de soutien à la liquidité du secteur bancaire européen (December).
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- Amenc, N., and V. Le Sourd. Socially responsible investment performance in France (December).

• Amenc, N., B. Maffei, and H. Till. Oil prices: The true role of speculation (November).
• Till, H. The oil markets: Let the data speak for itself (October).
• Sender, S. QIS4: Significant improvements, but the main risk for life insurance is not taken into account in the standard formula (February). With the EDHEC Financial Analysis and Accounting Research Centre.

2007
• Amenc, N. Trois premières leçons de la crise des crédits « subprime » (August).
• Amenc, N. Three early lessons from the subprime lending crisis (August).
• Amenc, N., W. Géhin, L. Martellini, and J.-C. Meyfredi. The myths and limits of passive hedge fund replication (June).
• Sender, S., and P. Foulquier. QIS3: Meaningful progress towards the implementation of Solvency II, but ground remains to be covered (June). With the EDHEC Financial Analysis and Accounting Research Centre.
• Hedge fund indices for the purpose of UCITS: Answers to the CESR issues paper (January).
• Géhin, W. The Challenge of hedge fund measurement: A toolbox rather than a Pandora’s box (January).
About Deutsche Bank
Deutsche Bank is a leading global investment bank with a strong and profitable private clients franchise. Its businesses are mutually reinforcing. A leader in Germany and Europe, the bank is continuously growing in North America, Asia and key emerging markets. With 81,929* employees in 72 countries, Deutsche Bank offers unparalleled financial services throughout the world. The bank competes to be the leading global provider of financial solutions, creating lasting value for our clients, our shareholders, our people and the communities in which we operate.

Despite turbulence in financial markets, Deutsche Bank maintained its capital strength. This gives the bank a firm foundation from which to focus on its responsibilities: responsibilities to its clients, who continue to look to the bank as a dependable business partner; responsibilities to its shareholders and staff, to whom the bank seeks to remain attractive in future; and finally, the responsibilities to the financial system, of which the bank is a part, and which now needs to be rigorously analysed and re-engineered.

Deutsche Bank comprises three Group Divisions: Corporate and Investment Bank (CIB); Private Clients and Asset Management (PCAM) and Corporate Investments (CI).

While this is a significant statement of intent, Deutsche Bank already has a world-class operation across Asia of significant scale, having invested in the region throughout many financial cycles. The strength of its CIB business is however a standout in terms of scale and success. As a result, the region has made a major contribution to the bank for a number of years, with EUR 2.96 billion in revenues derived from Asia Pacific in 2009. This year, the bank celebrates 30 years of business in India where it has a large and highly successful franchise; while in China, a priority growth market, the bank holds all the operating licenses required to compete in its core global business lines.

<table>
<thead>
<tr>
<th>The Group at a Glance</th>
<th>2009 in € m.</th>
<th>2008 in € m.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total net revenues</td>
<td>27,952</td>
<td>13,613</td>
</tr>
<tr>
<td>Total noninterest expenses</td>
<td>20,120</td>
<td>18,278</td>
</tr>
<tr>
<td>Income (loss) before income taxes</td>
<td>5,202</td>
<td>(5,741)</td>
</tr>
<tr>
<td>Net income (loss)</td>
<td>4,958</td>
<td>(3,896)</td>
</tr>
<tr>
<td>Dec 31, 2009 in € bn.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total assets</td>
<td>1,501</td>
<td>2,202</td>
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<tr>
<td>Shareholders’ equity</td>
<td>36.6</td>
<td>30.7</td>
</tr>
<tr>
<td>Tier 1 capital ratio**</td>
<td>12.6 %</td>
<td>10.1 %</td>
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<tr>
<td>Long-term rating</td>
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<td>Moody’s Investors Service</td>
<td>Aa1</td>
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<tr>
<td>Standard &amp; Poor’s</td>
<td>A+</td>
<td>A+</td>
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<tr>
<td>Fitch Ratings</td>
<td>AA-</td>
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