A Survey of the Literature on Hedge Fund Performance

October 2004

Walter Géhin
Engineer, Misys Asset Management Systems
Research Associate, Edhec Risk and Asset Management Research Centre
Abstract
The issue of performance measurement in the hedge fund industry has led to literature that is both abundant and controversial. The explanation for this complexity lies in the particular features of alternative funds. Hedge funds invest in a heterogeneous range of financial assets and cover a wide range of strategies that have different risk and return profiles. Even though the current studies on hedge fund performance appear to be confusing, due to conflicting conclusions and criticism of the methods employed in previous papers, they contribute to an improvement in the knowledge of alternative funds, and leading approaches are confirmed. The aim of this paper is to highlight some specific characteristics of hedge funds and their implications in terms of performance measurement. The most recent innovative contributions are reported.

EDHEC is one of the top five business schools in France. Its reputation is built on the high quality of its faculty and the privileged relationship with professionals that the school has cultivated since its establishment in 1906. EDHEC Business School has decided to draw on its extensive knowledge of the professional environment and has therefore focused its research on themes that satisfy the needs of professionals.

EDHEC pursues an active research policy in the field of finance. EDHEC-Risk Institute carries out numerous research programmes in the areas of asset allocation and risk management in both the traditional and alternative investment universes.
Introduction
The issue of performance measurement in the hedge fund industry has led to literature that is both abundant and controversial. The methods commonly used in the context of traditional investments such as mutual funds do not appear to be appropriate for hedge funds. The explanation for this complexity lies in the particular features of alternative funds.

Hedge funds invest in a heterogeneous range of financial assets: equities, bonds, swaps, sophisticated derivative securities, currencies, mortgage-backed securities, convertible debt and regulation D securities. Moreover, some asset prices are hard to determine, and the use of leverage and short positions complicates the calculation of returns. Managers often apply dynamic strategies, as opposed to the buy-and-hold strategies used, for example, in mutual funds. The presence of fees, based partly on fund performance, adds the requirement to distinguish between pre-fee returns and post-fee returns. A majority of hedge funds have “lockup periods”, during which the invested cash flows cannot be withdrawn. All of these characteristics give the return distribution a non-normal profile.

On the other hand, the hedge fund industry covers a wide range of strategies that have different risk and return profiles. Non-directional strategies (income, value, distressed securities, market neutral/convertible arbitrage, market neutral/securities hedging) have weak correlation with a related market. Directional strategies (macro, short-selling, market timing, opportunistic, emerging markets) try to benefit from market trends. Under this assumption, it is necessary to take into account the particularities of each strategy before investigating and analysing hedge fund returns.

Even though the current studies on hedge fund performance seem to be confusing, due to conflicting conclusions and criticism of the methods employed in previous papers, they contribute to an improvement in the knowledge of alternative funds, and leading approaches are confirmed.

The aim of this paper is to highlight some specific characteristics of hedge funds and their implications in terms of performance measurement. The most recent innovative contributions are reported.

In the first section, the importance of the quality of the database and the impact of the different biases on the returns are discussed. In the second section, the different hedge fund return factors are examined. The third part focuses on the advantages and drawbacks of the traditional and more recent performance indicators. The fourth section is devoted to performance evaluation models. The last section gives an overview of an interesting aspect of performance measurement, the persistence of returns.

1. Quality of the data
1.1. Accuracy of the database
Before investigating the problems of performance measurement, the choice of an accurate database is of major interest in the context of the hedge fund industry, where a lack of transparency is often observed. Performance measurement based on an inaccurate database is biased in all cases. Liang (2003b) focuses on this point.

1.1.1. Factors of accuracy
Some factors have an effect on the quality of the database:
- Auditing effectiveness. Funds that are audited effectively have lower absolute return discrepancies than those for which audit dates are missing.
- Transparency. This involves being listed on exchanges, for example.
- Verification of the returns by managers
- Ease of calculating the returns
1.1.2. Differences among database vendors and successive versions

When comparing the returns given by TASS and the U.S. Offshore Fund Directory with the percentage changes in the net asset values, the latter exhibits an average discrepancy of 0.29 points per year, while the former exhibits an average discrepancy of 0 points per year.

Nevertheless, this example does not signify that the TASS database is always better than the U.S. Offshore Fund Directory database. It indicates that at a given date the quality of the marketed databases is not homogeneous. The lack of constancy of each database is illustrated by the fact that for the same database vendor, the quality differs between versions. In comparing two different versions of TASS returns, one from July 31, 1999, and the other from March 31, 2001, 3,638 observations from 461 hedge funds are different across the two dates.

1.1.3. Other variables

According to Liang’s investigations, funds of hedge funds report returns more accurately than single hedge funds.

When comparing onshore funds with their offshore twins (the difference is solely the fund location), audited pairs reveal significantly less return discrepancy than non-audited pairs.

Finally, significant positive correlation between hedge fund size and the auditing variable appears: large funds are more frequently audited than small funds.

1.2. Biases

Hedge fund databases can potentially suffer from several biases which have a significant impact on the performance measures. The most common biases are survivorship bias, instant history bias, selection bias and stale price bias.

1.2.1. Survivorship bias

Survivorship bias occurs if the database only contains information on ‘surviving funds.’ Those funds are in operation and report information to the database vendor at the end of the data sample. The opposites of these are defunct funds. They stop reporting because of bankruptcy or liquidation, for example. Good funds that close generate a downward bias, while bad funds that fail generate an upward bias. Following Malkiel’s method (1995), the bias is evaluated via the difference in the performance of the “observable” portfolio (investment in each fund in the database from the beginning of the data sample) and the portfolio of surviving funds. Fung and Hsieh (2000) exhibit this to be about 3% per year. A similar result is found in Brown, Goetzmann and Ibbotson (1999). To correct this bias, TASS keeps the returns of defunct funds since 1994 in its database. The same method has been applied by MAR Hedge since 1995. Caglayan and Edwards (2001) include 496 defunct hedge funds in their sample.

Gregoriou (2003) conducts a survival analysis that focuses on funds of hedge funds (henceforth FoHFs). The data used, provided by ZCM, covers the period from January 1990 to December 2001. It contains 344 live and 191 defunct funds.

Endpoints occur if funds stop reporting to ZCM for three consecutive months. Censored funds are funds that are still alive at December 2001. FoHFs born from January 1990 onwards and dead before December 2001 are included in the survival analysis, as are censored funds, in order to avoid a downward bias of survival time.

The effects of several predictor variables on survival time are examined. These covariates are average monthly return, average millions managed, age, performance fees, management fees, leverage, redemption period and minimum purchase.
Three types of methods can be distinguished: non-parametric methods, semi-parametric methods and parametric methods. The non-parametric methods employed by the author are the Kaplan-Meier estimator and the life table method. One semi-parametric method is used, namely the Cox proportional hazards model. An example of a parametric model is the accelerated failure time model.

A life table exhibits a median survival time of 7.45 years at the beginning of the period. Using Kaplan-Meier estimates of survival times, it appears that the greater the assets under management, the longer the mean survival time. Focusing on the minimum purchase, the results are less homogeneous. Considering cut-offs of $25,000, $50,000 and $100,000, the larger the minimum purchase, the higher the mean survival, while an inverse relationship is observed when considering cut-offs of $250,000, $500,000, $1,000,000 and $2,000,000.

The hazard function shows that the risk of failure varies according to the survival time in years. There are peaks between 2 and 3 years, between 4 and 5 years, and between 7 and 8 years. A decreasing trend is observed from the first peak to 8 years, and after that the risk of failure increases.

Log-rank tests are conducted, on the basis of Kaplan-Meier estimates of survival times, for several covariates. Cut-offs are determined by the median. It appears that the funds that survive longer present the following characteristics: assets under management greater than $14.9 million, average monthly returns higher than 0.82%, performance fees higher than 20%, a minimum purchase higher than $250,000, low leverage and an annual redemption period. Management fees seem to have no impact on the survival times.

The results on assets under management are reinforced by the fact that the proportion of dead funds decreases when the assets under management increase. Using the Cox proportional hazards model and the accelerated failure time model with Weibull distribution leads to a conclusion on the significant impact of the amount of leverage, the level of minimum purchase and mean monthly returns on survival time. The higher these covariates, the longer the survival times.

As mentioned by Amin and Kat (2003), survivorship bias also introduces a downward bias in the standard deviation, an upward bias in the skewness, and a downward bias in the kurtosis.

1.2.2. Instant history bias
Instant history bias (or backfill bias) is the consequence of adding a hedge fund whose earlier good returns are backfilled between the inception date of the fund and the date it enters the database, while bad track records are not backfilled. This bias is evaluated by the difference between the return of an adjusted observable portfolio (the returns corresponding to the incubation period are dropped) and the return of a non-adjusted observable portfolio.

An instant history bias of 1.4% per year is calculated by Fung and Hsieh (2000) for the TASS database over the period 1994-1998. To calculate individual fund alphas without the impact of this bias, Caglayan and Edwards (2001) exclude the first twelve months of returns for all funds.

In a different approach, Posthuma and van der Sluis (2003) eliminate an individual incubation period fund by fund, for the TASS database over the period 1996-2002. In a first scenario based on the hypothesis that lockup periods and fund liquidation have no impact on the returns, a backfill bias of 4.35% per year is found (all strategies are considered). In a second scenario based on the hypothesis that lockup periods and fund liquidation engender an extra negative impact of 50% on the returns, the backfill bias is 7.24% per year. In a third scenario based on the hypothesis that lockup periods and fund liquidation engender an extra negative impact of 100% on the returns, the backfill bias is 10.13% per year.
1.2.3. Selection bias
Selection bias is explained by the fact that only funds with good performance want to be included in a database. However this upward bias is limited, due to managers who do not want to publish their performance because, for example, they have reached their goal in terms of assets under management or their target size. That is why Fung and Hsieh (2000) consider that this bias is negligible.

1.2.4. Stale price bias
Hedge funds invest in securities that cannot be liquidated easily (i.e. there is no market price available). In order to report returns at all dates, the last price of the security is often used. This is referred to as stale price bias.

2. Return factors
Hedge fund returns can be affected by both market factors (or macro-factors) and fund factors (or micro-factors).

2.1. Fund factors
Fund factors refer to the specific characteristics of individual funds, such as fund size, age or performance fees. The contradictory results presented here are in large part attributable to differences in database providers, periods and model specifications.

2.1.1. Size of the fund
Studying the relationship between size and performance can have two different implications. For the investor, it involves taking the size of the fund into account before investing. For the fund manager, it concerns the optimal size to be chosen.

Gregoriou and Rouah (2002) focus on the relationship between the size of hedge funds and their performance. The size of a fund is defined as the total asset amount at the start of the calculation period. The relationship between size and performance is tested by Pearson's correlation coefficient and Spearman's rank correlation, from January 1994 to December 1999, on the basis of databases obtained from ZCM and LaPorte. Using the geometric mean, the Sharpe ratio and the Treynor ratio, the correlations are not statistically significant. The authors conclude that the size of a hedge fund (and of a fund of hedge funds) has no impact on its performance. However, they suggest testing this relationship again over a longer period, because some size factors are liable to harm performance, for example the lower speed of operations due to administrative duties.

Koh, Koh and Teo (2003) study this relationship for Asian hedge funds. Their results corroborate the previous results, with a non-significant relationship.

Brorsen and Harri (2004) find that returns decrease when the market capitalisation increases. They provide the hypothesis that the funds are created to exploit market inefficiencies, and that the inefficiencies are finite. To maintain the performance, the managers have to close the funds to new investors.

De Souza and Gokcan (2003) exhibit through a regression on the TASS database that assets under management have a positive relationship with performance. According to them, this could imply that poor performing funds have difficulty attracting new contributions, or that large size allows lower average costs to be obtained.

Amenc, Curtis and Martellini (2003) study the impact of various fund characteristics on performance on the basis of several models, such as the standard CAPM, an adjusted CAPM for
the presence of stale prices and an implicit factor model extracted from a Principal Component Analysis. All models indicate that the mean alpha for large funds exceeds the mean alpha for small funds, with a large share of statistically significant differences.

Getmansky (2004) uses a regression on the TASS database that includes the size squared as a factor. A positive and concave relationship between current performance and past asset size is found. This suggests that an investor should select hedge funds that are near their optimal size.

2.1.2. Age of the fund
Howell (2001) investigates the relationship between the age of hedge funds and their performance, from 1994 to 2000. Young hedge funds are usually defined as those with a track record of less than three years. The first step was to adjust the returns by applying the probabilities of failure to report to the surviving funds. This gives ex-post returns, which correspond to the true costs and benefits of investing in funds with different maturities. The second step was to adjust the returns by applying the probability of future survival to the survivors’ returns by age decile. This gives ex-ante returns, which are the expected returns from investing in hedge funds with different maturities. Ex-ante returns infer that young funds' returns are superior to those of seasoned funds: the youngest decile exhibits a return of 21.5%, while the whole sample median exhibits a return of 13.9% (a spread of 760 basis points in favour of young funds). Moreover, the spread between the decile of youngest funds and the decile of oldest funds is 970 points, and the spread between the second youngest fund decile and the whole sample median is 290 points. The conclusion of this study is that hedge fund performance deteriorates over time, even when the risk of failure is taken into account. Consequently, the youngest funds seem particularly attractive.

In Amenc, Curtis and Martellini (2003), it appears that for all the models used newer funds (one or two years old) exhibit an alpha exceeding the alpha of the older funds. Nevertheless, the significance of the difference between the alphas varies across the models.

In opposition to these results, Koh, Koh and Teo (2003) find that fund age is not an explanatory factor for Asian hedge fund returns, in a cross-sectional Fama and MacBeth (1973) framework.

According to De Souza and Gokcan (2003), on the basis of a regression on the TASS database, older funds outperform younger funds on average.

2.1.3. Manager tenure
Boyson (2003a) analyses the relationship between hedge fund manager tenure and fund returns.

As far as the manager tenure is concerned, regressions show that each additional year of experience is associated with a statistically significant decrease in the annual returns of approximately -0.8%.

To explain the relationship between experience and performance in the light of risk-taking behaviour, Boyson successively examines the relationship between manager tenure and risk-taking behaviour and the relationship between risk-taking behaviour and returns.

Focusing on the relationship between manager tenure and risk-taking behaviour, three risk measures are used: the standard deviation of a portfolio’s return, a tracking error deviation\(^1\) and a beta deviation.\(^2\) It appears that an increase in manager tenure, fund size or tenure/size interaction engenders less risky behaviour. Concerning the relationship between the risk-taking behaviour and the returns, each of the three risk measures is positively related to the annual returns. In other words, when manager tenure increases, risk-taking decreases, and when risk-taking decreases, returns decrease.

---

1. Measure of how much a manager’s tracking error (i.e. the volatility in returns not explained by market volatility) differs from that of the average manager in the same style category.
2. Difference between the fund’s beta on the fund of funds index (i.e. each individual fund’s time-vary coefficient obtained from a regression of the fund’s returns on the fund of funds index) and the average beta on the fund of funds index for all other funds in the same style category.
These results highlight the impact on hedge fund returns of increasing career concerns over time, with risk-taking behaviour characterised by increasing risk aversion. Career concerns in the hedge fund industry are unique in that they change over time. This is due to the sources of the manager’s compensation, i.e. the assets under management and the returns. Young managers generally have a lower level of assets under management than older managers. Consequently, they take more risk to obtain good returns, while the large size of the fund provides older managers with their compensation. As a result, the risk level diminishes as the hedge fund manager’s age rises. Moreover, statistics show that failed hedge fund managers rarely start a new hedge fund, and if they move into the mutual fund industry, for example, this is associated with a pay cut. The amount of the pay cut is more significant for older hedge fund managers, and it is thus an incentive for them to mitigate their risk-taking behaviour. A final explanation for the lower level of risk taken by an older hedge fund manager is the large amount of personal assets invested in the fund.

2.1.4. Performance fees
Kazemi, Martin and Schneeweis (2002) study the impact of performance fees for Value, Growth and Small styles. From their data, fees have a poor effect on performance.

Koh, Koh and Teo (2003) find that funds with higher performance fees have smaller post-fee returns than funds with lower performance fees.

De Souza and Gokcan (2003) find that incentive fees and performance are positively correlated. Higher incentive fees generating higher performance can be explained by the fact that incentive fees are increased when a manager improves his performance, or by the fact that the best managers in terms of performance demand higher incentive fees.

In Amenc, Curtis and Martellini (2003), it appears that for all the models used, funds exhibiting high incentive fees (greater than or equal to 20%) obtain a better alpha than the funds with low incentive fees. However, the implicit factor model indicates a non-significant difference.

2.1.5. Other fund factors
Koh, Koh and Teo (2003) also examine other possible return factors. According to them, Asian hedge funds returns have a positive and significant relationship with the redemption period and the size of the holding company. It appears that the size of the minimum investment does not have significant explanatory power.

Similarly, according to Kazemi, Martin and Schneeweis (2002) the redemption period seems to affect the returns, since for a similar strategy, funds with a quarterly lockup have higher returns than funds with a monthly lockup.

De Souza and Gokcan (2003) exhibit that the investment of a manager’s own capital has a positive impact on performance, with regards to the lockup and redemption periods.

2.2. Market factors
Hedge fund returns are also exposed to economic factors. It is necessary to approach the different exposures strategy by strategy, because each strategy has particular trading methods. It appears that some hedge fund strategies have market factors in common with traditional stock and bond investments, whilst other parts of hedge funds are driven by factors that are not relevant in the context of stock and bond investments.

On the basis of an asset class factor model, Agarwal and Naik (1999) study the impact of various market factors on the returns of four directional strategies (Macro, Long, Hedge Long Bias, Short) and six non-directional strategies (Fixed Income Arbitrage, Event Driven, Equity Hedge, Restructuring, Event Arbitrage, Capital Structure Arbitrage).
The market factors are the following indices: S&P 500 Composite (factor 1 in the Table 1), MSCI World excluding US (factor 2), MSCI Emerging Markets (3), Salomon Brothers Government and Corporate Bond (4), Salomon Brothers World Government Bond Index (5), Lehman High Yield Composite (6), Federal Reserve Bank Trade-Weighted Dollar (7), and UK Market Price for Gold (8).

<table>
<thead>
<tr>
<th>[Table 1] Impact of market factors on returns of HFR indices, strategy by strategy, from January 1994 to September 1998</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Macro</td>
</tr>
<tr>
<td>Long</td>
</tr>
<tr>
<td>Hedge</td>
</tr>
<tr>
<td>Short</td>
</tr>
<tr>
<td>Fixed Income Arbitrage</td>
</tr>
<tr>
<td>Event Driven</td>
</tr>
<tr>
<td>Equity Hedge</td>
</tr>
<tr>
<td>Restructuring</td>
</tr>
<tr>
<td>Event Arbitrage</td>
</tr>
<tr>
<td>Capital Structure Arbitrage</td>
</tr>
</tbody>
</table>

Source: Agarwal and Naik (1999)

According to Agarwal and Naik, the negative factor loading of Fixed Income Arbitrage on the two bond indices indicates, for example, that these hedge funds short the overvalued fixed income securities.

3. Performance indicators

An initial step involves calculating a “raw” return, where contributions, withdrawals, interest, dividends accrued, gains/losses, accrued management fees and transactional fees are taken into account. For example, Hedgeworks’ methodology is as follows:

$$return = \frac{(i - e)^b(1 - ifa)}{b}$$

where $b$ is the basis (prior period ending capital plus capital contributed or withdrawn at beginning of period), $i$ is the income earned during the period (interest, dividends accrued, realized and unrealized gains/losses, other income), $e$ is expenses accrued during the period (interest, dividends (short), accrued management fees, transactional fees, other fees), and $ifa$ is the incentive fee adjustment (deduction if over high watermark; gross up or giveback of prior accrued if under high watermark).

However such a performance indicator is not sufficient, because it does not provide a risk-adjustment.

3.1. Traditional measures

3.1.1. Absolute risk-adjusted performance measures

These measures are considered “absolute” because no benchmark is used to calculate them. The most common indicators are the Sharpe ratio (1966) and the Treynor ratio (1965).

$$Sp = \frac{E(Rp) - Rf}{\sigma(Rf)}$$

where $E(Rp)$ is the expected return of the portfolio, $Rf$ is the risk-free rate, and $\sigma(Rf)$ is the standard deviation of the portfolio returns.

$$Tp = \frac{E(Rp) - Rf}{\beta p}$$

where $E(Rp)$ is the expected return of the portfolio, $Rf$ is the risk-free rate, and $\beta p$ is the beta of the portfolio.
3.1.2. Relative risk-adjusted performance measures
These measures are considered "relative" because a benchmark is used to calculate them. The most common indicator is Jensen's alpha (1968). It is obtained via a regression on:

\[ \text{Jensen's alpha} = \hat{\alpha} = \beta_p \times (\text{RM}_t - \text{RF}_t) - (\text{R}_p - \text{RF}_t) \]

where \( R_p \) is the return of the portfolio, \( R_{ft} \) is the risk-free rate, \( \beta_p \) is the beta of the portfolio, and \( R_{Mt} \) is the market return.

3.1.3. Theoretical problems
These indicators suffer from some theoretical drawbacks if they are applied to hedge funds:

- Hedge fund returns follow a hyperbolic (i.e. non-symmetric) distribution (Moix and Schmidhuber (2001)), partly due to the use of derivatives. Moreover the tails of the frequency distribution of hedge fund returns are “fatter" than those of a normal distribution. The traditional indicators are only appropriate if the returns follow a symmetrical distribution, by representing the risk through the standard deviation of the return.
- In a mean–variance framework, higher moments are not taken into account. It appears that higher moments are the source of underestimation or overestimation of the performance results via a Sharpe ratio in the context of hedge funds.
- The Sharpe ratio penalises high volatility and ignores correlation.
- Some investments may be mistakenly under or over-evaluated, because not all of the risk characteristics are captured.
- Lo (2002) concludes that there is an overstatement of the Sharpe ratio in the case of positive autocorrelation of the hedge fund returns. He documents the fact that the presence of a serial correlation in monthly returns generates an overestimation of as much as 65 per cent of the annual Sharpe ratio. That is why a ranking of hedge funds based on the Sharpe ratio can be dramatically wrong.

In order to test the normality of a distribution, a Jarque-Bera test can be conducted. A normal distribution has skewness = 0 and kurtosis = 3. The Jarque-Bera statistic is given by:

\[ JB = n \left( \frac{\text{skewness}}{6} + \frac{\text{kurtosis} - 3)^2}{24} \right) \]

where \( n \) is the number of observations in the sample period. This statistic has a chi-squared distribution (with two degrees of freedom) under the null hypothesis of normality.

The Sortino ratio (1994) provides a solution to the asymmetry of the return distribution by replacing the standard deviation with a downside deviation. This is the excess return over the risk-free rate over the downside semi-variance, so it measures the return to "bad" volatility.

\[ \text{Sortino ratio} = \frac{E(R_p) - MAR}{\sqrt{\frac{\sum_{t=0}^{T} (R_{p,t} - MAR)^2}{T \sum_{t=0}^{T} (\bar{R}_{p} - MAR)^2}}} \]

where \( R_{p,t} \) is the return of the portfolio in the sub-period \( t \), \( \bar{R}_{p} \) is the average of the returns of the portfolio over the whole period, \( MAR \) is the minimum acceptable return, and \( T \) is the number of sub-periods.

However, the Sortino ratio does not solve the problem of higher moments. Like the Sharpe ratio, the Treynor Index and Jensen's alpha, when returns are asymmetric and mean-variance rules no longer efficient, these measures cease to capture the essential features of the distribution.

Other measures such as the Omega, Kappa and AIRAP have been introduced more recently to attempt to evaluate the performance of hedge funds.
3.2. Attempts to improve performance measurement for alternative investments

3.2.1. Adjustment of the Sharpe ratio

3.2.1.1. Adjusted Sharpe ratio on the basis of excess downside deviation

**Presentation**

Downside deviation measures the degree to which overall return distribution is attributable to returns that are below a threshold level. According to Johnson, MacLeod and Thomas (2002), “when returns are not normally distributed, downside deviation introduces information beyond that contained in the Sharpe ratio”.

Johnson et al. show that in most strategies, for hedge funds exhibiting a high Sharpe ratio, the downside deviation per unit of standard deviation is higher than it would be if the return distribution was normal. This additional downside deviation is called “excess downside deviation”. It implies that the Sharpe ratio overestimates performance when returns are not normally distributed, by underestimating risk.

To take the downside risk into account, Johnson et al. propose an adjusted Sharpe ratio denoted \( \lambda \) and defined as the solution to the following equation:

\[
\frac{\mu}{\sigma} = [1 + \lambda^2] \Phi(\lambda) \phi(\lambda)
\]

The adjusted Sharpe ratio is lower than the standard Sharpe ratio if non-normality is associated with excess downside deviation. This adjusted measure allows hedge funds with occasional negative returns to be penalised.

**Empirical results**

It is illustrated by the fact that a fund with a standard Sharpe ratio of 2.56 displays an adjusted Sharpe ratio of 0.79, while another fund with a higher Sharpe ratio (greater than 2.7) displays an adjusted Sharpe ratio lower than 0.6. The second fund is penalised by a higher ratio of downside deviation to standard deviation (in other words a higher "excess downside deviation").

3.2.1.2. Autocorrelation-adjusted Sharpe ratio

**Presentation**

This indicator is recommended by Lo (2002) to avoid the overestimation of the Sharpe ratio due to the autocorrelation of the hedge fund returns. Liang (2003a) uses the autocorrelation-adjusted Sharpe ratio with the following terms:

\[
\eta(q)_{\text{SR with } \eta(q)} = \frac{q}{\sqrt{q + 2 \sum_{k=1}^{q-1} (q - k) \rho_k}}
\]

where SR is the regular Sharpe ratio on a monthly basis, \( \rho_k \) is the \( k \)th autocorrelation for hedge fund returns, and \( \eta(q)_{\text{SR with } \eta(q)} \) is the annualised autocorrelation-adjusted Sharpe ratio with \( q = 12 \).

**Empirical results**

On the basis of a database provided by Zurich Capital Markets, Liang (2003a) observes an annualised Sharpe ratio of 1.2505 and an annualised autocorrelation-adjusted Sharpe ratio of 1.0743 from 1998 to 1999 (corresponding to a bull market), while from 2000 to 2001 (corresponding to a bear market) the annualised Sharpe ratio is 0.0918 and the annualised autocorrelation-adjusted Sharpe ratio 0.1417.

These results do not indicate that in bull markets (respectively in bear markets) the standard Sharpe ratio is always greater (less) than the autocorrelation-adjusted Sharpe ratio, but according
to the period where the performance is measured, the autocorrelation of the hedge fund returns can have various impacts on the Sharpe ratio.

3.2.1.3. Modified Sharpe ratio

Presentation

Gregoriou and Gueyie (2003) propose an improvement to the original Sharpe ratio through the use of the Modified Value-at-Risk (MVaR). The new performance measure is named the Modified Sharpe ratio.

In the equation of the modified Sharpe ratio, the modified VaR is introduced instead of the standard deviation. It is defined as follows:

$$\text{Modified Sharpe Ratio} = \frac{(R_p - R_f)}{\text{MVaR}}$$

where $R_p$ is the return of the portfolio (i.e. a hedge fund or a fund of hedge funds), $R_f$ is the risk-free rate, and MVaR is the modified VaR.

The replacement of the standard definition by the MVaR is justified by the fact that the latter takes into account skewness and kurtosis in addition to mean and standard deviation. It is of particular interest in the case of hedge funds in order to avoid underestimating risk. It should be noted that from this angle the VaR exhibits the same shortcomings as the standard deviation.

Empirical results

An empirical application of the modified Sharpe ratio is examined. The data, provided by Zurich Capital Markets, covers the period from January 1997 to December 2001. The whole sample contains monthly returns of 90 live funds of hedge funds, but only 30 funds are studied: the 10 funds with the largest assets under management, the middle 10 and the bottom 10 funds. The risk-free rate $R_f$ is assumed to be nil to simplify the ranking. The MVaR is calculated at a 95% confidence level.

Comparing the average of mean returns in each of the three groups, the top group (respectively bottom funds) exhibits the highest (lowest) mean return average. On the other hand, the most negative skewness is in the bottom group, where the standard deviation is also the highest. Considering the MVaR, the bottom funds display the highest in absolute value. In short, bottom funds are more frequently affected by extreme negative returns. Mostly, empirical results for the 30 selected funds confirm that a normal Sharpe ratio overestimates the performance in comparison with the modified Sharpe ratio, except when the normal Sharpe ratio is negative.

3.2.1.4. Adjusted Sharpe ratio

Presentation

Mahdavi (2004) introduces a performance measure called the Adjusted Sharpe Ratio (henceforth ASR). The advantage of the ASR is that it provides the possibility of being directly compared to the Sharpe ratio of the benchmark in the context of non-normality in the return distribution. Nevertheless, the author highlights the fact that the ASR and the standard Sharpe ratio are based on a mean-variance framework for ranking portfolios or funds.

The distribution of the returns of a fund is adjusted in order to match the distribution of a benchmark, such that:

$$F(1 + R) \sim (1 + B)$$

where $R$ is the rate of return on a fund and $B$ is the rate of return on a benchmark.
After that, the ASR can be calculated as follows:

\[
ASR = \frac{E[R] - r_f}{\text{Std}[B]} + \frac{(1 + r_f)(1 - P)}{\text{Std}[B]}
\]

where \( P \) is the current value of \( F(1 + R) \), and \( r_f \) is the risk-free rate.

The second part of the equation corresponds to the difference between the Sharpe ratio of the benchmark and the ASR of the fund.

**Empirical results**

As a first step, the methodology is applied from January 1990 to September 2002 to a subset of indices provided by CISDM and HFR (\( R \) will be the return on one of these indices), and the benchmarks are the Lehman Aggregate Bond index and the S&P500 index. Descriptive statistics show that in most cases returns on the indices are not normally distributed. It is stated that the transformed distributions of four indices match the distribution of the selected benchmark, i.e. the Lehman Aggregate Bond index. Even if the ASRs obtained are generally superior to the standard Sharpe ratio, the difference is not significant. Similar results are exhibited if other indices and the S&P500 index are used.

As a second step, the methodology is applied to a group of 30 hedge fund managers selected from the CISDM database (\( R \) will be the return on this group of managers), and the benchmark used. In most strategies, ASR is higher than the standard Sharpe ratio, but the difference is not significant.

Consequently, it would be interesting to implement ASR in the context of a more pronounced non-normality of the distribution of returns.

### 3.2.2. Alternative measures not based on the Sharpe ratio

#### 3.2.2.1. Stutzer index

**Presentation**

The Stutzer index was introduced by Stutzer (2000). It is based on the behavioural hypothesis that investors aim to minimise the probability that the excess returns over a given threshold will be negative over a long time horizon. When the portfolio has a positive expected excess return, this probability will decay to zero at an exponential decay rate as the time horizon increases. It is equal to the maximum decay rate to zero of the expected excess return: the higher the Stutzer index, the longer the time horizon, and the better the hedge fund.

The effects of the higher moments (skewness and kurtosis) are included in this performance index. The Stutzer index penalises negative skewness and high kurtosis. It displays risk and return components and it takes into account returns that are non-normal or suffer kurtosis because of asymmetrical economic shocks or investments in options and other derivative securities. Even though the Stutzer index is based on the Sharpe ratio, the relative skewness of excess returns is impacted. By comparing two distributions with the same levels of mean and variance, the non-normal distribution with negative skewness and high kurtosis has a lower Stutzer index than the normal distribution.

**Empirical results**

Bacmann and Scholz (2003) compare the rankings of 44 hedge fund indexes with the Stutzer index and the Sharpe ratio. The database used, provided by CSFB/Tremont, HFR and Stark, covers the period from January 1994 to February 2003. Four indices are drawn from the traditional universe (MSCI World Index, Russell 2000, S&P 500 and the Salomon World Government Bond Index). 15 indices are normally distributed according to the Jarque-Bera statistic at the 5% significance level.
In comparison with the Sharpe ratio, 37 funds have the same ranking according to the Stutzer index. However, if we consider the higher moments for the indices whose rank improves, the negative skewness turns positive in the case of the Stutzer index. The positive kurtosis decreases from 7.22 to 3.69. For the indices whose rank deteriorates, the negative skewness significantly increases from -0.82 to -2.95. The positive kurtosis increases strongly from 7.22 to 19.17.

In contrast to the previous results, ranks are similar when the authors only consider the traditional indices, whatever the performance measure applied. It appears that higher moments are the source of the mismatch between the Sharpe ratio and the Stutzer index.

The Stutzer index downgrades the ranking of funds whose skewness is strongly negative and whose kurtosis is strongly positive, while it upgrades the ranking of funds whose skewness is near zero and whose kurtosis is not strongly positive.

### 3.2.2.2. Omega

#### Presentation

The Omega measure was introduced by Shadwick and Keating (2002). It reflects all the statistical properties of the return distribution, i.e. all the moments of the distribution are embodied in the measure. It requires no assumptions on the return distribution or on the utility function of the investor. It is represented by the ratio of the gain with respect to the threshold and the loss with respect to the same threshold.

Kazemi, Schneeweis and Gupta (2003) give an intuitive expression of Omega:

\[ \Omega(L) = \frac{C(L)}{P(L)} \]

where \( C(L) \) is essentially the price of a European call option written on the investment and \( P(L) \) is essentially the price of a European put option written on the investment.

The main advantage is that this measure incorporates all the moments of the return distribution, including skewness and kurtosis. Moreover, in contrast to the Sharpe ratio, ranking is always possible, whatever the threshold.

De Souza and Gokcan (2004) express Omega as follows:

\[ \Omega(L) = \frac{\int_a^b (1 - F(r)) dr}{\int_a^b F(r) dr} \]

where \( L \) is the required return threshold, \( a \) and \( b \) are the return intervals, and \( F(r) \) is the cumulative distribution of returns below threshold \( L \).

They also provide the Omega formula in a discrete case:

\[ \Omega(L) = \frac{\sum_a^b \max(0, R^+)}{\sum_a^b \max(0, R^-)} \]

where \( R^+ \) (\( R^- \)) is the return above (below) a threshold \( L \).

At a defined threshold level, the higher the Omega the better.
Empirical results

In a methodology similar to that of the Stutzer index, Bacmann and Scholz (2003) compare the rankings of 44 hedge fund indexes with the Omega and the Sharpe ratio. In comparison with the Sharpe ratio, 36 funds have the same ranking according to the Omega, but if we consider the higher moments for the indices whose rank improves, the negative skewness decreases from -0.75 to -0.45. The positive kurtosis decreases from 7.18 to 4.09. For the indices whose rank deteriorates, the negative skewness significantly increases from -0.75 to -2.60. The positive kurtosis increases strongly from 7.18 to 16.85 in the case of the Omega.

As in the case of the Stutzer index, ranks are similar when the authors only consider the traditional indices. It tends to indicate that the Sharpe ratio tends to underestimate or overestimate the performance results in the context of hedge funds.

3.2.2.3. Sharpe-Omega

Presentation

Kazemi, Schneeweis and Gupta (2003) also present the Sharpe-Omega. This measure has identical features to the Omega, whilst keeping the same risk approach as the Sharpe ratio. It is introduced in the following way:

$$\text{Sharpe-Omega} = \frac{\bar{x} - \mu - (\text{expected return} - \text{threshold})}{P(L) \text{ put option price}}$$

This indicator has the particularity of being proportional to (1 - Omega). Consequently it provides strictly the same rankings as the Omega. Through numerical examples in the case of changes in the distribution of an investment’s return, the authors show that the Sharpe-Omega is most sensitive to the mean and the variance, and is less impacted by skewness and kurtosis.

Empirical results

Using monthly data from January 1994 to May 2003, Gupta et al. estimate the Omega and Sharpe-Omega for the S&P 500 index, the CSFB convertible arbitrage index and the CSFB equity market neutral index. For different levels of threshold, the two indicators give the same rankings of the three indices.

Sharpe-Omega is successively calculated by successively modifying only the mean and the threshold (while standard deviation=5%, skewness = 0, kurtosis = 3), only the standard deviation and the threshold (while mean=1%, skewness = 0, kurtosis = 3), only the skewness and the threshold (while mean=1%, standard deviation=5%, kurtosis = 3), and only the kurtosis and the threshold (while mean=1%, standard deviation=5%, skewness = 0). It appears that changes in mean and standard deviation have the most pronounced impact on the Sharpe-Omega, confirming Keating and Shadwick’s (2002) conclusions on Omega.

3.2.2.4. Q-return

Presentation

According to Gulko (2003), performance metrics that are based on the hedge fund’s return and volatility alone are not efficient. An example of this mistake is to consider short selling funds as unattractive, even though their high volatility and large negative correlations with the stock market allow portfolio volatility to be decreased. It is therefore important to evaluate the effects of combining a hedge fund with other investments through correlation. Gulko presents an ex-post performance evaluation method that takes the return and volatility of the hedge funds and their correlations with stocks and bonds into account, in the Markowitz mean-variance framework. The advantage of this method is that it gives the contribution of a hedge fund style to a market portfolio.

---

4 - This performance measure is based on the Sharpe ratio, but it is inserted in this section because it is a specific form of the Omega.
The first step is to construct a test portfolio, by combining an investment in hedge funds with a market portfolio, with the latter made up of 65% stocks and 35% bonds. In the test portfolio, the hedge funds represent 20% of the assets, and the market portfolio 80%.

The second step is to measure the risk-adjusted return for test portfolios, which evaluates the hedge fund’s contribution to the market portfolio. The author uses the quadratic utility:

\[ Q(r, \sigma) = r - \lambda \sigma^2 \]

where \( \lambda \) is the risk aversion coefficient. The maximisation of this quadratic utility function is the goal in the mean-variance framework.

**Empirical results**

Q-returns are calculated from July 1, 1997 to June 30, 2000, with an average risk aversion valuation of 3.75. Hedge fund statistics are given by the CSFB/Tremont indices. By comparing the Q-return of the test portfolio with the Q-return of 10.21% in the market portfolio, three hedge fund styles have a positive contribution: Long/Short (+2.23 points), Market Neutral (+1.4) and Convertible Arbitrage (+0.95). Six hedge fund styles have a negative contribution: Event Arbitrage (-0.03), Fixed Income Arbitrage (-0.69), Futures (-0.74), Macro (-0.98), Short Bias (-1.59) and Emerging Markets (-4.98).

When the Sharpe ratio and Q-return are calculated for hedge funds only, style by style, Long/Short displays the highest Q-return (15.47%), but the second highest Sharpe ratio (1.25%), while Market Neutral displays the second highest Q-return (14.04%) and the highest Sharpe ratio (2.97%). These results show that differences in rankings between the Sharpe ratio and Q-return occur in some cases.

**3.2.2.5. AIRAP**

**Presentation**

Sharma (2003) introduces an innovative risk-adjusted performance measure that is specially designed to be applied to hedge funds. The new measure is called the Alternative Investments Risk Adjusted Performance (AIRAP).

AIRAP is constructed on the basis of the Expected Utility theory. The selected form of utility is a Constant Relative Risk Aversion (CRRA). AIRAP is formulated as follows:

- when \( c \) (Arrow-Pratt coefficient) is different to 1 and greater than or equal to 0:

\[ AIRAP = \left( \prod_i (1 + TR_i) \right)^\frac{1}{p_i} - 1 \]

where \( TR = \frac{dNAV}{NAV_{t-1}} \) and \( p_i \) is the frequency of % returns.

- when \( c \) is equal to 1:

\[ AIRAP = \left[ \prod_i p_i * (1 + TR)^{(1-c)} \right]^{\frac{1}{1-c}} - 1 \]

Sharma recommends an Arrow-Pratt coefficient (represented by \( c \)) from 1 to 10. Because a geometric mean is used to measure the average performance, \( c=1 \) corresponds to risk neutrality (in this case the risk premium is nil).\(^5\) Cases with \( c \) comprised between 0 and 1 assume that rational investors accept the risk of insolvency, and according to the author this is implausible. In a cautious view, the author assumes \( c=4 \). This corresponds to a case where investors accept a risk of a maximum loss of 20.7% of their wealth.

---

\(^5\) When a geometrically compounded arithmetic mean is used, \( c>0 \) always represents risk-aversion.
An approach that only involves using the ratio of gross and net assets is inadequate to take into account the impact of leverage on the performance of hedge funds, because of the presence of derivatives. This justifies a risk-based approach. AIRAP captures the impact of leverage through a credit for the higher mean and a penalty for the higher volatility as a function of the CRRA parameter. The optimal leverage, which maximises AIRAP for a range of CRRA, can be defined by standard optimisation techniques.

According to Sharma, AIRAP presents several advantages. It takes into account leverage, investor preferences, the non-normality of the return distribution, negative mean excess returns and higher moments. Unlike traditional risk-adjusted performance measures, AIRAP penalises negative skewness and positive kurtosis. Moreover, it is scale invariant and can be used for non-directional strategies, unlike the Treynor ratio. Another advantage is the intuitive interpretation of this performance measure.

Empirical results
Using data that covers the period from January 1997 to December 2001 (at the index level, the data is provided by EACM, and at the individual fund level, the data is provided by HFR), rank reversals between Sharpe and AIRAP and between Jensen’s alpha and AIRAP are presented, for 19 different levels of Constant Relative Risk Aversion, for the HFR universe. The percentage of Sharpe ratio rank reversals is between 99% and 100%, while the percentage of Jensen’s alpha rank reversals is between 98% and 100%. The Spearman rank correlation confirms the lack of correlation between standard measures and the AIRAP. At the intra-strategy level, even if the rank reversal is somewhat lower, it also indicates discrepancies between the Sharpe ratio and AIRAP.

3.2.2.6. Kappa
Presentation
Kappa, introduced by Kaplan and Knowles (2004), is presented as a generalised downside risk-adjusted performance measure. "Generalised" means that this indicator can become any risk-adjusted return measure, through a single parameter.

\[ K_n(\tau) = \frac{\mu - \tau}{\sqrt{nLPM_n(\tau)}} \]

where \( \mu \) is the expected periodic return, \( \tau \) is the investor’s minimum acceptable or threshold periodic return and LPM is the lower partial moment.

It becomes apparent that the Sortino ratio is equal to \( K_2 \), and Omega to 1. \( K_1 + "n" \) is strictly greater than 0.

Kappa can be calculated in two ways: it can use discrete return data or a parameter-based calculation. A discrete calculation gives robust results, but it is a strict requirement. The other method involves deriving a continuous return distribution from the values of the first four moments, i.e. mean, standard deviation, skewness and kurtosis.

Empirical results
Kaplan and Knowles test Kappa on a database provided by HFR that covers January 1990 to February 2003 and focuses on 11 hedge fund indexes. Firstly, for each hedge fund strategy, Kappa is calculated with "n" equal to 1 or 2, with a successive threshold of 0% or 1%. It is stated that the difference between the results obtained through the two methods (discrete or parameter-based) increases when the threshold decreases. In such cases, Kappa has to be handled cautiously. Secondly, the rankings obtained through the two methods are compared, successively with n=1, 2 and 3, and with a threshold of −1%, −0.5% and 0%. In terms of ranking, the parameter-based method gives similar results to the discrete method. The parameter "n" has a most important
impact on the ranking: only two strategies (Emerging Markets and Event-Driven) have the same ranking whatever "n" is, for a threshold of 0%.

With "n" equal to 1, 2 or 3, an inverse relationship between the threshold and the value of Kappa appears. The steepness of the Kappa curve decreases when the parameter "n" increases.

Considering the sensitivity of Kappa to skewness, when the threshold is above (below) the mean return, it is insensitive (sensitive). When Kappa is sensitive, it is a negative function of "n".

3.2.2.7. Calmar ratio and Sterling ratio
The Calmar ratio allows returns to be determined on a downside risk-adjusted basis. The higher the Calmar ratio the better. Some funds have high annual returns, but they also have extremely high drawdown risk.

The Calmar ratio compares the opportunity of gain to the potential loss. It is expressed as follows:

\[
\text{Calmar ratio} = \frac{\text{compounded annual return}}{\text{maximum drawdown}}
\]

where the compound annualised rate of return is typically over the last 3 years, and the maximum drawdown is in absolute value.

The Sterling ratio is a risk-reward measure which determines the advisors that have the highest returns with the least amount of volatility. The higher the Sterling ratio the better, because it means that the investments are receiving a higher return relative to risk.

The formula uses the average for risk (drawdown) and return over the past 3 years. It is presented as follows:

\[
\frac{\text{compounded annual return}}{(\text{average maximum drawdown}-10\%)}
\]

Nevertheless, these two measures have not been specifically introduced to measure the performance of hedge funds. Moreover, to our knowledge, they are not used in academic or practitioner studies on hedge fund performance.

4. Higher-moment-adjusted CAPM, multi-factor models and conditional approaches
The primary model is the Capital Asset Pricing Model (CAPM) which is a single-factor model. The CAPM implies that security prices are governed by their market risks and not their firm-specific risks. Based on a simple statistical regression framework using T historical returns:

\[
R_{it} = \alpha_i + \beta_i R_{mt} + \epsilon_{it}
\]

where \( R_{it} \) is the return on a given portfolio (or fund) \( i \), \( \alpha_i \) is the abnormal performance of the portfolio (or fund) \( i \), \( \beta_i \) is the sensitivity of the portfolio (or fund) \( i \) and \( R_{mt} \) is the market return for the period.

Several attempts have been made to extend the standard CAPM, in order to consider the impact of higher moments on excess returns. For example, Favre and Ranaldo (2003) and Fung, Xu and Yau (2004) apply this method.

On the other hand, in most cases, hedge fund strategies are exposed to a number of market and micro variables. Consequently a multi-factor model may be more appropriate than a single-factor model for capturing hedge fund returns.
In 1976 Ross developed the Arbitrage Pricing Theory (APT) as an alternative to the CAPM. This proposed a set of macroeconomic factors which pervasively explain stock returns and which are “priced” (a matter of expected return relative to risk). This explicit macro-factor model is completed by an implicit factor model based on Principal Component Analysis (Roll and Ross (1980)) and Factor Analysis (Chen (1983), Connor and Korajczyk (1988)), and by several explicit micro-factor models such as the Fama-French three-factor model (1993), the Carhart four-factor model (1997) and the Barra E2 model (1980).

In another way, Kat and Miffre (2002), Kazemi and Schneeweis (2003) and Cerrahoglu, Daglioglu and Gupta (2003) introduce a conditional approach to increase the efficiency of the models by taking the dynamic strategies of hedge funds into account.

4.1. Higher-moment-adjusted CAPM

Presentation

The standard CAPM can be modified in order to consider the impact of higher moments on excess returns. This is a higher-moment-adjusted CAPM.

As an example, Favre and Ranaldo (2003) examine the relevance of cos-kewness and co-kurtosis in an extensive CAPM. They use the monthly returns of 16 hedge fund indices provided by HFR, from January 1990 to August 2002.

Firstly, they employ the Market Model to study the standard CAPM. This is expressed as follows:

\[ R_{i,t} - R_{f,t} = \alpha_1 + \alpha_2 R_{m,t} - R_{f,t} + \epsilon_t \]

where \( R_{i,t} \) is the return of the hedge fund (or the index), \( R_{f,t} \) is the return of the risk-free rate, \( R_{m,t} \) is the return of the market portfolio. \( \alpha_1 \) means the excess return (in other words the alpha), and \( \alpha_2 \) expresses the covariance of hedge fund returns with the market portfolio.

After that, in a Quadratic Model corresponding to a three-moment CAPM, they add a relation to the third moment through the following term:

\[ \alpha_3 (R_{m,t} - E(R_m))^2 \]

where \( \alpha_3 \) is a proxy of co-skewness, and \( E(R_m) \) is the expected return of the market portfolio.

Finally, in a Cubic Model corresponding to a four-moment CAPM, they add a relation to the fourth moment through the following term:

\[ \alpha_4 (R_{m,t} - E(R_m))^3 \]

where \( \alpha_4 \) is a proxy for co-kurtosis.

Fung, Xu and Yau (2004) implement a higher-moment-adjusted CAPM on equity-based style hedge funds, on the basis of a database provided by CISDM, from 1994 to 2000. Results are presented below.

Empirical results

Favre and Ranaldo (2003) find that the Quadratic Model permits the adjusted \( R^2 \) (in comparison to the Market Model) in most hedge fund styles to be increased, in particular for Convertible Arbitrage (from 0.154 to 0.198), Distressed Securities (0.194 – 0.339), Event Driven (0.469 – 0.579), Emerging Markets (0.368 – 0.406), Fund of Funds (0.231 – 0.266), Market Timing (0.254 – 0.360), Merger Arbitrage (0.468 – 0.509), Relative Value Arbitrage (0.180 – 0.318) and Weighted Composite (0.585 – 0.624) indices. For these indices, \( \alpha_3 \) exhibits a high significance level.
Focusing on the Cubic Model, the adjusted $R^2$ is increased only for Convertible Arbitrage (from 0.198 to 0.211), Emerging Markets (0.406 – 0.429), Market Timing (0.360 – 0.374) and Merger Arbitrage (0.509 – 0.541) indices. $\alpha_4$ is only significant for these four strategies. However, the authors stress the fact that the high significance level of the co-kurtosis comes at the cost of a decrease in the significance level of the co-skewness coefficient. This seems to indicate collinearities between the co-moments, limiting the explanatory power of the model.

Fung et al. (2004) obtain similar excess returns with the standard CAPM and the extended CAPM. Such results suggest that in the case of equity-based style hedge funds, the inclusion of higher moments in the performance measure is not mandatory. It can be explained by the fact that the non-normality of the return distribution is not pronounced enough to obtain results that are significantly different.

4.2. Multi-factor models
There are three major multi-factor models based on the Jensen measure:
- Implicit factor model
- Explicit macro-factor model
- Explicit micro-factor model
The general presentation is as follows:

$$R_{it} = \alpha_i + \sum_{k=1}^{K} b_{ik} F_{kt} + \varepsilon_{it}$$

where $R_{it}$ is the return on a given portfolio (or fund) $i$, $\alpha_i$ is the abnormal performance of the portfolio (or fund) $i$, $b_{ik}$ the sensitivity of the portfolio (or fund) $i$ and $F_k$ is the return on factor $k$ for the period.

4.2.1. Implicit factor model
Presentation
Implicit factors are obtained through Principal Component Analysis. This is a purely statistical approach. The aim is to explain the return series of observed variables through a smaller group of non-observed implicit variables. From a mathematical point of view, each implicit factor is defined as a linear combination of the primary variables. The implicit factors are extracted from the time-series of returns.

The advantage is that it solves the problem of the choice of factors, because they are drawn from the series of returns. That avoids the risk of under-specifying the model (omitting true factors) or over-specifying the model (including spurious factors).

The drawback relates to the economic significance of the implicit variables obtained. Apart from the first variables, which are strongly correlated with the market index, the implicit factors are not easy to interpret.

Empirical results
Amenc, Curtis and Martellini (2003) use a Principal Component Analysis (PCA) to extract a set of implicit factors. While a CAPM gives an annualised mean alpha of 5.83% from a CISDM database, the implicit factor model gives an annualised mean alpha of ~1.04%. Such results highlight the impact of the model specification on results.

4.2.2. Explicit macro-factor model
Presentation
In this approach macroeconomic variables are used as factors. The sensitivity of the factors is estimated via regressions.
The most common factors added to the market factor are the inflation rate, industrial production, interest rates, the long-short treasury spread and the price of oil.

Nevertheless, the accuracy of a model depends on the factors chosen, as the selection of factors may lead to model misspecification.

Agarwal and Naik (1999) use an asset class factor model, denoted as follows:

\[ R_t = \alpha + \sum_{k=1}^{K} b_k F_{kt} + u_t \]

where:
- \( R_t \) is the return on the HFR index for a particular strategy for period \( t \),
- \( \alpha \) is the abnormal return,
- \( b_k \) is the factor loading,
- \( F_{kt} \) is the return on the \( k \)th asset class factor (or index) for period \( t \), (\( k=1, \ldots, 8 \)),
- \( u_t \) is the error term.

To avoid correlation between the returns of the different asset class factors, variables are selected through a stepwise regression. As mentioned by the authors, “the stepwise method involves entering the independent variables into the discriminant function one at a time, based on their discriminating power. The single best variable is chosen first; the initial variable is then paired with each of the other independent variables, one at a time, and a second variable is chosen, and so on.”

The asset class factors (i.e. the market factors) are selected among the following indices: S&P 500 Composite, MSCI World excluding US, MSCI Emerging Markets, Salomon Brothers Government and Corporate Bond, Salomon Brothers World Government Bond Index, Lehman High Yield Composite, Federal Reserve Bank Trade-Weighted Dollar and the UK Market Price for Gold.

**Empirical results**

From January 1994 to September 1998, Agarwal and Naik apply the model to four directional strategies (Macro, Long, Hedge Long Bias and Short) and six non-directional strategies (Fixed Income Arbitrage, Event Driven, Equity Hedge, Restructuring, Event Arbitrage and Capital Structure Arbitrage). Confirming that non-directional strategies are less correlated with the market, the non-directional strategies (from 0.38 to 0.73) exhibit lower R² than the directional strategies (from 0.49 to 0.83).

For eight strategies, alpha is significant at the 5% level, and for two strategies, it is significant at the 10% level. It is comprised between 0.53 (Macro strategy) and 1.25 (Short strategy).

**4.2.3. Explicit micro-factor model**

**Presentation**

The selected factors depend on the specific features of the funds.

The most famous explicit micro-factor model is the Barra E2 model (1980), which uses 13 factors. In the Barra-type model, factor loadings \( b_i \) are firm attributes and are observed. The factor variables \( F_t \) must be estimated. Thirteen variables are constructed:
- variability in market - momentum - size
- trading activity (turnover) - growth in earnings - earnings/price ratio
- book/price ratio - earnings variability - financial leverage
- foreign income - labor intensity - yield
- LOCAP (extension of size)

As with the other models, the Barra-type model has been subject to criticism. A disadvantage of this method is the complexity of the construction of some factors and the non-independence of the factors.
Empirical results
The results obtained by De Souza and Gokcan (2003) show that the specification of an explicit micro-factor model is a difficult and hazardous task. Setting a requirement level of 5% significance, six variables are kept in the model: managed assets, age of the fund, partner-capital participation, lockup, required redemption notice period and incentive fees. This model displays poor explanatory power with an adjusted-$R^2$ of 0.14.

4.3. Adaptation of the models to the non-linearity of hedge fund returns
Traditional multi-factor models seem inappropriate in the context of the wide range of dynamic trading strategies applied by hedge funds. Fung and Hsieh (2000) show that hedge fund returns exhibit non-linear option-like exposures to standard asset classes.

Therefore, new models have been constructed in two different ways: the first consists of introducing non-linear regressors into a linear model, the second approach is followed in most recent studies which investigate the notion of "conditional performance".

4.3.1. Linear models including non-linear regressors
To reproduce the dynamic trading strategies of hedge funds, two different non-linear variables are used in the literature: option portfolios and hedge fund indexes.

4.3.1.1. Option portfolios as non-linear regressors
Presentation
Agarwal and Naik (2000a) propose a general asset class factor model comprised of excess returns on passive option-based strategies and on buy-and-hold strategies to benchmark the performance of hedge funds. The authors analyse hedge funds that follow ten commonly used strategies. The "Generalized Asset Class Factor Model with Passive Option-based Strategies and Buy-and-Hold Strategies" presented evaluates the performance of hedge funds through the following regression:

$$ R_i^t = \alpha_i + \sum_{k=1}^{K} b_k^i F_{k,t} + U_i^t $$

where $R_i^t$ is the net-of-fees excess return (in excess of the risk free interest rate) on an individual hedge fund $i$ for month $t$, $b_k^i$ is the average factor loading of an individual hedge fund $i$ on the $k^{th}$ factor during the regression period, $\alpha_i$ is the value added by hedge fund over the regression time period, $F_{k,t}$ is the excess return (in excess of the risk free interest rate) on the $k^{th}$ factor for month $t$, ($k=1,\ldots,K$) where the factor could be a Trading Strategy factor (an option-based strategy) or a Location factor (long position in an index), and $U_i^t$ is the error term.

Empirical results
Agarwal and Naik consider an at-the-money option trading strategy (where the present value of the exercise price equals the current index value), an out-of-the-money option trading strategy (where the exercise price is half a standard deviation away from that of the at-the-money option) and a deep-out-of-the-money option trading strategy (where the exercise price is one standard deviation away from that of the at-the-money option) on the Russell 3000 index.

Using the generalised asset class factor model, they obtain $R^2$ values that are dramatically higher than the ones obtained by Fung and Hsieh (2001) using Sharpe's (1992) asset class factor model. In the case of the hedge funds that follow non-directional strategies, the proportion of observed $R^2$ attributable to trading strategies is, on average, 71% of the total $R^2$.

In the case of the hedge funds that follow directional strategies, the average proportion of observed $R^2$ due to trading strategies is 51% of the total $R^2$. These results tend to prove the importance of including trading in performance evaluation models for hedge funds.
4.3.1.2. Hedge fund indexes as non-linear regressors

Presentation

Lhabitant (2001) implements nine hedge fund indexes as non-linear variables. The hedge fund indexes, which are provided by CSFB/Tremont, have the advantage of being asset-weighted. The model is presented as follows:

\[ R_t = \alpha + \sum_{i=1}^{9} \beta_i \cdot I_{i,t} + \epsilon_t \]

where \( I_{i,t} \) is one of the 9 CSFB/Tremont sub-indexes selected. The strategies are convertible arbitrage, short bias, event driven, global macro, long short equity, emerging markets, fixed income arbitrage, market neutral and managed futures. The sub-indexes explain the first part of the return, and the other part is explained by \( \alpha \).

Empirical results

Using the generalised asset class factor model, the index-style model obtains an average \( R^2 \) value of 0.56, indicating that this model captures a significant share of the profile of hedge funds.

4.3.2. Conditional approaches

4.3.2.1. Methodologies employed

In opposing static models, some authors consider the following issue to be important: static asset pricing models imply that risk and performance are constant over time. Due to investment decisions based on public information and dynamic trading strategies, in the case of hedge funds, static models present the risk of being misspecified. If the risk profile is modified over the calculation period, it can have a strong impact on abnormal performance. This assumption goes against several studies which use multi-factor asset pricing models, where the risk exposure remains constant. For this reason, Kat and Miffre (2002), Kazemi and Schneeweis (2003) and Cerrahoglu, Daglioglu and Gupta (2003) attempt to improve the statistical significance of the performance evaluation by constructing a time varying expected return asset pricing model.

Kat and Miffre (2002) estimate three conditional models. The set of risk factors is composed of a market factor, two microeconomic factors (size and book-to-market value) and five macroeconomic factors (exchange rate risk, term structure of interest rates, international risk of default on short maturity securities, inflation risk and industrial risk). The three conditional models are a market model, the Fama-French three-factor model comprising the two microeconomic factors, and an explicit macro-factor model that considers the market factor and the five macroeconomic factors. In Jensen’s traditional approach and on the basis of Ferson and Schadt (1996), the authors replace \( \alpha \) and \( \beta \) with \( (\alpha_0, \beta_0) \) where \( (\beta_0, f_t) \) denotes a parameter that is conditional upon \( (f_t) \) in order to obtain:

\[ p_t = \alpha_0 + \alpha_{t-1} z_{t-1} + \beta_{t-1} f_t + \beta_{t-1} f_{t-1} z_{t-1} + \epsilon_t \]

where \( p_t \) is the excess portfolio return on a constant and \( \alpha_0 \) is the conditional counterpart of the Jensen measure of abnormal performance.

The regressors pick up the variations through time in the performance and risk measures that are related to changing economic conditions.

Kazemi and Schneeweis (2003) argue that the distributions of hedge fund returns are neither normal nor identical through time. They propose a conditional model of performance, based mainly on previous work by Chen and Knez (1996) and Cochrane (2001): the Stochastic Discount Factor model. The SDF model has a principal advantage in that it takes the time-varying nature of the relationship between hedge fund returns and primary asset classes into account. The major assumption behind the SDF approach is the absence of arbitrage in financial markets. Under this
condition, the SDF is a positive random variable which adjusts future payoffs for the passage of time and uncertainty.

Cerrahoglu, Daglioglu and Gupta (2003) consider that in the context of the dynamic trading strategies pursued by hedge fund managers, the introduction of time variation to Jensen’s model (1968) is a potential source of a more accurate estimation of the parameters. A linear relationship between beta and a set of mean zero information variables available at time ‘t−1’ enables a conditional performance evaluation model, in contrast to static models, to be obtained.

In their study, stochastic discount factors (SDF) are used as a linear function of the excess market return. On the basis of a multi-factor model which accounts for time-varying betas and the non-normality of returns, two methods are applied: the Generalised Method of Moments (GMM) and the Ordinary Least Squares (OLS) method.

4.3.2.2. Results

In Kat and Miffre (2002), results highlight the fact that both the abnormal performance and risk measures are time dependent. The hypothesis of constant regression is rejected for about 79% of the funds for the market model, and for all the funds for the two multi-factor models. When considering the conditional six-factor model, the best predictor of abnormal hedge fund performance is its own return (39% of the time). Next come the default spread (22.1%), the dividend yield (16.9%), the term structure (13%) and the Treasury bill (11.7%). The impact of these variables is different in periods of expansion and recession. The default spread, dividend yield and term structure have a positive (negative) impact in a down (up) market. The Treasury bill has a negative (positive) impact in an up (down) market. Moreover, abnormal performance is counter-cyclical (in a recessionary period, it is above average 75% of the time). This reveals that hedge funds are attractive in down markets.

The statistical significance of abnormal performance is raised by conditional models, by consulting the average t-ratio of the conditional models versus static models (0.72 on average, 0.98 for the six-factor model). This is confirmed by the following result: static models capture on average between 13.2% and 20% of the variation in hedge fund returns, versus a range of between 24.3% and 35.1% for conditional models.

In economic terms, the significance of the improvement in performance evaluation through the conditional models is tested via the annually compounded average measure of abnormal performance. On average, abnormal performance is increased by 1.25% through the conditional models.

Concerning hedge fund indexes, Kazemi and Schneeweis (2003) find that the risk-adjusted returns obtained through Jensen’s model are significantly positive and the estimated alphas are significant. Measures based on the Stochastic Discount Factor approach show similar results for all strategies.

The other results relate to hedge fund managers. Using a multi-factor model and an SDF approach, the estimated alphas are close to the alphas given by the single factor model, except for three strategies (large hedged equity, small hedged equity and large convertible arbitrage funds), whose alphas are more significant using the SDF model. Kazemi and Schneeweis therefore conclude that the set of primitive assets and conditioning variables that they use are not capable of capturing the type of trading strategies followed by most hedge fund strategies.

Cerrahoglu, Daglioglu and Gupta (2003) highlight the fact that the estimations provided by both the single-factor model and the multi-factor model are the same. According to them, and in
agreement with Kazemi and Schneeweis (2003), this firstly indicates that the variables used in this study have weak explanatory power for the type of trading strategies. Secondly, estimated alphas are not explained by static or dynamic strategies, but by managers’ skills. It would appear that a conditional model does not improve the estimations of the excess return, relative to a static model.

5. Performance persistence
Performance persistence measurement is of major interest because many investors allocate to different hedge funds on the basis of their track record, which implies that the performance of hedge funds is stable over time. From this angle, the measure of performance persistence is a quantitative feature of the manager selection process.

The methods and results reported by the studies that focus on hedge fund performance persistence are addressed in the following sections.

5.1. Test methods
The issue of performance persistence can be examined through two approaches. The first approach consists of measuring the persistence of relative returns. For example, the funds can be ranked in comparison to the median return in a given period, or be ranked into deciles based on the previous sub-period return.

The second approach measures the persistence of individual returns directly, without a comparison to a median.

On the other hand, it is important to examine whether persistence is sensitive to the length of return measurement intervals, due to the lockup periods applied by the managers. If only short-term persistence that is shorter than the lockup period is found, it is not possible to profit from this persistence in order to allocate their assets.

5.1.1. First approach: persistence of relative returns
The persistence of relative returns can be tested in two ways. The first way is in a two-period framework, and the second in a multi-period framework.

5.1.1.1. Two-period framework
This is the most common method employed. A distinction has to be made between non-parametric and parametric methods.

5.1.1.1.1. Non-parametric methods
Non-parametric methods are based on the construction of a two-way winner-and loser contingency table. Winners are funds whose return is higher than the median return of all the funds following the same strategy over this period, and losers are funds whose alpha is weaker than the median return of all the funds following the same strategy. Consequently, persistence refers to funds which are winners over two consecutive periods, denoted WW, and funds which are losers over two consecutive periods, denoted LL. In the absence of persistence, winners during the first period and losers during the second period will be denoted WL, and LW if the opposite is the case.

5.1.1.1.1.1. Cross Product Ratio test
The numerator of CPR corresponds to the funds which persist, and the denominator corresponds to the funds which do not persist:

\[ CPR = \frac{WW \times LL}{WL \times LW} \]
Under the null hypothesis of no persistence, the ratio is equal to 1. This implies that each of the four categories WW, LL, WL and LW represent 25% of all the funds.

The statistical significance of CPR is tested via the calculation of the Z-statistic, corresponding to the ratio of the natural logarithm of the CPR to the standard error of the natural logarithm of CPR, expressed as follows:

\[ Z - \text{statistic} = \frac{\ln(CPR)}{\alpha_{\ln(CPR)}} \]

where \( \alpha_{\ln(CPR)} \) is the standard error of the natural logarithm of CPR, equal to:

\[ \alpha_{\ln(CPR)} = \sqrt{\frac{1}{WW} + \frac{1}{WL} + \frac{1}{LW} + \frac{1}{LL}} \]

For example, a Z-statistic greater than 1.96 indicates significant persistence at a 5% confidence level.


5.1.1.1.1.2. Chi-square test
The chi-square test is carried out by comparing the distribution of the observed frequencies for the four categories WW, LL, WL and LW with the expected frequencies of the distribution. This test is always unilateral. The chi-square measurement allows the level of independence of the results to be evaluated between two periods. It is then possible to construct, for each sub-period, different rankings according to the number of years. The chisquare is equal to:

\[ \chi^2 = \frac{\sum (O_i - E_i)^2}{E_i} \]

where \( O_i \) is the observed number of funds in each case of the contingency table, and \( E_i \) is the expected number of funds in each case. The degree of freedom is equal to 1 in the case of a table with 2 lines and 2 columns, and to \([(\text{Line}-1)\times(\text{Column}-1)]\) in the other cases.

Carpenter and Lynch (1999) consider that tests based on the chi-square are more robust than the CPR in the presence of survivorship bias.


5.1.1.1.1.3. Spearman rank correlation test
Spearman (1904) proposes a non-parametric (distribution-free) rank as a measure of the strength of the associations between two variables.

Assume a set of funds \((1, 2, 3, \ldots, n)\), which have been ranked by two regimes \((x \text{ and } y)\). In the case of performance persistence, the two regimes are the two different periods. Let \(x(i)\) and \(y(i)\) be the rank (rank one is the highest and rank \(n\) is the lowest) of fund \(i\) in the two regimes respectively and define \(d_i = x(i) - y(i)\) as the distance between these rankings.

Spearman's rank correlation is obtained with the following formula:

\[ r_s = 1 - 6 \left[ \frac{\sum_{i=1}^{n} d_i^2}{n^3 - n} \right] \]
The result will always be between 1 (a perfect positive correlation, i.e. a perfect positive persistence of the performance) and minus 1 (a perfect negative correlation, i.e. a perfect negative persistence of the performance). A coefficient close to 0 indicates an absence of performance persistence over two periods.

The Spearman rank correlation test is used by Park and Staum (1998) and Brorsen and Harri (2002).

5.1.1.1.2. Parametric method, cross-sectional regression

To test the value of a parameter, one possibility is to use a consistent estimator of this parameter as a statistical test. A consistent estimator is a statistic (function of the sample) which takes values that are ever closer to \( \theta \) the larger the sample size. If \( T \) is a consistent estimator of \( \theta \), then under the assumption \( H_0: \theta = \theta_0 \), \( T \) must take values close to \( \theta_0 \). One will reject \( H_0 \) when \( T \) takes values that are too distant from \( \theta_0 \).

One will use a unilateral test to test: \( H_0: \theta = \theta_0 \) against \( H_1: \theta = \theta_1 \). To test \( \theta_0 < \theta_1 \), the test will be unilateral on the right (rejection of the values of \( T \) that are too large). To test \( \theta_0 > \theta_1 \), the test will be unilateral on the left (rejection of the values of \( T \) that are too small).

One will use a bilateral test in order to test \( H_0: \theta = \theta_0 \) against \( H_1: \theta <> \theta_0 \).

In the context of performance persistence measurement, the parametric test will relate to the significance of the sign of the coefficient obtained via a cross-sectional regression. If the estimated slope coefficient is significantly greater than zero, this is evidence of persistence.

The return of the current period (explained variable) is regressed onto the return of the previous period (explanatory variable). In other words, returns are regressed against lagged returns. A positive coefficient applied to the explanatory variable indicates that a hedge fund that performs well over the previous period will also obtain a positive result at the time of the current period, which would testify to performance persistence.


5.1.1.2. Multi-period framework, Kolmogorov-Smirnov goodness-of-fit test

The Kolmogorov-Smirnov test (K-S test) tries to determine whether two data sets differ significantly. A multi-period test has the advantage of proposing a more marked robustness of the results.

The Kolmogorov-Smirnov test is a goodness-of-fit test to a continuous law, which takes all of the quantiles into account. The model is a sample \((X_1,...,X_n)\) of an unknown law \(P\).

The Kolmogorov-Smirnov test is defined by \(H_0: \) the data follows a specified distribution, and \(H_1: \) the data does not follow a specified distribution.

To apply the Kolmogorov-Smirnov test, the cumulative frequency (normalised by the sample size) of the observations is calculated as a function of class. Then the cumulative frequency for a true distribution (most commonly, the normal distribution) is computed. The greatest discrepancy between the observed and expected cumulative frequencies, which is called the "D-statistic", has to be found. Finally it is compared to the critical D-statistic for that sample size.

In the context of the performance persistence of hedge funds, this test is used in order to check whether the distributions of winning funds and losing funds are statistically different from the
theoretical distribution. Observed frequencies of wins and losses are recorded. This frequency
distribution is compared with that generated from a normal distribution and the maximum
difference in cumulative densities between the observed and the normal distribution is used to
construct the Kolmogorov-Smirnov statistic.

An attractive feature of this test is that the distribution of the K-S test statistic itself does not
depend on the underlying cumulative distribution function being tested. Another advantage is
that it is an exact test (the chi-square goodness-of-fit test depends on an adequate sample size
for the approximations to be valid). The K-S test is generally more efficient than the chi-square
test for goodness-of-fit for small samples and can be used for very small samples where the chi-
square test does not apply.

Despite these advantages, the K-S test has the following important limitation: it can only be
applied to continuous distributions.

The Kolmogorov-Smirnov test is used by Agarwal and Naik (2000a) and Koh, Koh and Teo (2003).
Agarwal and Naik argue that the K-S test reduces the likelihood of observing large numbers of
consecutive wins or losses due to the chance factor. It permits discrimination between the chance
and skill factors.

5.1.2. Second approach: Hurst exponent as a pure persistence measure

The Hurst exponent is presented by De Souza and Gokcan (2004) as a pure persistence measure.
The Hurst exponent directly indicates the managers that persistently display positive or negative
returns. It is the solution to the following equations:

\[ RS_t = (c(t))^H \]

or

\[ \ln RS_t = \ln(c) + H \ln(t) \]

where \( RS_t \) is the range of cumulative deviations from the mean divided by the standard deviation,
and \( H \) is the Hurst exponent.

An advantage of the Hurst exponent is that its efficiency is not related to an assumption on the
return distribution. A Hurst exponent comprised between 0 and 0.5 indicates reverse persistence.
An exponent of 0.5 indicates random performance. An exponent comprised between 0.5 and 1
indicates positive persistence.

The studied period is divided into two sub-periods. The first period is considered as the in-sample
period, and the second period is considered as the out-of-sample period. The funds are divided
into several groups according to the level of the Hurst exponent exhibited by each fund.

Nevertheless, a high Hurst exponent does not indicate whether it is negative returns or positive
returns that persist. Therefore a D-Statistic can be calculated, for the managers included in the
groups exhibiting Hurst exponents greater than 0.5. It is calculated as follows:

\[
D\text{-statistic} = \frac{\text{sum negative returns}}{\text{sum all returns}}
\]

Ranging from 0 to 1, the lower the D-statistic, the more favourable it is. Managers of the high
Hurst group are ranked into three portfolios according to their D-statistic calculated during the
in-sample period.
5.2. Results

Table 2 shows the heterogeneity of the studies on the performance persistence of the hedge funds. The databases, periods, performance measures, test methods and persistence horizons differ between the studies.

<table>
<thead>
<tr>
<th>Performance measure</th>
<th>Period</th>
<th>Database (Year of the first version)</th>
<th>Test methods</th>
<th>Relative persistence</th>
<th>Absolute persistence</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specific appraisal ratio</td>
<td>1992-1998</td>
<td>Ibbotson (1999)</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Appraisal ratio</td>
<td>1990-2001</td>
<td>HFR (2000)</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Appraisal ratio</td>
<td>1977-1999</td>
<td>LaPorte (2001)</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Appraisal ratio</td>
<td>1992-2000</td>
<td>Barres (2002)</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Estimated alpha using a multi-factor model</td>
<td>1994-2000</td>
<td>Baquerizo et al. (2002)</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

6. - Here, alpha is equal to the return of a hedge fund using a particular strategy minus the average return for all hedge funds following the same strategy.
7. - Here, the appraisal ratio is equal to the previously cited alpha divided by the residual standard deviation resulting from a regression of the hedge fund return onto the average return of all the hedge funds following that strategy.
30

With regard to the database used, for example, Brown, Goetzmann and Ibbotson (1999) concentrate on offshore hedge funds, whereas Agarwal and Naik (2000b) cover offshore as well as onshore hedge funds. Unlike these US-centric studies, Kok, Koh and Teo (2003) focus on Asian hedge funds only.

Concerning the performance measure, some studies, such as Caglayan and Edwards (2001) and Capocci, Corhay and Hübner (2003) use a multi-factor model to estimate the alpha, while Brown, Caglayan and Edwards (2001) and Capocci, Corhay and Hübner (2003) use a multi-factor model to estimate the alpha, while Brown,

<table>
<thead>
<tr>
<th>Authors (year of the first version)</th>
<th>Database</th>
<th>Period</th>
<th>Performance measure</th>
<th>Test methods</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boyson (2003a,b)</td>
<td>TASS</td>
<td>1994-2000</td>
<td>estimated alpha using a multi-factor model including style factors</td>
<td>significance of the spread in alpha between worst and best deciles</td>
</tr>
</tbody>
</table>
Goetzmann and Ibbotson (1999) and Agarwal and Naik (2000b) define alpha as the return of a hedge fund using a particular strategy minus the average return of all hedge funds following the same strategy. The choice of the appraisal ratio as a performance measure in Park and Staum (1998) is justified by the fact that this measure is leverage-invariant and is interpretable as the ratio of excess reward to risk. The risk-free rate is not subtracted as in the Sharpe ratio, leading the Sharpe ratio to vary with leverage. It means that the Sharpe ratio does not separate persistence due to the mean from persistence due to the variance of returns.

Brown, Goetzmann and Ibbotson (1999) consider the possibility that performance persists on the pre-fee basis and that managers can extract their full value-added through fees. Since fees are imputed but not paid intra-year, a return adjustment for fees may introduce spurious persistence in returns measured at horizons of less than a year. That is why in some studies such as Agarwal and Naik (2000b) or Koh, Koh and Teo (2003), the tests are conducted on pre-fee and post-fee return bases.

Boyson (2003a) analyses persistence on the basis of a previous study (see Section 2.1.3. of our paper), leading her to introduce manager tenure as a proxy in the tests. Tests of performance persistence are made on an alpha obtained through a multi-factor model including style factors.

To our knowledge, De Souza and Gokcan (2004) are the only ones to have implemented the Hurst exponent in order to study the pure persistence of hedge fund performance. This is carried out in addition to tests based on the cross product ratio and regressions.

5.2.1. Overview of the studies on the relative performance persistence of hedge funds

5.2.1.1. Short-term persistence
Agarwal and Naik (2000b) use quarterly and half-yearly returns combined with alpha and the appraisal ratio as a performance measure. Parametric and non-parametric tests show significant persistence in pre-fee and post-fee returns. Directional and non-directional strategies have a similar degree of persistence. Conducting the K-S test, the level of persistence is dramatically lower, with only one case of persistence in losers and none in winners at a six-month horizon.

Brorsen and Harri (2002) conduct regression-based tests which indicate significant persistence for all styles (except short sales) for one-month, two-month and three-month horizons. For longer horizons the significance decreases, and the lagged values become negative after 11 months. Using the Spearman rank correlation, significant levels of persistence are found for all styles considered as a group, market neutral, event driven, short sales and funds of funds, contrary to sector and long only styles. Nevertheless the authors explain these opposite conclusions by the possibly low power of the Spearman test in the context of the performance persistence tests.

Baquero, ter Horst and Verbeek (2002) find, in raw returns and at a quarterly horizon, positive persistence in hedge fund returns, particularly for the best four deciles.

In order to check whether the presence of a cross-sectional variation in expected returns due to style or risk characteristics explains the observed persistence patterns in raw returns, persistence in relative returns is examined. On a risk-adjusted basis, at a quarterly horizon, strong persistence of the relative returns is found.

In Koh, Koh and Teo’s study (2003), the CPR and chi-square tests indicate persistence for horizons from one month to nine months, for pre-fee and post-fee returns. The K-S test corroborates these results: as the horizon lengthens to six months and beyond, the persistence weakens.
In Boyson (2003b), at a quarterly time horizon, evidence of persistent underperformance among old, past poor performers is found. Using a time-varying proportional hazards model, it appears that young and middle tenure managers are more exposed to the risk of termination due to poor performance than older managers. More accurately, to “survive” a young hedge fund manager has to be in the top third of performers.

5.2.1.2. Long-term persistence
Park and Staum (1998) use a Spearman rank correlation test which shows that a trader’s skill relative to his peers tends to persist at an annual horizon. Nevertheless, the strength of the persistence seems to vary substantially from year to year. The authors give the following reason for this variability: the best managers during periods of specific market conditions such as a stock market crash are unlikely to outperform their peers during periods of normal market conditions. The chi-square test confirms the previous results at an annual horizon.

Brown, Goetzmann and Ibbotson (1999) find no persistence in raw and risk-adjusted returns at an annual horizon. It should be noted that the database only contains offshore funds.

Agarwal and Naik (2000b) increase the horizon with yearly returns. It appears that the extent of the return persistence diminishes according to the chi-square, the CPR and the parametric regressions. The K-S test concludes that there is an absence of persistence at a one-year horizon, at the 5% and 10% confidence levels.

Caglayan and Edwards (2001) conduct non-parametric tests which reveal the existence of both winner and loser persistence for all hedge funds, funds of funds, global macro and market neutral funds at one-year and two-year horizons. The results displayed by the parametric tests are similar.

According to Barès, Gibson and Gyger (2002), on a risk-adjusted basis, for all the different strategies at an aggregated level, the abnormal performance of the 5 best portfolios decreases from the formation period to the holding period. In lower proportions, the opposite phenomenon is observed for the 5 worst portfolios. In panel A, where the formation period is from January 1992 to December 1994, and the holding period is from January 1995 to December 1997, the best portfolio (containing the best 20 funds during the formation period) remains the best in the holding period. That is not the case for panel B, where the formation period is from January 1995 to December 1997, and the holding period is from January 1998 to December 2000. The best portfolio in the formation period is the worst portfolio in the holding period. This may indicate that the selection of persistently winning funds depends on the formation period.

Baquero, ter Horst and Verbeek (2002) show that in raw returns, at the annual horizon, the top three deciles show persistence. At a biannual level, a clear positive persistence is found, with the exception of the top decile. On a risk-adjusted basis, at an annual horizon, a strong persistence of the relative returns to style benchmarks for the top three deciles. At a biannual horizon, no persistence is observed.

Kat and Menexe (2002), on the basis of the mean returns from the June 1994-November 1997 and December 1997-May 2001 periods, and according to the CPR test, find no evidence of persistence for all the hedge funds considered as a group and for the strategies analysed one by one. Parametric tests indicate significant persistence for funds of funds and emerging market strategies.

Capocci, Corhay and Hübner (2003) find no persistence in annual mean returns for best and worst performing funds. It appears that if persistence exists, it is in the bull market, and more accurately for the medium performers. This suggests that many hedge fund managers follow less risky strategies, which allow them to outperform the market for longer periods of time.
Kouwenberg (2003) finds some evidence of performance persistence on the basis of the alpha and the Sharpe ratio, from the period January 1995-December 1997 to the period January 1998-November 2000, mainly for event driven, market neutral and global funds, which constitute about 90% of the funds in the database. Sector funds show reverse performance persistence. Emerging markets do not report significant persistence. The results for short seller funds cannot be interpreted due to the low number of funds.

In Koh, Koh and Teo (2003), the CPR, chi-square and K-S tests indicate an absence of significant persistence at an annual horizon.

Chen and Passow (2003) examine, through cross-sectional regressions (for this purpose, the track record from January 1990 to September 2002 is split into two sub-periods of equal length), the performance persistence of hedged equity funds. More accurately, the persistence of selected funds is compared to the persistence of the other funds. The selected funds are those which maintain a moderate exposure to the factors of a multi-factor model. These funds exhibit better performance persistence. On the other hand, it is stated that outperformers do not show significant performance persistence.

Significance tests applied by De Souza and Gokcan (2004) to a cross product ratio and regressions lead to the same conclusion at a three year horizon. A lack of significant return and Sharpe ratio persistence is observed on all seven strategies (convertible arbitrage, distressed securities, merger arbitrage, fixed income arbitrage, equity market neutral, equity long/short and global macro), except for the tests on regressions, which exhibit significant Sharpe ratio persistence for two strategies (convertible arbitrage and equity market neutral).

5.2.1.3. Comments
Even though some studies present conflicting conclusions, the main characteristics of hedge fund performance persistence can be highlighted.

Firstly, short-term persistence, for horizons of up to six months, is reported by non-parametric and parametric tests in the two-period framework. Using a multi-period test (the Kolmogorov-Smirnov test), the persistence levels are less significant. A multi-period framework allows for better discrimination between persistence due to chance and persistence due to manager skill.


The sensitivity of the persistence to the test horizon becomes important when the lockup periods of the hedge funds are considered: if the lockup period is greater than the persistence horizon, the investor is exposed to a potential reversal in the trend of the returns reported by the hedge funds.

5.2.2. Pure persistence of hedge fund performance
On the basis of the Hurst exponent, De Souza and Gokcan (2004) examine the pure persistence of the performance of seven strategies, namely convertible arbitrage, distressed securities, merger arbitrage, fixed income arbitrage, equity market neutral, equity long/short and global macro. The period is from January 1997 to December 2002. The sample contains 314 hedge funds provided by HFR.
The period from January 1997 to December 2002 is divided into two sub-periods of three years. The first period is considered as the in-sample period, and the second period is considered as the out-of-sample period. The funds are divided into three groups according to the level of the Hurst exponent exhibited by each fund. To determine the weights of each manager in the groups, two methods are used. The simpler one consists of equal weighting. The second method introduces the notion of risk budgeting. Risk budgeting is "an asset allocation technique where [...] capital is allocated to risk buckets with no consideration of associated returns."

The "low Hurst" group contains 105 managers, where exponents range from 0.32 to 0.58. The "medium Hurst" group contains 105 managers, where exponents range from 0.59 to 0.69. The "high Hurst" group contains 104 managers, where exponents range from 0.70 to 0.98. Except for the distressed securities strategy, which does not appear in the low Hurst group, all the strategies are represented in each group. Using an equal-weighting scheme, during the in-sample period returns, standard deviations and Sharpe ratios do not differ significantly among Hurst groups. During the out-of-sample period, the high Hurst group displays the highest rate of return, the lowest volatility (thus automatically the highest Sharpe ratio), the highest Calmar ratio and the highest number of months with consecutive gains. Using the risk budgeting approach, the high Hurst portfolio presents the best statistics too. In other words, persistent managers outperform non-persistent managers during the out-of-sample period.

A D-Statistic is calculated, for the managers included in the high Hurst group only, in order to filter negative persistence. The 104 managers of the high Hurst group are ranked into three portfolios according to their D-statistic calculated during the in-sample period. The equal-weighting scheme and risk budgeting are successively used to construct the three portfolios. During both periods, using either equally-weighted portfolios or a risk budgeting approach, the low D-statistic portfolio exhibits the lowest standard deviation, the highest Sharpe ratio, the highest Calmar ratio, and the highest number of months with consecutive gains.

In other words, funds exhibiting the highest Hurst exponent and lowest D-Statistic are more liable to have persistent performance.

De Souza and Gokcan stress the ability of such a method based on the Hurst exponent and the D-statistic "to identify a sample of 35 managers with a "future" return of 9.12% and a volatility of 2.57% (on an equally weighted basis) from a pool of 314 managers with an average return of 6.16% and a volatility of 4.52%."  

5.3. Persistence of risk profile, rather than returns?
According to Kat and Menexe (2002), one possible explanation for the contradictory conclusions on hedge fund performance persistence lies in an incorrect adjustment for risk. Because of the shape of the hedge fund return distribution, traditional measures such as the Sharpe ratio and the alpha would indicate the presence of greater persistence. If the persistence test is based on these measures, its results lead to a conclusion on the persistence of the fund’s superior performance, when it is in fact a persistence of the risk profile. To illustrate this, they find evidence of persistence in the higher moments of hedge fund returns, especially in the standard deviation, while they find no evidence of the persistence of mean returns. This may suggest that the risk profile persists, rather than the performance.

Herzberg and Mozes (2003) deal with the persistence of the success of hedge fund strategies over time and the use of quantitative techniques to identify the best managers on the basis of past performance.
They measure the persistence of basic fund attributes, namely the return, the Sharpe ratio, the maximum drawdown, the standard deviation and the correlation with the S&P 500 Index, the Russell 2000 Index, the Lehman Aggregate Bond Index and the Goldman Sachs Commodity Index. The method is based on the calculation of a Rank Information Coefficient. It measures the correlation between the value of a given variable for a period 1 (here the prior 36-month period) and its value for a period 2 (here the subsequent 12-month period). The results confirm those from Kat and Menexe (2002): the risk profile persists (maximum drawdown, standard deviation, correlation with the previously mentioned indexes), rather than the performance (returns and Sharpe ratio) which tends to revert. The authors explain these results through the link between the risk profile and the fund’s style, which is more stable over time than its performance. Another explanation for the persistence of the standard deviation is the persistence of the correlation of the fund with various market indexes, mainly with the S&P 500 Index and the Russell 2000 Index.

De Souza and Gokcan (2004) find significant persistence of standard deviation, by testing persistence through a cross product ratio and regressions.

**Conclusion**

Several articles have been written highlighting the difficulties one encounters in practice when evaluating the performance of hedge funds.

A first potential source of error in the measurement of performance is related to the database. The quality of the data marketed by the database vendors is in fact heterogeneous. Moreover, this data has to be corrected by some biases which have a strong impact on the results.

The application of the traditional measures of performance to hedge funds does not take the higher moments of the hedge fund return distributions into account. Alternative measures such as the Omega have been introduced in recent studies, but their contributions are not widely documented at present, so it is not possible to conclude that they are superior.

The identification of the source of returns is of major interest in the context of performance measurement based on explicit macro-factor models and explicit micro-factor models, in order to obtain well-specified models. The use of an implicit factor model permits the risk of misspecification of the model to be avoided, but the interpretation of the variables is more difficult. Some studies attempt to take into account the specific characteristics of the hedge fund return distributions by introducing non-linear regressors into the models or by constructing non-linear models. Recent studies obtain satisfactory explanatory power with these models. Conversely, conditional approaches to the dynamic trading strategies of the hedge funds are more prone to controversy.

The issue of hedge fund performance persistence is also examined. In spite of the heterogeneity of the performance measurement methods and the various statistical tests used, it appears that it is a short-term phenomenon, which is significant at quarterly and half yearly horizons. Studies using the most relevant test methods find no evidence of persistence at a yearly horizon.

**References**


• Schneeweis, T., "Alpha, Alpha, who’s got the Alpha?", CISDM, October 1999.


EDHEC-Risk Institute is part of EDHEC Business School, one of Europe’s leading business schools and a member of the select group of academic institutions worldwide to have earned the triple crown of international accreditations (AACSB, EQUIS, Association of MBAs). Established in 2001, EDHEC-Risk Institute has become the premier European centre for applied financial research.

In partnership with large financial institutions, its team of 90 permanent professors, engineers and support staff implements six research programmes and ten research chairs focusing on asset allocation and risk management in the traditional and alternative investment universes. The results of the research programmes and chairs are disseminated through the three EDHEC-Risk Institute locations in London, Nice, and Singapore.

EDHEC-Risk Institute validates the academic quality of its output through publications in leading scholarly journals, implements a multifaceted communications policy to inform investors and asset managers on state-of-the-art concepts and techniques, and forms business partnerships to launch innovative products. Its executive education arm helps professionals to upgrade their skills with advanced risk and investment management seminars and degree courses, including the EDHEC-Risk Institute PhD in Finance.

Copyright © 2012 EDHEC-Risk Institute

For more information, please contact:
Carolyn Essid on +33 493 187 824
or by e-mail to: carolyn.essid@edhec-risk.com

EDHEC-Risk Institute
393-400 promenade des Anglais
BP 3116
06202 Nice Cedex 3 - France

EDHEC Risk Institute—Europe
10 Fleet Place - Ludgate
London EC4M 7RB - United Kingdom

EDHEC Risk Institute—Asia
1 George Street - #07-02
Singapore 049145

www.edhec-risk.com